BNL-105119-2014-TECH

Booster Technical Note No. 72;BNL-105119-2014-IR

ON THE OPERATIONAL WINDOW OF BOOSTER LATTICE

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January 1987

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

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ABSTRACT

Analytic method is used to analyze the tracking results for the AGS-BOOSTER lattice. We found that the effect of the amplitude dependent tune is very important in understanding characteristic of the tracking result. With the perturbed tune, the effective second order perturbation theory works very well. The method can be used to analyze the optimized operational condition for the lattice. For the Booster, the analysis suggests that chromaticity of -2 to -5 and a minimum unperturbed tune split $|\Delta\nu^{o}| > 0.01$. At the zero chromaticity point, the required unperturbed tune split is considerably larger in order to obtain a similar performance.

1. Introduction

The beam size of the particles in the accelerator is very important to the performance of the accelerators. For instance, when the beam size is increased, the effect of the nonlinearities increases. The effect of these nonlinearities gives rise to nonlinear resonance to the Hamiltonian. When the particles is tracked through the accelerators, they experience many resonances. A special kind of resonance is the nonlinearcoupling resonance due to the nonlinear magnet elements which exist in the accelerator. Some of these nonlinear magnetic elements are sextupoles due to eddy current, saturation and the chromatic corrections. Because of these nonlinear magnet elements, the beam emittance is not a constant of motion. If the important resonance is a sum resonance, the emittance may grow without bound. If the only important resonance is a difference resonance, the sum of emittances of horizontal and vertical planes remains constant. The Hamiltonian of the system near to a resonance line can be expressed as,[1-3]

$$H = H_o(I_x , I_y) + b_{m,n}(I_x,I_y) \cos(m\phi_x+n\phi_y-p\vartheta+\psi) \tag{1}$$

where I_x and I_y are the action variables or the emittances of the beam and ϕ_x ,ϕ_y are the conjugate angle variables. When the resonance term of eq.1 is zero, the ϕ_x,ϕ_y are cyclic variables and the actions becomes constant of motion. The equation of motion of the particles becomes

$$\nu_x = \dot{\phi}_x \simeq \partial H/\partial I_x =\nu_x^o + \alpha_{xx} I_x + \alpha_{xy} I_y \tag{2}$$

$$\nu_y = \dot{\phi}_y \simeq \partial H/\partial I_y =\nu_y^o + \alpha_{xy} I_x + \alpha_{yy} I_y \tag{3}$$

$$\dot{I}_x = -\partial H/\partial\phi_x = m\ b_{m,n} \sin(m\phi_x+n\phi_y-p\vartheta+\psi) \tag{4}$$

$$\dot{I}_y = -\partial H/\partial\phi_y = n\ b_{m,n} \sin(m\phi_x+n\phi_y-p\vartheta+\psi) \tag{5}$$

From eqs. 4 and 5, one obtains the condition,

$$n\ I_x - m\ I_y = \text{constant} \tag{6}$$

Note here that the invariant of the Hamiltonian becomes the linear combination of action variables. For a sum resonance, where m and n are positive, the action variables of each plane may become unbounded while eq.(6) remain satisfied. For a different resonance, the emittances of x and y plane are bounded while the magnitudes oscillate around its mean value. We are interested mainly in the difference resonance. To take into account the contribution of tune versus amplitude modulation, eqs.1 and 2 would include the amplitude dependence. It is interesting to note that the condition ,

$$\partial(m\phi_x+n\phi_y)/\partial\vartheta = m\nu_x+n\nu_y \simeq \text{constant} \tag{7}$$

can be approximately satisfied, , Eqs.4 and 5 can be solved as,

$$I_x = I_x^o + \frac{m\ b_{m,n}}{m\nu_x+n\nu_y-p}\cos(m\phi_x+n\phi_y-p\vartheta+\psi) \tag{8}$$

$$I_y = I_y^o + \frac{n\ b_{m,n}}{m\nu_x+n\nu_y-p}\cos(m\phi_x+n\phi_y-p\vartheta+\psi) \tag{9}$$

Note here that the actions or the emittances of the system would oscillate around the average values. The amplitude of the oscillation is proportional to $b_{m,n} / (m\nu_x+n\nu_y-p)$. The smear defined by,

$$S_x = \frac{(I_x^{max} - I_x^{min})}{\sqrt{3}\,(I_x^{max} + I_x^{min})} = \frac{1}{\sqrt{3}} \left| \frac{m\ b_{m,n}}{m\nu_x+n\nu_y-p} \right| \tag{10}$$

is a measure of the amplitude of oscillations. When the smear is large the particle is strongly influenced by the resonance. It is noted that the smear can be large when the residue of the resonance $b_{m,n}$ is large or when the particle is very near to the resonance line, i.e. $m\nu_x+n\nu_y-p \simeq 0$. It is important to note that the tunes of the machine ν_x and ν_y depend on the emittance of machine. A self-consistant calculationis is needed to evaluate the important of the resonance.

2. The coupling resonance of the AGS Booster due to eddy current.

AGS Booster is a rapid cycling machine (10 Hz) for proton operation. The fast acceleration rate creates also sizable sextupole contribution to the accelerator.[4] In this section, we shall apply the theory in section 1 to the coupling resonance in the booster. The important coupling resonance is $2\nu_x-2\nu_y=0$ systematic resonance. The tracking study of Dell and Parzen[5] shows that there exist a window in the chromaticity space where the coupling is fairly small. When the machine is corrected to a zero chromaticity, the coupling becomes large. The smear factor has a peak at the chromaticity of 5. This effect is important in choosing the operating point of the machine. In this section, we shall use this analytic theory to study the effect.

Figure 1 shows the sextupole strength needed as a function of the chromaticity with two families of sextupoles and with the eddy current sextupoles present in the dipole. Because of the presence of sextupoles, the tune of the machine depends on the amplitude. The coefficient of the amplitude dependent tune is shown in Fig.2. We note here that the coefficients are rather large at large positive and negative chromaticities. Since these coefficients are large, the tune depends sensitively on the amplitude or the emittancesof the particles. Figure 3 shows an example of the amplitude dependent part of $\nu_x - \nu_y$, $\Delta\nu_{NL}$

$$\Delta\nu_{NL} = \alpha_{xx}\varepsilon_x + \alpha_{xy}\varepsilon_y - \alpha_{yx}\varepsilon_x - \alpha_{yy}\varepsilon_y \tag{11}$$

as a function of the chromaticity at the emittance of 50 π mm-mrad for both planes.Let us define the enhancement factor due to the tune difference for the $2(\nu_x - \nu_y)$ resonance as,

$$f = \left| \frac{1}{\nu_x - \nu_y} \right| = \left| \frac{1}{\nu_x^o - \nu_y^o + (\alpha_{xx}\varepsilon_x + \alpha_{xy}\varepsilon_y - \alpha_{yx}\varepsilon_x - \alpha_{yy}\varepsilon_y)} \right| \tag{12}$$

Note here that the tune vs. amplitude dependence of eqs. 2 and 3 has been included in eq.(11). Fig.4 shows the enhancement factor for the unperturbed tune $\Delta\nu^o = \nu_x^o - \nu_y^o = 0.01$. We observe that the enhancement of the coupling resonance peaks at the chromaticity 5 due mainly to cancellation between the unperturbed tune and the amplitude dependence tune shifts. Indeed the tracking result of Parzen shows a peak in the smear at the chromaticity 5. Fig.5 shows the similar tracking result by using the TEAPOT program. The peak of the smear at the chromaticity 5 is clearly seen for the vertical emittance. Using eq.(10), we can deduce the effective coupling resonance strength $b_{2,-2}$ from the tracking result. Fig.6 compares the results of an analytic calculation obtained from the second order perturbation theory[2] with that obtained from the tracking analysis. The agreement is good.

