BNL-103701-2014-TECH

AGS/RHIC/SN 076;BNL-103701-2014-IR

Effect on Spin of Systematic Twist iin RHIC Dipole Magnets

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September 1998

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Spin Note

AGS/RHIC/SN No. 076

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Magnetic measurements in some of the earlier RHIC dipole magnets indicated a twist of the dipole field along the body of the magnets. In one case in particular, a strong systematic twist through an angle of about 10 mrad over the length of the 9.5 m magnet was observed. While the dipole can be set so that the average field direction is vertical, thus self-canceling (or at least greatly reducing) the effects of a twisting field on the orbit, such a twisting field could in principle have an undesirable effect on the stable spin direction within the machine.

The average field direction was measured and noted in the magnet database for each magnet to be used for surveying purposes. This direction was determined from 10 field direction measurements along the length of the magnet. Recently, this data has been re-examined to look for systematic twist along the length of about 300 RHIC dipole magnets. For each of the magnets, the central 8 measurements in the straight section were used, and a straight line was fit to the field directions as a function of the axial position. The slope of this line is a measure of the twist in the magnet. This slope multiplied by the magnetic length, 9.45 m, is referred to as the "average twist" in the magnet. Fig. 1 shows the data and fitted line for a particular magnet in RHIC (one with a rather large average twist).

The output of the investigation is shown in Fig. 2. Some of the earlier magnets showed large twisting of the field, most likely generated by the welding process of the magnet cold mass shell. This process was subsequently altered, and the typical average twist angle through a magnet was kept below 2.2 mrad rms. The final distribution of twist angles is shown in Fig. 3. As can be seen, the distribution has a systematic twist error of 0.575 mrad.

1 Effects on Spin

Suppose that a magnet's field $\vec{B}_0$ is systematically twisted along its length ℓ through an angle α. The field is oriented at an angle $-\alpha/2$ at the entrance of the magnet, and $+\alpha/2$ at the exit. We assume at this point that the effects on the orbit are

