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Spin Note

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Analytical and Numerical Study of Spin Depolarization in Particle Collider

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Analytical and Numerical Study of Spin Depolarization in Particle Collider

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Abstract

Analytical and numerical treatment of spin depolarization due to ion - ion collisions is given. It is shown, that in a collider with two Siberian Snakes in each ring spin depolarization due to beam-beam collision is suppressed. Degree of depolarization can be controlled by an appropriate choice of working point of betatron particles motion. In the absence of Siberian Snakes, beam-beam collision results in monotonous spin depolarization.

1. Introduction

Particle colliders with polarized beams require careful control of spin depolarization. During acceleration spin is subjected to intrinsic and imperfection resonances, resulting in depolarization. Extra source of depolarization is beam-beam collisions. In present paper effect of beam-beam collision on spin depolarization in an ion - ion collider is studied. Analytical treatment of the problem provides choice of the collider operation point, where depolarization is minimized. It also indicates zone of relatively strong depolarization.

2. Collider Model With Polarized Beams

Let us consider a collider ring with two installed Siberian Snakes [1]. We use a two-dimensional particle model in phase space (x, p_x), (y, p_y), where x and y are particle positions, $p_x = \beta_x^*(dx/dz)$ and $p_y = \beta_y^*(dy/dz)$) are particle momentum, $\beta_x^* = R/Q_x$ and $\beta_y^* = R/Q_y$ are average values of beta-functions of the ring, R is a ring radius, Q_x and Q_y are betatron tunes. Particle motion between subsequent collisions combines linear matrix with betatron angles $\bar{\theta}_x = 2\pi Q_x$, $\bar{\theta}_y = 2\pi Q_y$, perturbed by beam-beam interaction:

$$\begin{pmatrix} x_{n+1} \\ p_x^{n+1} \\ y_{n+1} \\ p_y^{n+1} \end{pmatrix} = \begin{pmatrix} \cos\bar{\theta}_x & \sin\bar{\theta}_x & 0 & 0 \\ -\sin\bar{\theta}_x & \cos\bar{\theta}_x & 0 & 0 \\ 0 & 0 & \cos\bar{\theta}_y & \sin\bar{\theta}_y \\ 0 & 0 & -\sin\bar{\theta}_y & \cos\bar{\theta}_y \end{pmatrix} \begin{pmatrix} x_n \\ p_x^n + \Delta p_x^n \\ y_n \\ p_y^n + \Delta p_y^n \end{pmatrix} . \quad (1)$$

Beam-beam kicks Δp_x^n, Δp_y^n are expressed as a result of interaction of particles with opposite beam with Gaussian distribution function

$$\Delta p_x^n = 4\pi\xi x_n \frac{1 - \exp[-r_n^2/(2\sigma_n^2)]}{[r_n^2/(2\sigma_n^2)]} , \quad (2)$$

and similar for Δp_y^n. Parameter ξ is a beam-beam parameter, which characterizes the strength of interaction:

$$\xi = \frac{N r_o \beta^*}{4\pi \sigma^2 \gamma} , \quad (3)$$

where N is a number of particles per bunch, $r_o = q^2/(4\pi\varepsilon_o mc^2)$ is a classical particle radius, σ is a transverse standard deviation of the opposite beam size and γ is a particle energy.

Rotation of spin vector $\vec{S} = (S_x, S_y, S_z)$ is described by subsequent spin matrix transformation [2, 3]. Matrix of spin advance in an lattice arc is described as a matrix of spin rotation in dipole magnet with bending angle υ:

$$D_\upsilon = \begin{vmatrix} \cos(\omega\delta z) & 0 & \sin(\omega\delta z) \\ 0 & 1 & 0 \\ -\sin(\omega\delta z) & 0 & \cos(\omega\delta z) \end{vmatrix} , \quad (4)$$

where $\omega\delta z = (1+G\gamma)\upsilon$ and G = 1.7928 is the anomalous magnetic moment of the proton. Matrixes of Siberian Snakes are given by

$$S_1 = \begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}, \quad S_2 = \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix} . \quad (5)$$

Spin advance after crossing the interaction point is described as follow :