This procedure of integrating the amplitude dependent tune into the perturbation expansion is to some sense similar to the superconvergent perturbation expansion[3]. At or near a resonance, the effective Hamiltonian of eq.(1) can be used to described the motion of the particles. The effect of the tune becomes the important determine factor on the effect of the resonance. It is natural that the perturbed tune should be used in the calculation.

We have established that the perturbation expansion of the nonlinear theory should take the tune vs amplitude modulation into account in the second order calculation to account for the strange behavior of the peak in the smear function at the chromaticity 5 for the initial emittances 50 π mm-mrad for both planes. There exists a window in the chromaticity degree of freedom where the smear is relatively small. Since the nonlinear tune is a very sensitive function of the emittance, we may ask whether the window would change from particle to particle in the phase space. The particles in the bunch may be described by a Gaussian distribution in the coordinate space. This correspond to the exponential distribution in the emittance space. The particles may be assumed to fill up to the maximum emittance. Fig.7a,7b shows the shows the nonlinear tune split, $\Delta\nu_{NL}$ of eq.(11) for $\varepsilon_x + \varepsilon_y = 100$ and 80 πmm-mrad respectively. Note here that the enhancement factor is proportional to

$$f = \left| \frac{1}{\Delta\nu^{o} + \Delta\nu_{NL}} \right|$$

Fig.7a shows that when $\Delta\nu^{o}$= 0.01 and chromaticity -1, the resonance $2\nu_x - 2\nu_y = 0$ would be important for the particles at the emittance ε_x=100 and ε_y=0 πmmmrads respectively. To operate the machine at chromaticity 0, a minimum tune split of $\Delta\nu^{o}$ = -0.04 or +0.05 is needed to obtain a smear of 15% or less. Fig.7 show that the $\Delta\nu^{o} < 0$ has slight advantage over $\Delta\nu^{o} > 0$ if we are forced to operate tha machine at or near to the zero chromaticity region. The above statement depends on the choice of the sextupole configuration. The least sensitivity of the amplitude dependent effect is located at chromaticity of -4, where the magnitude of $\Delta\nu^{o}$ should be larger than 0.01.

3. Coupling resonance due to magnet saturation.

Since the booster is also designed to accelerate the heavy ions to high energy, the magnet has the effect of iron saturation. The magnitude of the sextupole component due to the iron saturation is given by Danby et.al.[6]. In this section, we shall evaluate the strength of the coupling resonance, and analyze the accelerator to obtain the best operation point.

Fig. 8 shows the sextupole strength requirement as a function of the machine chromaticity. The corresponding coefficients for the tune amplitude dependence part is shown in Fig. 9. We can use these coefficients to find the nonlinear tune split $\Delta\nu_{NL}$ of eq.12. Fig 10 shows the nonlinear tune split as a function of the chromaticity for $\varepsilon_x + \varepsilon_y$ = 100 π mm-mrad. Various phase space points in the bunch are shown on Fig.10. Note here (as in the case with eddy current sextupoles) that the best operating point is located at chromaticity of -5. If the chromaticity is chosen to be 0 the minimum unperturbed tune split required is $\Delta\nu^{o} \geq 0.04$ or ≤ -0.05. Figure 10 indicates that $\nu_x < \nu_y$ would work better in this case.

Fig.11 shows the tracking result of the smear function for the magnet saturationsimilar to that of Fig.5. Fig.12 shows the effective $b_{2,-2}$ strength obtained from the procedure discussed in previous section (see Fig. 6 for comparison). The effective strength is compared with the analytic calculation[2] on the Fig.12 It is interest to note that the comparison is also good. Therefore the best operational chromaticity is also around -2 to -5.

4. Conclusion

We have used the analytic theory to understand the tracking result. The existance of the operational window in the chromaticity degree of freedom observed by Dell and Parzen is useful in controling the coupling resonance. Based on the analytic study, a minimum unperturbed tune spread can be derived to obtain the best performance for the entire phase space. We have found that the effective second order perturbation method agrees very well with the tracking calculation. The derived phenomenological residue from the tracking calculation agrees well with the analytic second order resonance strength.

For the AGS Booster, the best operational chromaticity, from the beam dynamic point of view, is around −2 to −5. The minimum unperturbed tune split $|\Delta\nu^{o}|$ needed to minimize the coupling resonance is about 0.01 for the entire available phase space.

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3. R. Warnock, and R.D. Ruth, SLAC preprint, 1987
4. G. Morgan, and S. Kahn, Booster tech-note 4.
5. F.G. Dell, Booster report
   G.Parzen, Booster report
6. G.T. Danby, and J.W. Jackson, Booster Tech-note 5.

Figure Captions:

Fig.1 Sextupole strength for the chromaticity correction as a function of chromaticity

Fig.2 The coefficients of tune vs. amplitude is shown as a function of chromaticity.

Fig.3 $\Delta\nu_{NL}$ of eq.11 is shown for $\varepsilon_x=\varepsilon_y=50$ πmmmrad as a function of the chromaticity. Note here that if $\Delta\nu^o=-.01$, there exist two resonances at chromaticities of -10 and $+15$ respectively. If the unperturbed tune is $\Delta\nu^o=0.01$, the resonance will be most important at the chromaticity of 5.

Fig.4 The enhancement factor f of eq.(12) is shown for $\Delta\nu^o=\nu^o_x-\nu^o_y=0.01$.

Fig.5 The tracking result of the smear factor defined in eq.10 is shown as a function of chromaticity. Note here the apparent peak at chromaticity of around 5.

Fig.6 The reduced coupling resonance strength b(2,-2) is compared with the analytic calculation of ref.2.

Fig.7a The nonlinear tune difference $\Delta\nu_{NL}$ is ploted for different region of the phase space with $\varepsilon_x+\varepsilon_y = 100\ \pi$ mm-mrad. Note here that when the emittances of the particles oscillate, the effect of the resonance also vary.

7b Similar to that of 7a for $\varepsilon_x+\varepsilon_y = 80\ \pi$ mm-mrad.

Fig.8 The sextupole strength needed for the chromatic correction for the magnet with iron saturation at high energy.

Fig.9 The coefficients of tune vs. amplitude is shown as a function of chromaticity for the magnet saturation at high energy.

Fig.10 The nonlinear tune split $\Delta\nu_{NL}$ for the phase space points of the bunch.

Fig.11 The smear factor is ploted as a function of the chromaticity.

Fig.12 The deduced effective coupling strength is compared with the analytic calculation of ref.2.