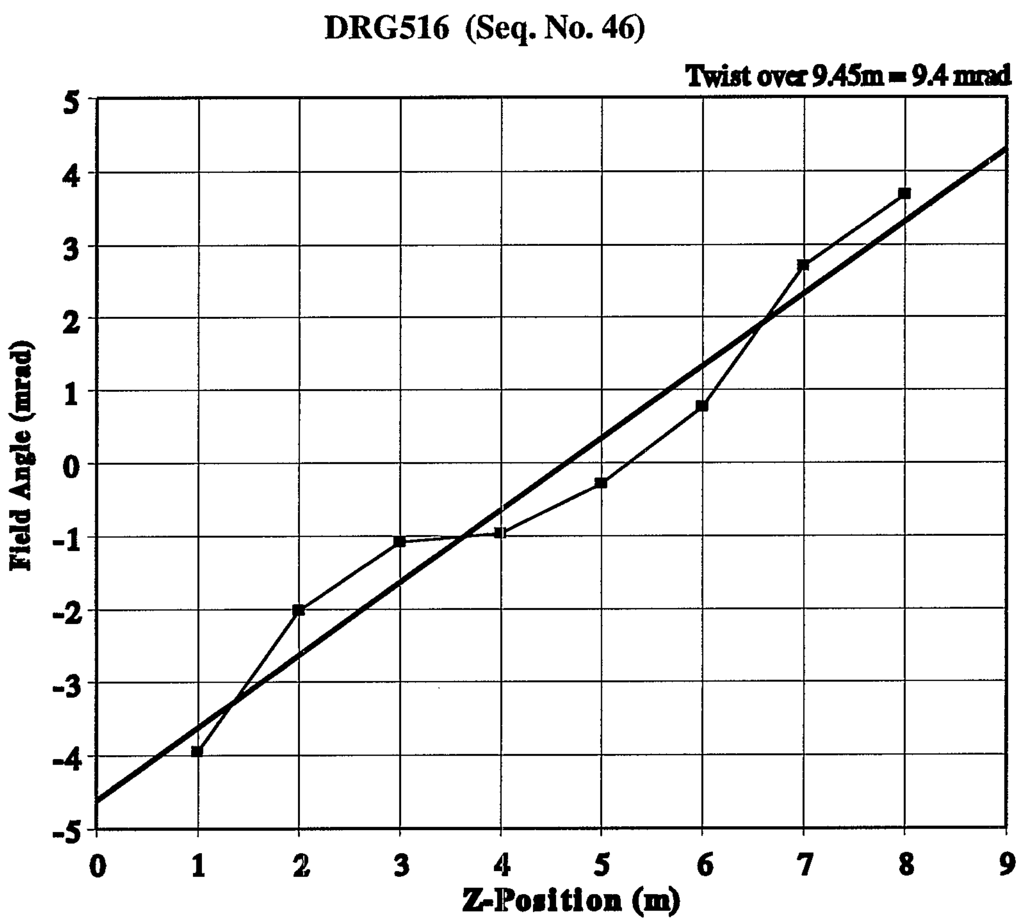

Figure 1: Field direction, in mrad from vertical, versus axial location along the length of RHIC dipole DRG516. The average twist of the field is 9.4 mrad.

DRG(8) Dipoles -- Twist over 9.45 m (Warm)

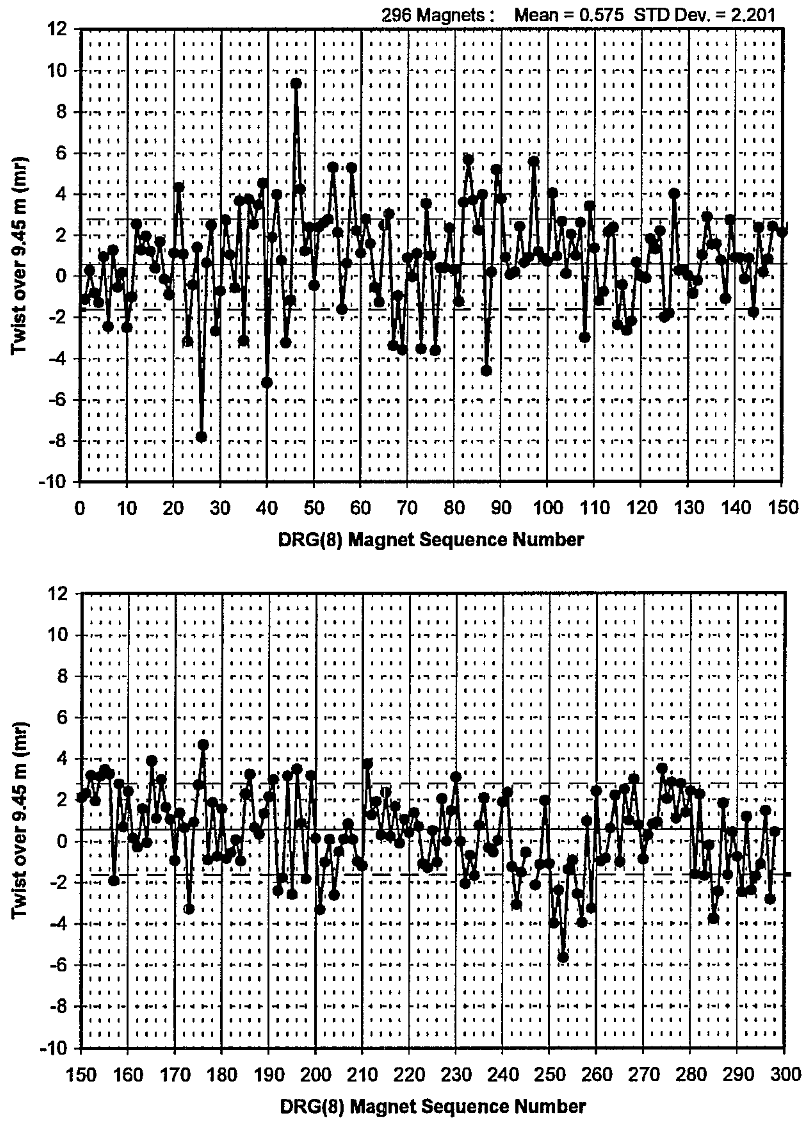

Figure 2: Average twist angle over the length of RHIC dipole magnets versus dipole magnet number. Larger twists were observed in some of the earlier magnets, which was kept better under control for the remainder of the production.