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix} = \begin{vmatrix} 1 - a(B^2+C^2) & ABa + Cb & ACa + Bb \\ ABa - Cb & 1 - a(A^2+C^2) & BCa - Ab \\ ACa - Bb & BCa + Ab & 1 - a(A^2+B^2) \end{vmatrix} \begin{vmatrix} S_{x,o} \\ S_{y,o} \\ S_{z,o} \end{vmatrix} , \quad (6)$$

$$A = \frac{P_x}{P_o}, \quad B = \frac{P_y}{P_o}, \quad C = \frac{P_z}{P_o}, \quad P_o = \sqrt{P_x^2 + P_y^2 + P_z^2} , \quad (7)$$

$$a = 1 - \cos(P_o \delta z) , \quad b = \sin(P_o \delta z) , \quad (8)$$

where $\delta z = l/2$ is an interaction distance and l is a bunch length. Vector $\vec{P} = (P_x, P_y, P_z)$ is given as follow:

$$P_x = \frac{1}{B\rho} [(1+G\gamma)B_x + (G\gamma + \frac{\gamma}{1+\gamma})\frac{\beta E_y}{c}], \quad (9)$$

$$P_y = \frac{1}{B\rho} [(1+G\gamma)B_y - (G\gamma + \frac{\gamma}{1+\gamma})\frac{\beta E_x}{c}], \quad P_z = 0, \quad (10)$$

where $B\rho$ is a rigidity of particles, $\vec{E} = (E_x, E_y, 0)$ is an electrical field and $\vec{B} = (B_x, B_y, 0)$ is a magnetic field of the opposite bunch. Taking into account, that particles are relativistic $\beta \approx 1$, $\gamma >> 1$ and electromagnetic field in the interaction point is created by an opposite beam with round Gaussian space charge distribution, vector $\vec{P}$ is simplified:

$$P_x = 4G \frac{I}{I_c} \frac{y}{r^2} [1 - \exp(-\frac{r^2}{2\sigma^2})], \quad (11)$$

$$P_y = -4G \frac{I}{I_c} \frac{x}{r^2} [1 - \exp(-\frac{r^2}{2\sigma^2})], \quad (12)$$

where I is a beam current and $I_c = 4\pi \varepsilon_0 m_o c^3/q = (A/Z) \cdot 3.13 \cdot 10^7$ Amp is a characteristic value of the beam current.

3. Analytical Treatment of Spin Depolarization

To make an analytical treatment of spin depolarization, let us consider a collider with two installed Siberian Snakes and one interaction point. Matrix of spin advance after one revolution in the ring between beam-beam interaction is

$$M_{ring} = D_{\pi/2} \cdot S_2 \cdot D_\pi \cdot S_1 \cdot D_{\pi/2} = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} . \quad (13)$$

Suppose, the betatron tunes in x and y directions are equal each other $\bar{\theta}_x = \bar{\theta}_y = \bar{\theta}$. We will consider particle motion far enough from low order resonances, therefore, particle trajectory can be expressed as a linear oscillator with perturbed betatron tune θ:

$$x = r\cos(n\theta + \Psi), \quad y = r\sin(n\theta + \Psi), \quad \theta = \bar{\theta} + \Delta\theta , \quad (14)$$

where Ψ is an initial phase of betatron particle oscillations and $\Delta\theta << 2\pi$ is tune perturbation due to beam-beam collisions. Parameters A and B in eq. (6) can be expressed as follow:

$$A = \frac{P_x}{P_o} = \sin(n\theta + \Psi), \qquad B = \frac{P_y}{P_o} = -\cos(n\theta + \Psi) . \quad (15)$$

Then, let us take into account , that parameter $\varphi = P_o \cdot \delta z$ in eq. (8) is small:

$$\varphi = P_o \delta z = \frac{2I\,l}{I_c\, r} G[1 - \exp(-\frac{r^2}{2\sigma^2})] \approx \frac{I\,l\,rG}{I_c\,\sigma^2}(1 + ..) << 2\pi . \quad (16)$$

Hence, the matrix parameters a and b in eq. (8) can be approximated as follow:

$$a = 1 - \cos\varphi \approx \frac{\varphi^2}{2}, \qquad b = \sin\varphi \approx \varphi . \quad (17)$$