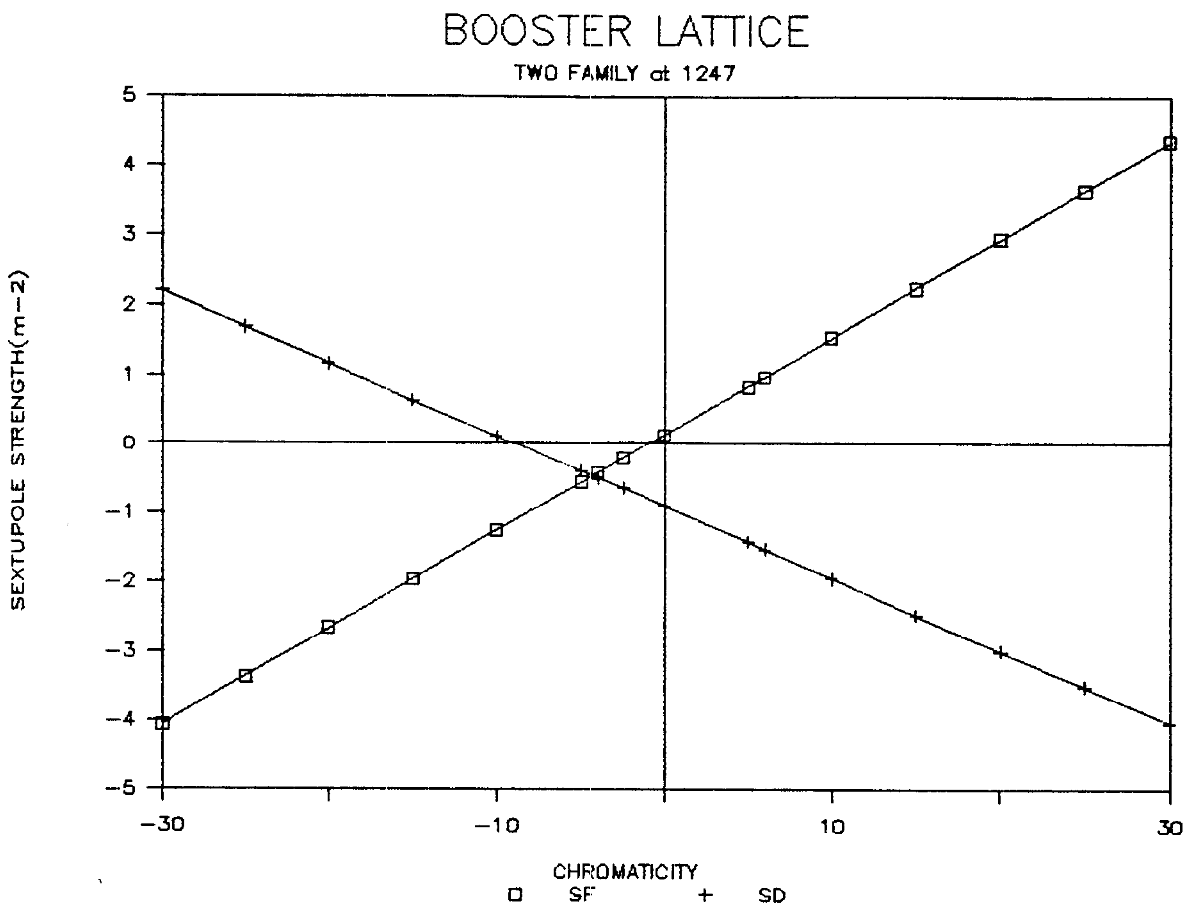

BOOSTER LATTICE
TWO FAMILY at 1247
SEXTUPOLE STRENGTH(m-2)
5
4
3
2
1
0
-1
-2
-3
-4
-5
-30
-10
10
30