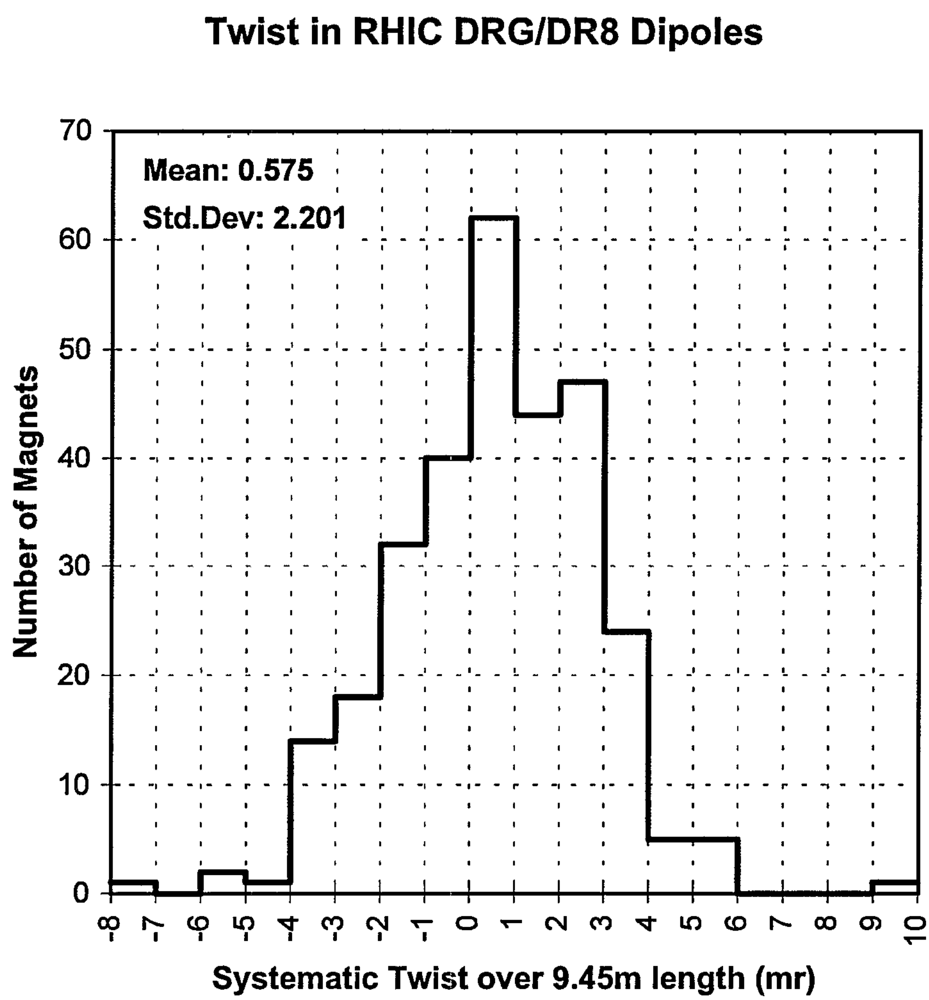

Figure 3: Distribution of average twist angles for 296 RHIC dipole magnets. The mean of the distribution is 0.58 mrad, with a standard deviation of 2.2 mrad.

minimal. (See Section 3.) The magnet acts as a helical dipole field of length ℓ and pitch $k = \alpha/\ell$. The spin vector $\vec{S}$ is then transformed according to

$$\begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}_{out} = M_\alpha \, M_{dpl} \, M_\alpha \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}_{in} \tag{1}$$

where the matrix for the "helical" field is given by [1]

$$\begin{aligned} M_{dpl} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\mu & -\sin\mu & 0 \\ \sin\mu & \cos\mu & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} \\ &= \begin{pmatrix} \cos\mu & -\cos\phi\sin\mu & \sin\phi\sin\mu \\ \cos\phi\sin\mu & \cos^2\phi\cos\mu + \sin^2\phi & \cos\phi\sin\phi(1-\cos\mu) \\ -\sin\phi\sin\mu & \cos\phi\sin\phi(1-\cos\mu) & \sin^2\phi\cos\mu + \cos^2\phi \end{pmatrix}, \end{aligned}$$

and where

$$M_\alpha = \begin{pmatrix} \cos(\alpha/2) & \sin(\alpha/2) & 0 \\ -\sin(\alpha/2) & \cos(\alpha/2) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