CHROMATICITY
SF
SD

Fig. 1

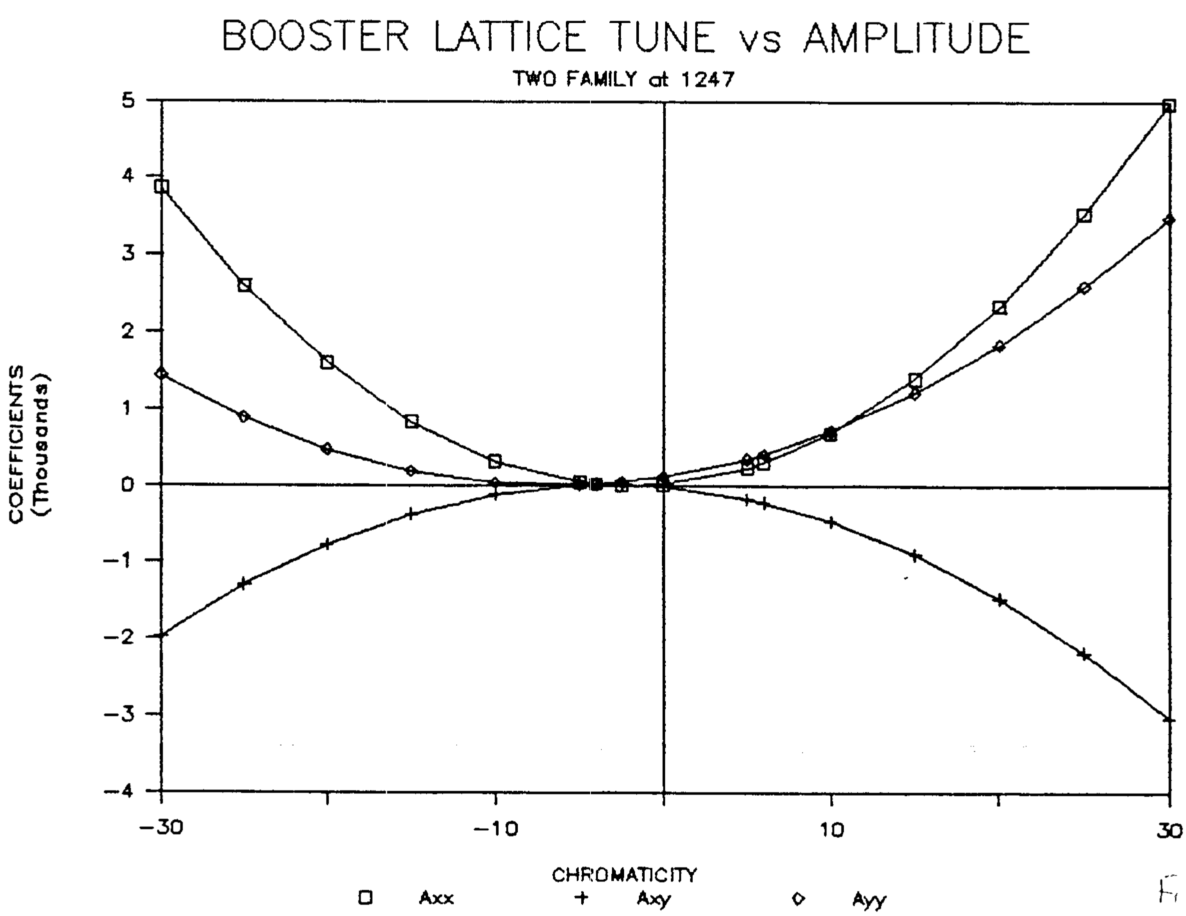
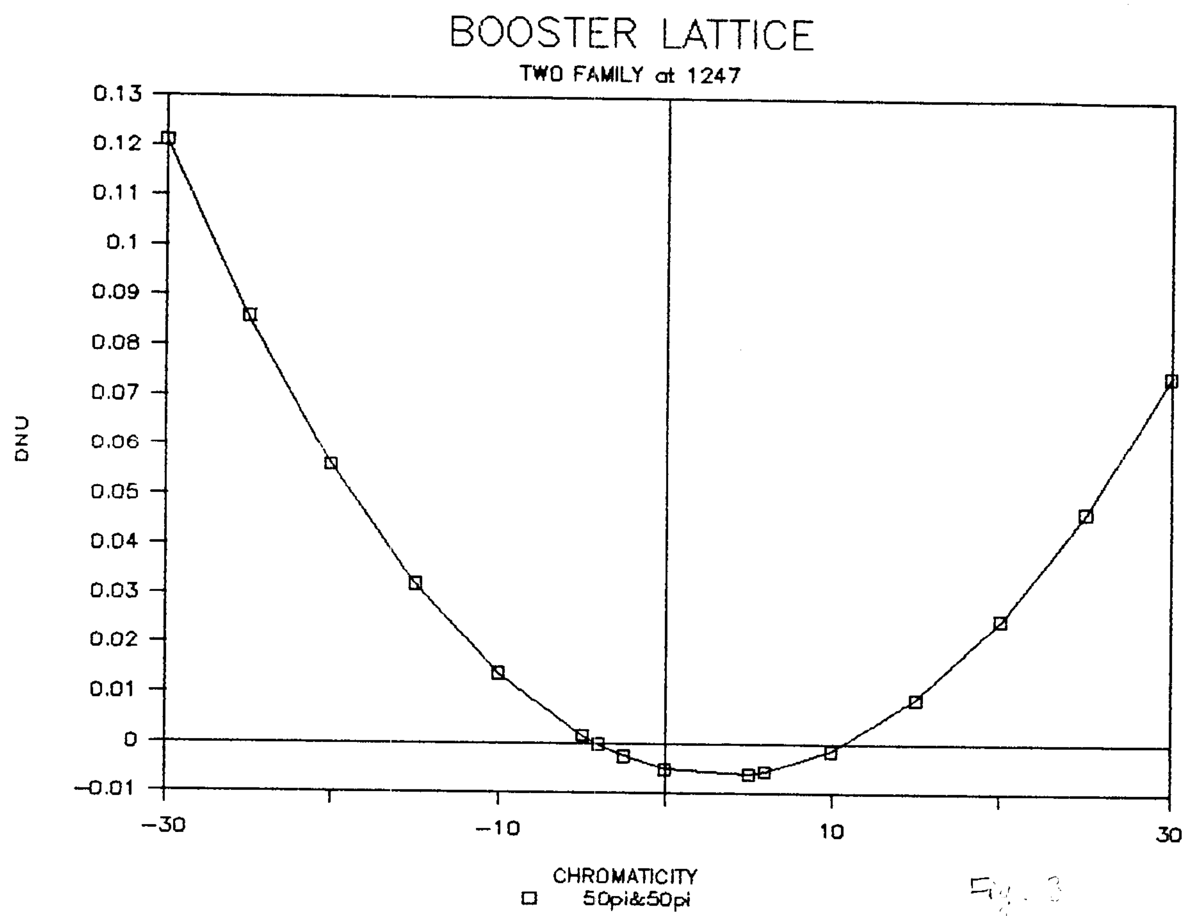
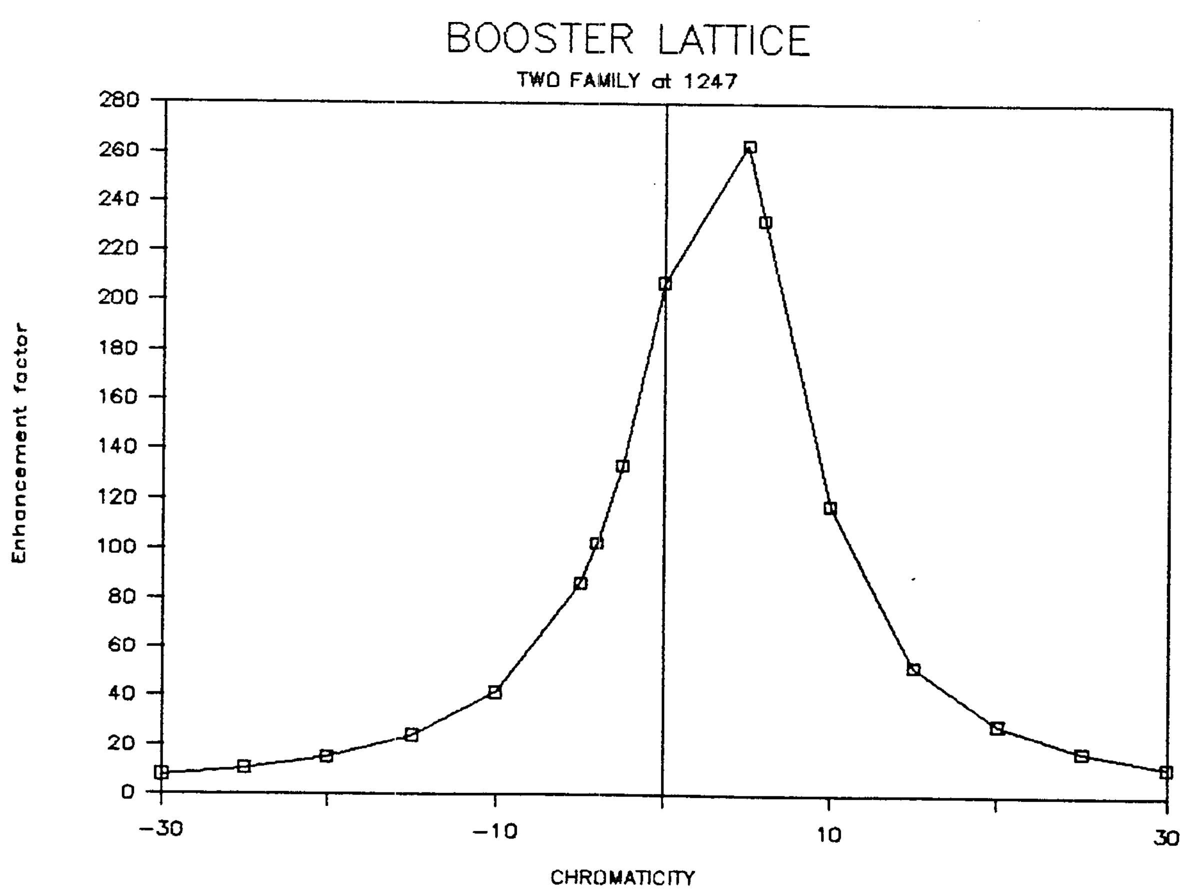

BOOSTER LATTICE TUNE vs AMPLITUDE
TWO FAMILY at 1247
COEFFICIENTS
(Thousands)
5
4
3
2
1
0
-1
-2
-3
-4
-30
-10
10
30
CHROMATICITY
Axx
Axy
Ayy

Fig. 2



BOOSTER LATTICE
TWO FAMILY at 1247
DNU
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
−0.01
−30
−10
10
30
CHROMATICITY
50pi&50pi

Fig. 3



BOOSTER LATTICE
TWO FAMILY at 1247
Enhancement factor
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
−30
−10
10
30
CHROMATICITY

Fig. 4

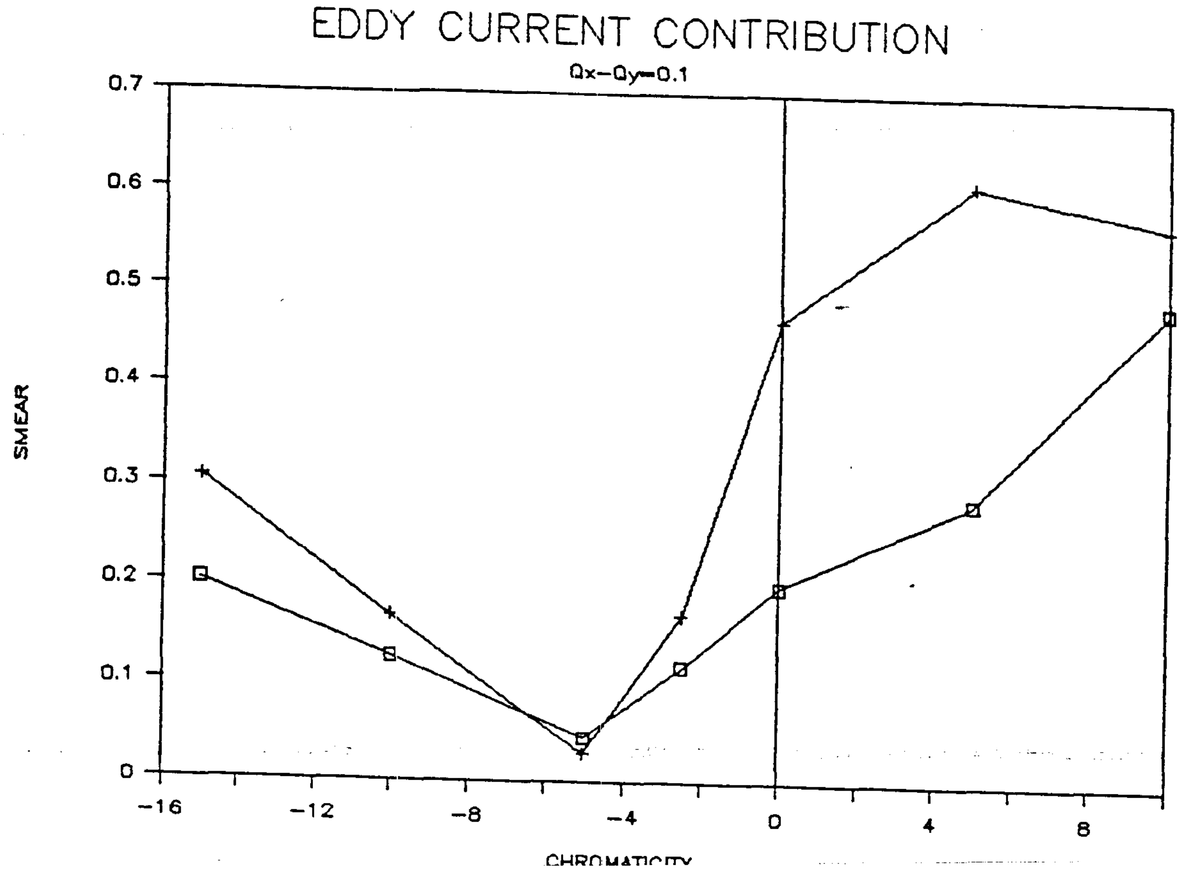

EDDY CURRENT CONTRIBUTION
Qx-Qy=0.1
SMEAR
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-16
-12
-8
-4
0
4
8
CHROMATICITY

Fig. 5

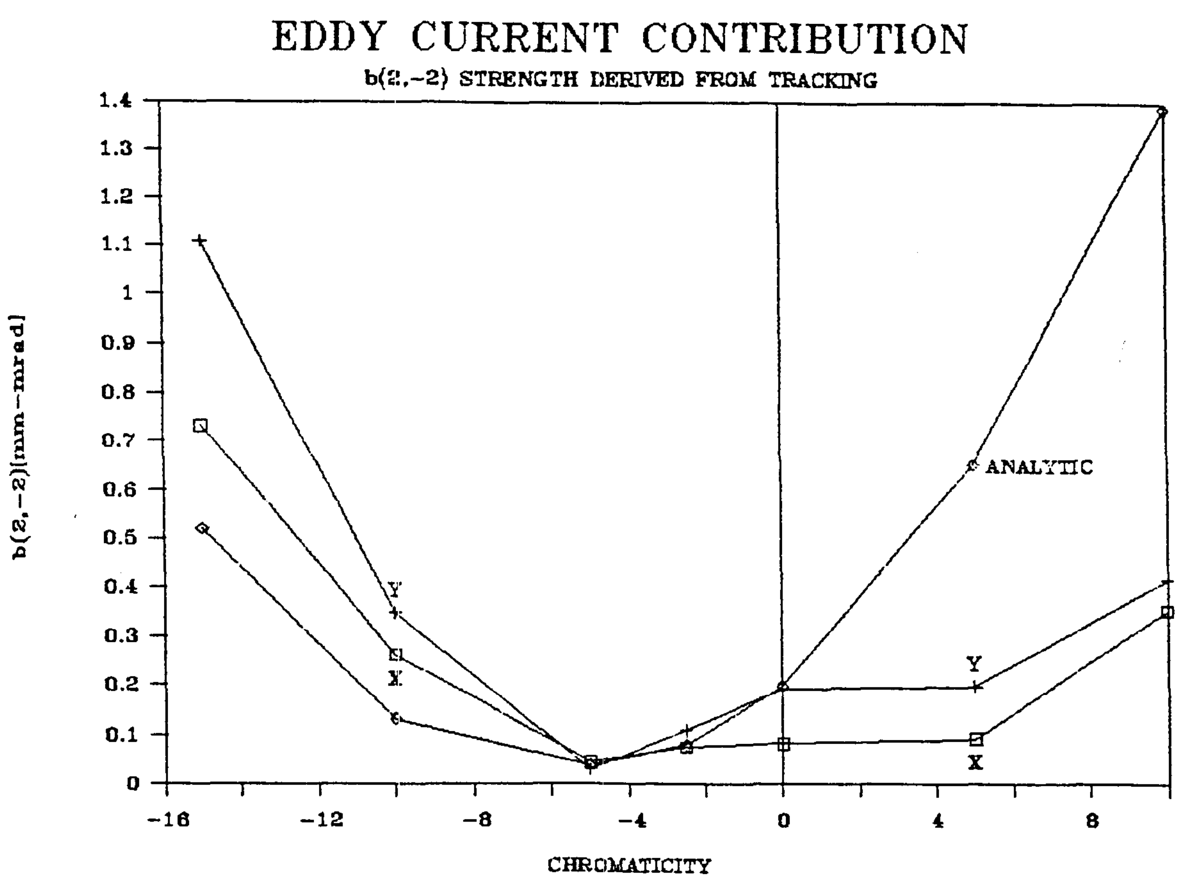

EDDY CURRENT CONTRIBUTION
b(2,-2) STRENGTH DERIVED FROM TRACKING
b(2,-2)(mm-mrad)
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-16
-12
-8
-4
0
4
8
CHROMATICITY
ANALYTIC
Y
X
Y
X