$$\mu = -k\ell\sqrt{1 + \left(\frac{\kappa}{k}\right)^2}, \tag{3}$$

$$\phi = \tan^{-1}(\kappa/k), \tag{4}$$

$$\kappa \equiv (1 + G\gamma)B_0/(B\rho). \tag{5}$$

2 Estimates for RHIC

Ideally, if α were zero for each RHIC dipole, then the total spin matrix for RHIC would just be the matrix for a rotation of $2\pi(1+G\gamma)$ about the vertical axis. For RHIC high energy (250 GeV) and a field of 3.45 T, this rotation angle modulo 2π is 137.53°. If each dipole in RHIC has a systematic twist of $\alpha = 0.58$ mrad, then subsequent matrix for RHIC is altered, and the resulting spin rotation angle is 137.54°. The change in the spin tune of RHIC is thus only 2×10^{-5}.

The above calculation was in the absence of Siberian Snakes. With the two Snakes in RHIC, the spin tune is altered from 0.5 by only about 2×10^{-6} due to the systematic twist error.

Since the twist angle has a distribution with rms 2.2 mrad, an ensemble of 500 RHIC rings was looked at, each ring containing dipoles generated randomly from a Gaussian distribution of twist errors with mean 0.575 mrad and rms 2.2 mrad. For each "ring" the difference in the spin tune from 0.5 was computed. The results are

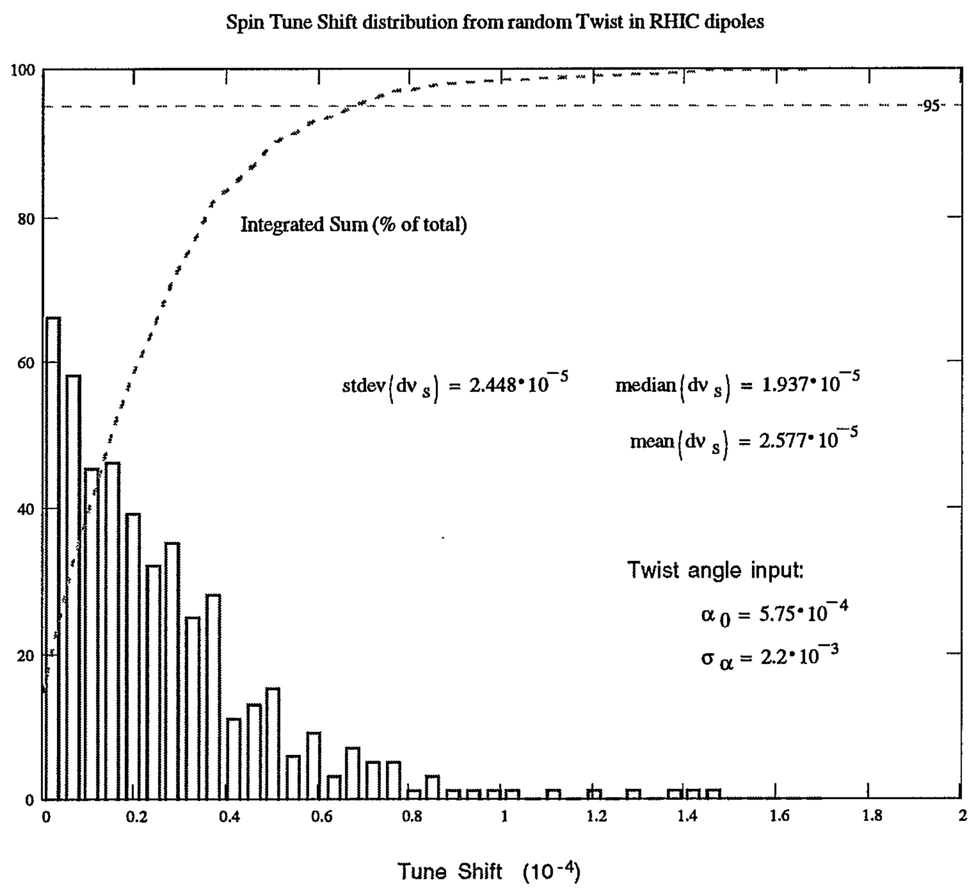

Figure 4: Distribution of spin tune shift (from the ideal value of 0.5) for an ensemble of 500 "RHIC rings." The dotted curve is the normalized integrated sum.

shown in Fig. 4. Here, we see that the chances of the spin tune shift due to this effect being greater than about 7×10^{-5} is only 5%.