Fig. 6

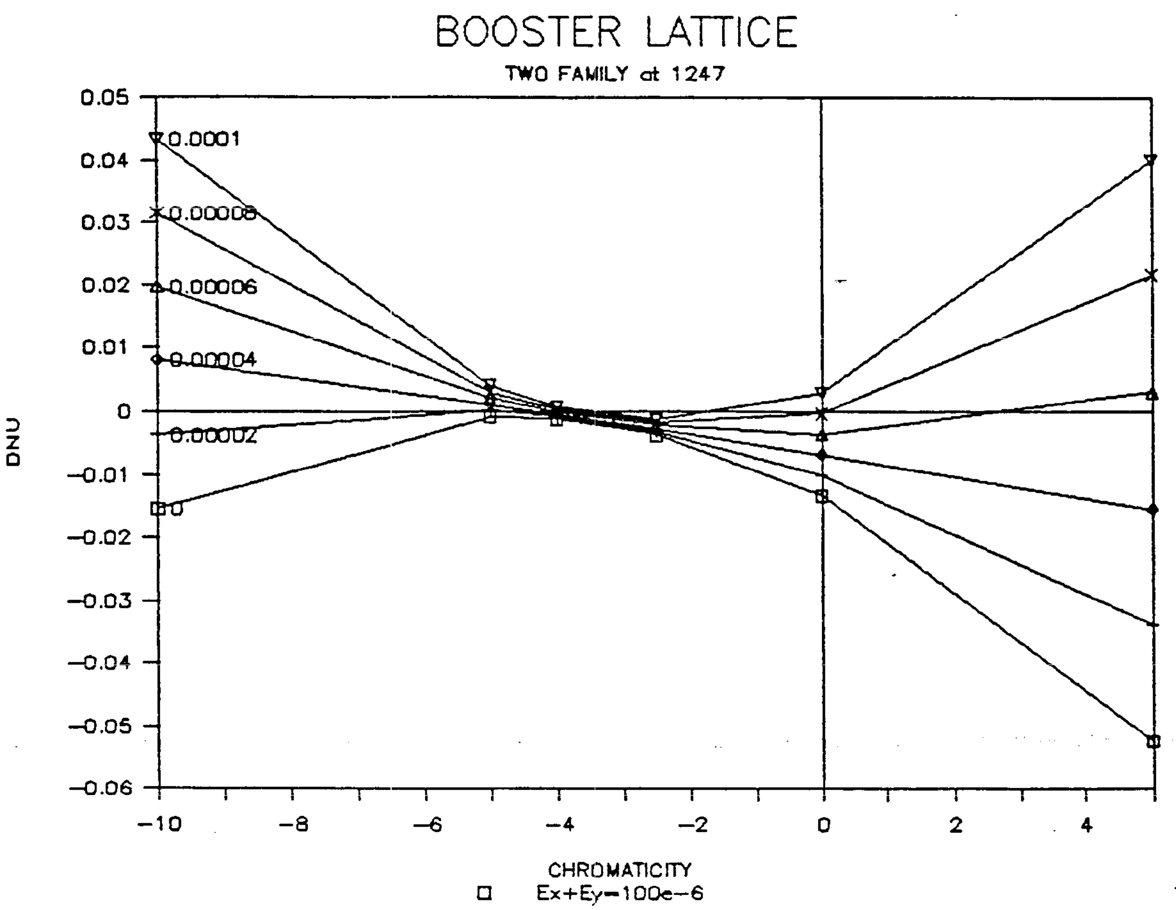

BOOSTER LATTICE
TWO FAMILY at 1247
0.0001
0.00008
0.00006
0.00004
0.00002
0
DNU
CHROMATICITY
Ex+Ey=100e-6

Fig. 7a

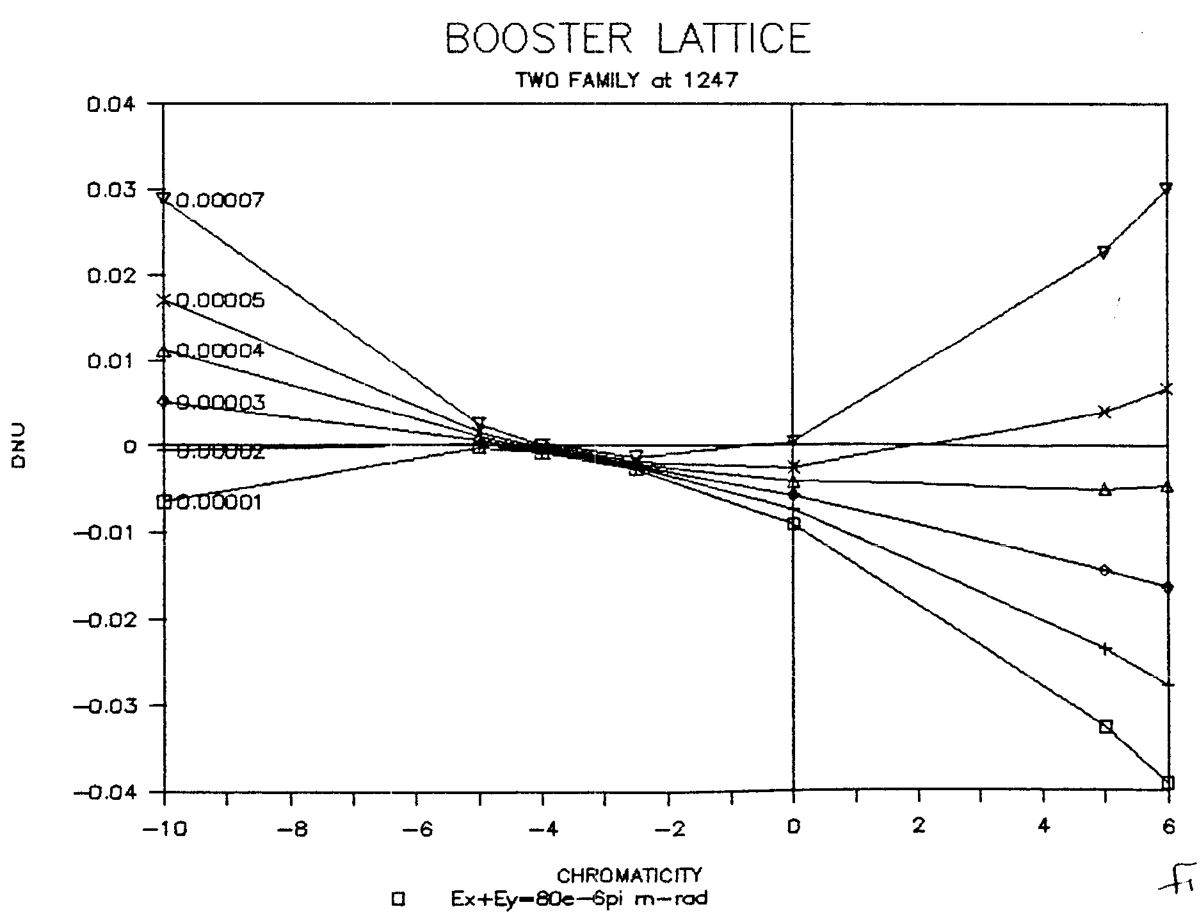
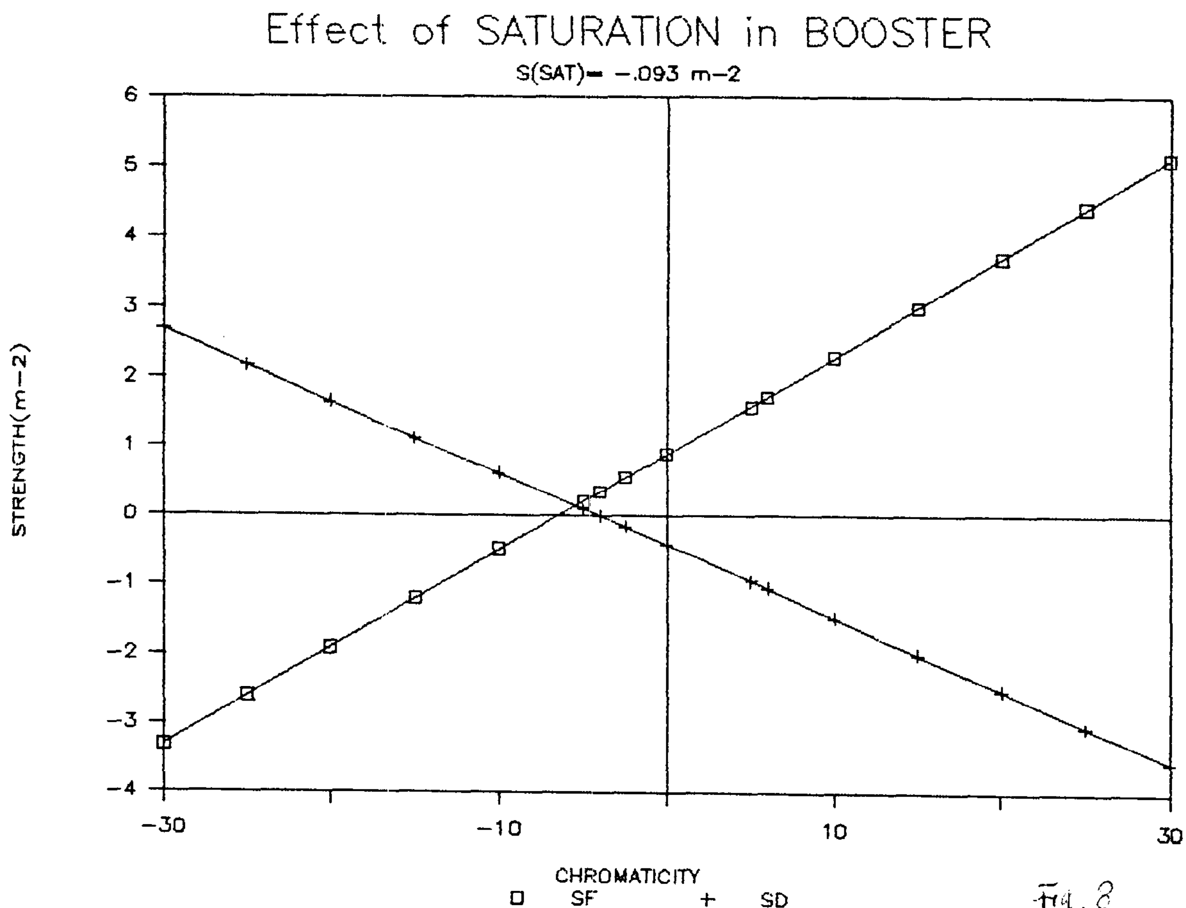
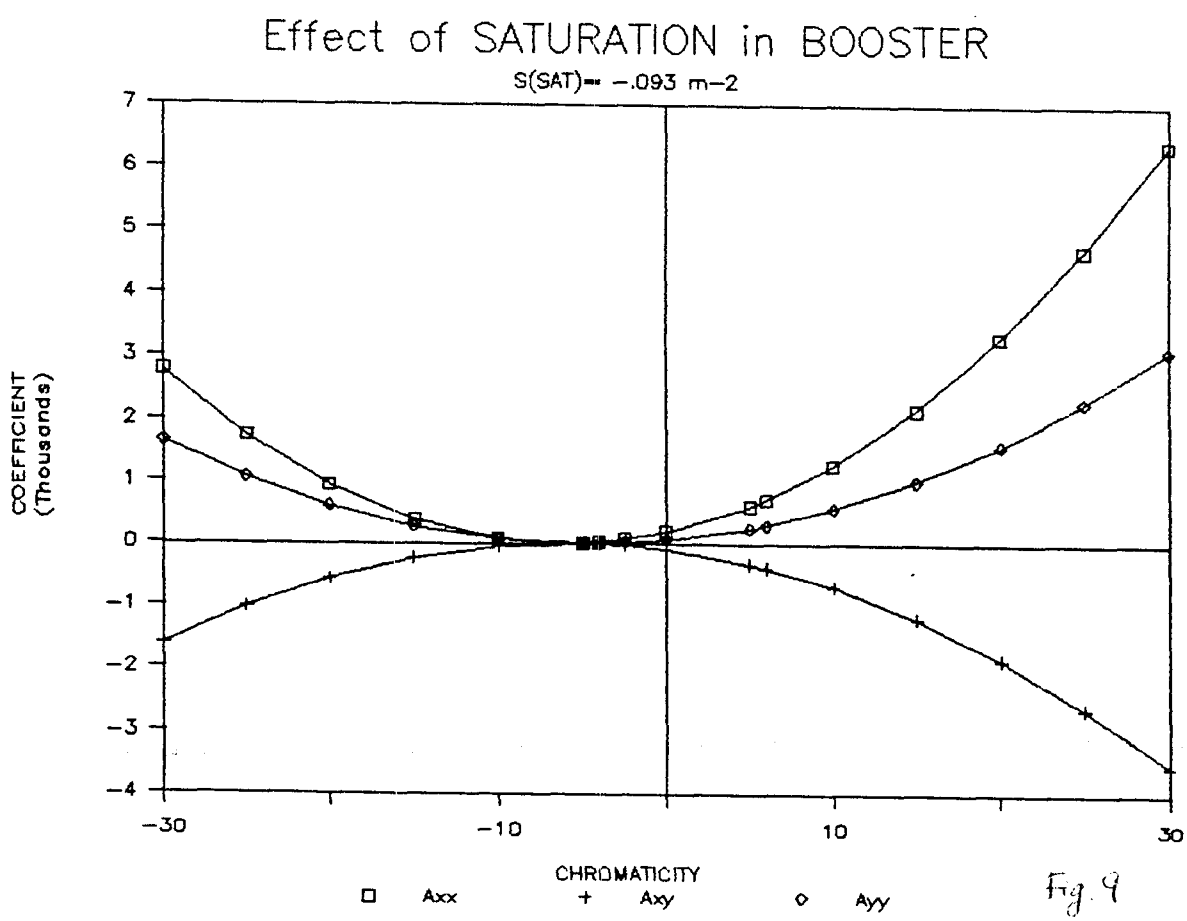

BOOSTER LATTICE
TWO FAMILY at 1247
0.00007
0.00005
0.00004
0.00003
0.00002
0.00001
DNU
CHROMATICITY
Ex+Ey=80e-6pi m-rad

Fig 7b



Effect of SATURATION in BOOSTER
S(SAT)= -.093 m-2
STRENGTH(m-2)
6
5
4
3
2
1
0
-1
-2
-3
-4
-30
-10
10
30
CHROMATICITY
SF
SD

Fig. 8



Effect of SATURATION in BOOSTER
S(SAT)= -.093 m-2
COEFFICIENT
(Thousands)
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-30
-10
10
30
CHROMATICITY
Axx
Axy
Ayy

Fig. 9

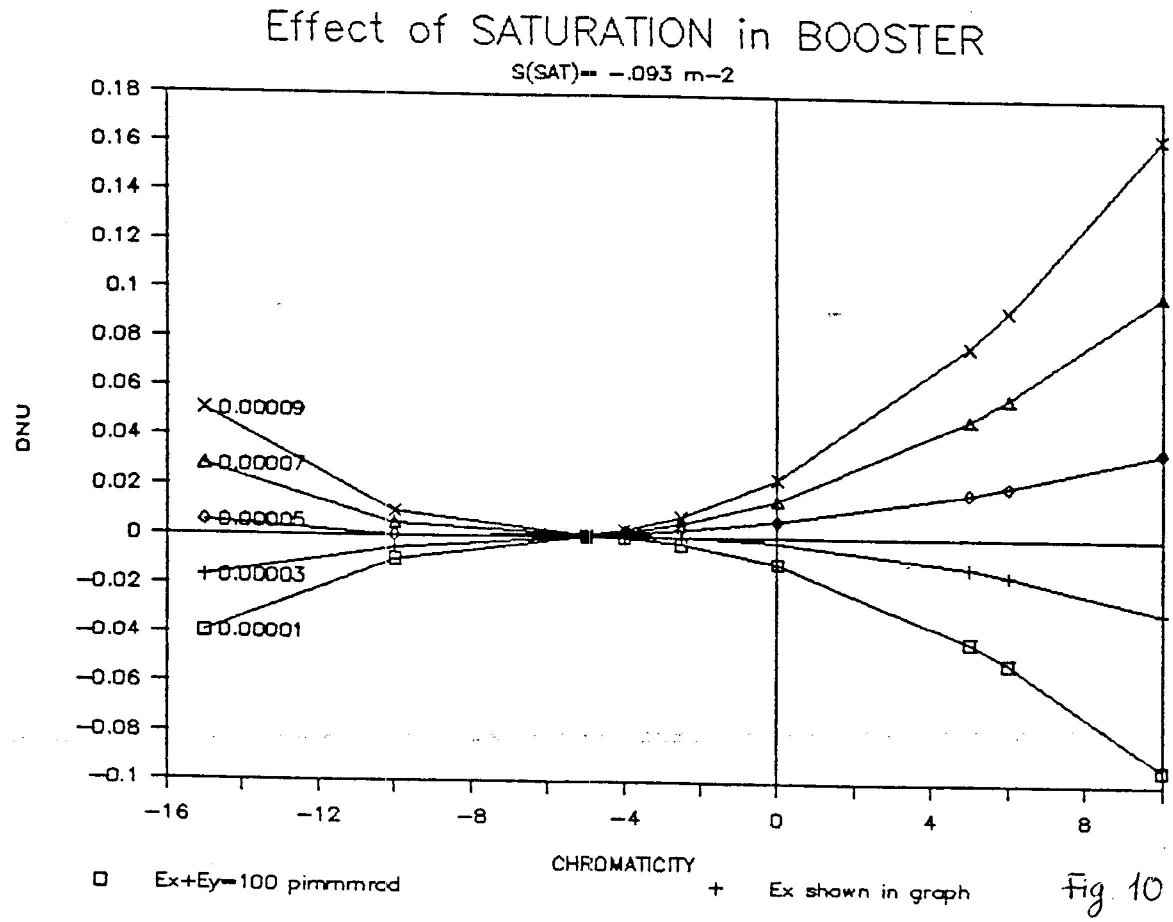

Effect of SATURATION in BOOSTER
S(SAT)= -.093 m-2
DNU
0.00009
0.00007
0.00005
0.00003
0.00001
CHROMATICITY
Ex+Ey=100 pimmmrad
Ex shown in graph