The twisting of the dipole field in the RHIC dipole magnets should not cause a significant effect on the proton spin at all.

3 Effects on Orbit

A twisting of the field through a dipole magnet in the RHIC lattice will contribute to the closed orbit distortion around the accelerator and must be corrected with the dipole corrector system. A changing orientation of the main dipole field, to lowest order in the twist angle, will cause no distortion of the vertical field component, however horizontal field errors will be generated. For a helical twist with pitch α, where the field begins and ends with angles $\pm\alpha/2$ to the vertical, the horizontal field component will be

$$B_x(s) = -B_0 \sin(ks - \alpha/2)$$

along the length ℓ of the magnet. Thus, the vertical equation of motion will be

$$y'' = \frac{B_x}{B\rho} = -\frac{B_0}{B\rho} \sin(ks - \alpha/2)$$

and upon passing through the twisted field of the magnet, the resulting vertical displacement and slope is given by

$$y(\ell) = y_0 + y_0'\ell - \frac{B_0\ell}{k(B\rho)}\cos(\alpha/2) + 2\frac{B_0}{k^2(B\rho)}\sin(\alpha/2) \quad (6)$$

$$y'(\ell) = y_0' \quad (7)$$

where y_0 and y_0' are the initial positions and slope coming into the magnet.

As can be seen, the action of the twisted dipole is equivalent to passing through a drift of length ℓ and subsequently being given a step in the vertical direction of displacement

$$\Delta y = \frac{1}{12}\theta_0\alpha\ell$$

where here, θ_0 is the horizontal bending angle of the dipole magnet and α is considered to be small.

A "step" in the particle trajectory generates a closed orbit distortion which is given by

$$y(s) = \sqrt{\frac{\beta(s)}{\beta_0}}\frac{\Delta y}{2\sin\pi\nu}\left[\alpha_0\cos(\psi(s) - \pi\nu) - \sin(\psi(s) - \pi\nu)\right]$$

with α and β being the Courant-Snyder parameters of the lattice, the subscript "0" designating the location at the end of the helical field.

For a distribution of N errors around the accelerator, the expected rms closed orbit can then be estimated by

$$\hat{y}_{rms} = \sqrt{\hat{\beta}}\frac{\Delta y_{rms}}{2|\sin\pi\nu|}\sqrt{\langle\frac{\alpha^2}{\beta}\rangle + \langle\frac{1}{\beta}\rangle}\sqrt{\frac{N}{2}}$$

Noting that the combination $(1+\alpha^2)/\beta \equiv \gamma$ (another Courant-Snyder parameter), the expression becomes

$$\hat{y}_{rms} = \sqrt{\hat{\beta}\langle\gamma\rangle}\frac{\Delta y_{rms}}{2\sqrt{2}|\sin\pi\nu|}\sqrt{\frac{N}{2}}$$

For our final expression, we use the fact that γ is a constant within all the arc dipoles in the FODO cells, and so $\hat{\beta}\langle\gamma\rangle = 1+\hat{\alpha}^2 = 2/[1-\sin(\mu/2)]$, where μ is the cell phase advance. Thus, we have finally

$$\hat{y}_{rms} = \frac{\Delta y_{rms}}{2|\sin\pi\nu|\sqrt{1-\sin(\mu/2)}}\sqrt{N}.$$

We can now put in some numbers. For the RHIC distribution of twist angles α, we get

$$\Delta y_{rms} = \frac{1}{12}\theta_0\ell\alpha_{rms} = \frac{1}{12}(40\text{ mrad})(9.5\text{ m})(2.2\text{ mrad}) = 0.07\text{ mm}.$$

And, assuming approximately 160 RHIC dipole magnets per ring,

$$\hat{y}_{rms} \approx 1.5\text{ mm}.$$

Thus, the orbit distortion due to the twist is much smaller than the orbit distortion due to the quadrupole alignment errors, which generate uncorrected rms orbit excursions of 10-15 mm or more.

References

[1] M. J. Syphers, "Spin Motion through Helical Dipole Magnets," BNL Internal Report AGS/RHIC/SN-020, February, 1996.