Fig. 10

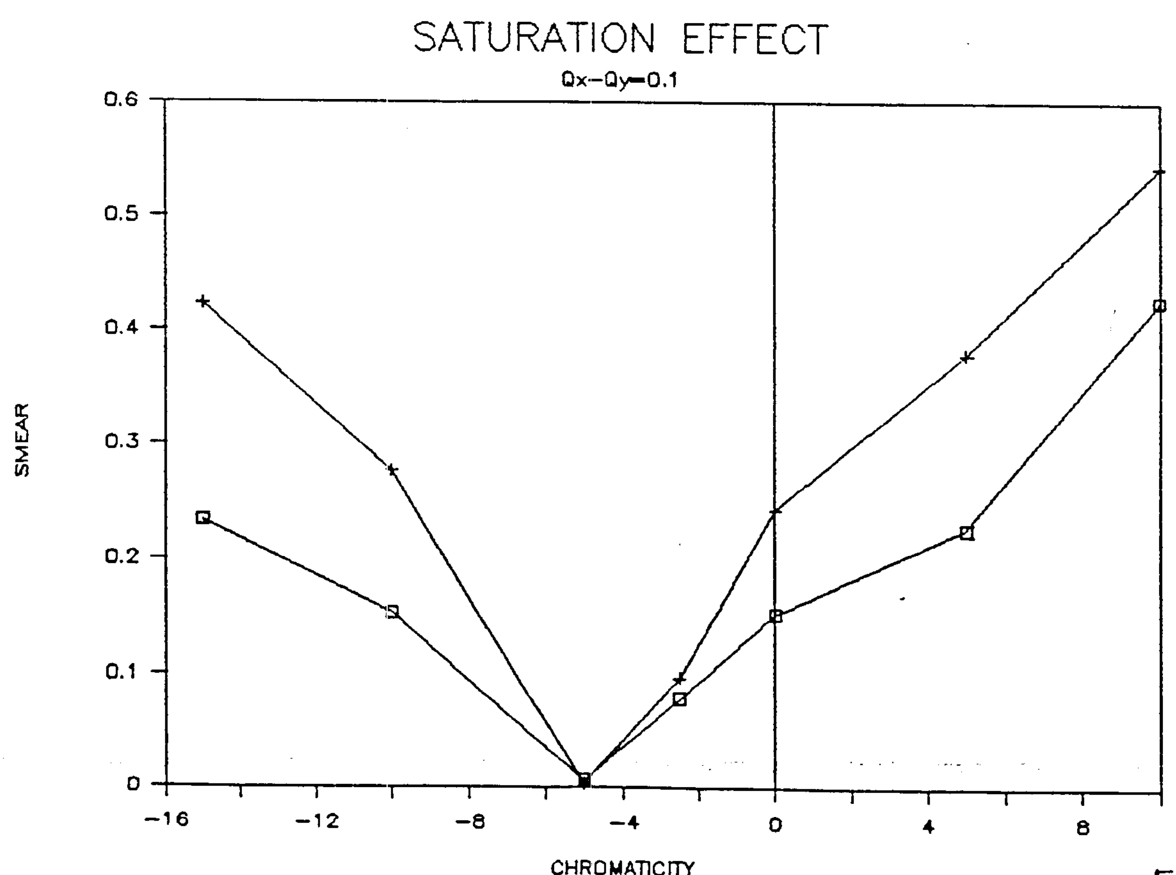
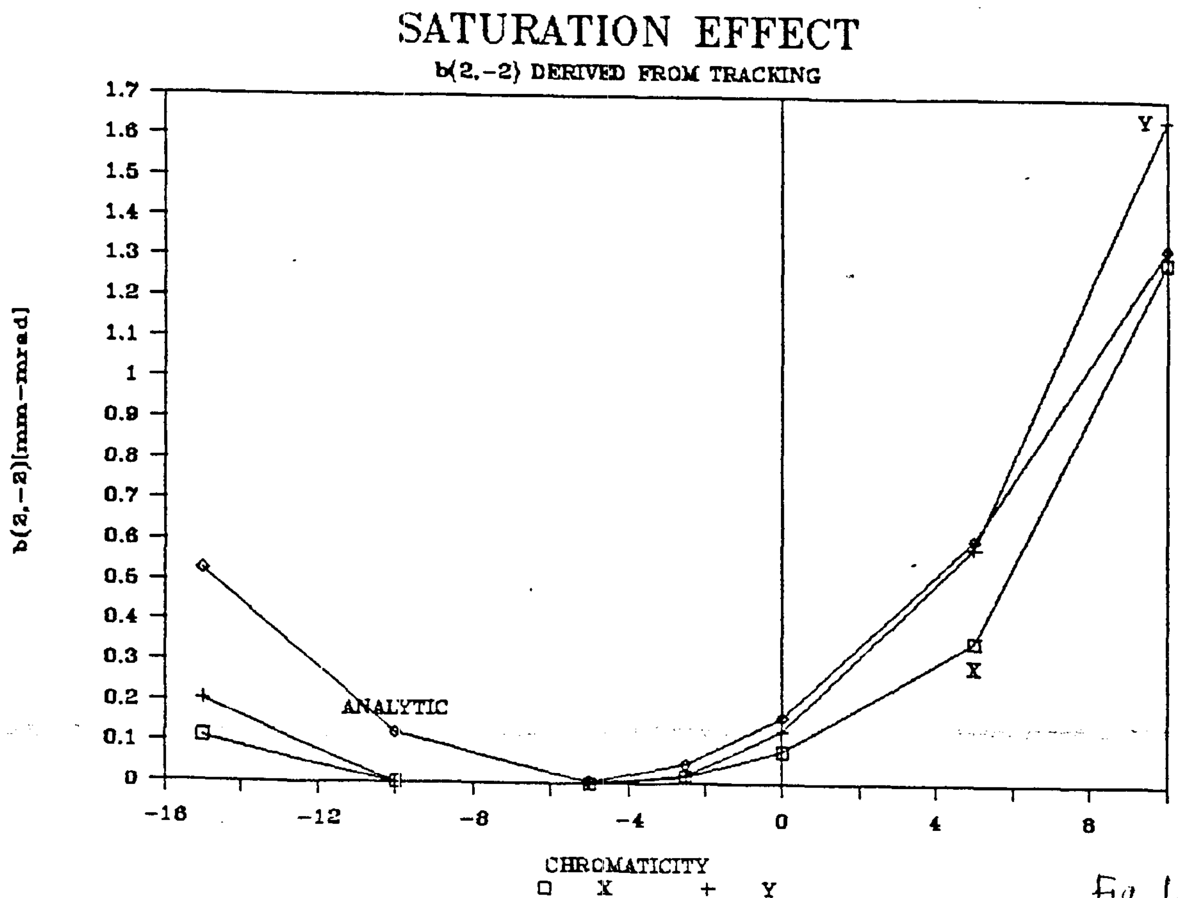

SATURATION EFFECT
Qx-Qy=0.1
SMEAR
CHROMATICITY

Fig. 11



SATURATION EFFECT
b(2,-2) DERIVED FROM TRACKING
b(2,-2)[mm-mrad]
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-16
-12
-8
-4
0
4
8
ANALYTIC
X
Y
CHROMATICITY
X
Y

Fig. 12