Finally, matrix of spin advance of a particle in interaction point at the n-th turn is:

$$\begin{vmatrix} [1 - \frac{\varphi^2}{2}\cos^2(n\theta+\Psi)] & [-\frac{\varphi^2}{4}\sin 2(n\theta+\Psi)] & [-\varphi\cos(n\theta+\Psi)] \\ [-\frac{\varphi^2}{4}\sin 2(n\theta+\Psi)] & [1 - \frac{\varphi^2}{2}\sin^2(n\theta+\Psi)] & [-\varphi\sin(n\theta+\Psi)] \\ \varphi\cos(n\theta+\Psi) & \varphi\sin(n\theta+\Psi) & 1 - \frac{\varphi^2}{2} \end{vmatrix} . \quad (18)$$

It is easy to verify, that determinant of the matrix (18) equals unit, as it is required by conservation of absolute value of spin $|\vec{S}| = 1$.

Subsequent multiplication of matrixes (13) and (18) gives us possibility to predict effect of beam-beam interaction on spin depolarization after large number of turns. Suppose, initial spin vector has only one transverse component $S_y = 1$ and other components are equal to zero $S_x = S_z = 0$. Spin advance after n turns is as follow

$$S_x = \frac{\varphi^2}{4} [\sum_{i=0}^{n-1} (-1)^{i+n-1} \sin 2(i\theta+\Psi) + ...] , \quad (19)$$

$$S_y = 1 - \frac{\varphi^2}{2} [\sum_{i=0}^{n-1} (-1)^i \sin(i\theta+\Psi)]^2 , \quad (20)$$

$$S_z = (-1)^n \varphi \sum_{i=0}^{n-1} (-1)^i \sin(i\theta+\Psi), \quad (21)$$

Average values of spin components are achieved by integration of eqs. (19-21) over all initial phases and averaging over turn number:

$$\bar{S}_x = 0, \quad \bar{S}_z = 0, \quad \bar{S}_y = 1 - \frac{1}{8} (\frac{\tilde{\varphi}}{\cos(\tilde{\theta}/2)})^2 , \quad (22)$$

where $\tilde{\varphi}$ and $\tilde{\theta}$ are average values of parameters φ, θ among all particles:

$$\tilde{\varphi} = 4\pi G \gamma \xi \frac{\sigma}{\beta^*}, \qquad \tilde{\theta} = 2\pi (Q - \frac{\xi}{2}) . \quad (23)$$

Averaged root-mean-square values of spin components are given by

$$\overline{<S_x^2>} = [\frac{\tilde{\varphi}^2}{8(\cos\tilde{\theta})}]^2, \quad \overline{<S_y^2>} = \frac{3}{512} [\frac{\tilde{\varphi}}{\cos(\tilde{\theta}/2)}]^4 , \quad (24)$$

$$\overline{<S_z^2>} = [\frac{\tilde{\varphi}}{2\cos(\tilde{\theta}/2)}]^2 . \quad (25)$$

Attained formulas indicate, that spin depolarization due to beam-beam collisions is suppressed and depends on betatron tune in the ring. The most dangerous working point is close to half-integer value of betatron tune, because in that case the value of cos $(\tilde{\theta}/2)$ is close to zero and spin depolarization becomes large. Due to small value of φ, all effects, proportional to φ^4 are negligible as compare with that, proportional to φ^2. Therefore, among possible depolarization effects, the most pronounced are change of average value of S_y component and rms value of $<S_z^2>$.

4. Numerical Simulation of Beam-Beam Effect on Spin Depolarization

Computer simulations utilizing numerical model of Section 2 were performed for the beam parameters, presented in Table 1.

Table 1. Parameters of the interacted beams

Particle energy, γ	260
Rms beam size at interaction point (IP), σ	0.08 mm
Beam-beam tune shift per collision ξ	- 0.0125
Beta function of a ring, β^*,	0.65 m
Number of modeling particles	5000

For that combination of collider parameters the value of matrix parameters are as follow:

$$\tilde{\varphi} = 4\pi G \gamma \xi \frac{\sigma}{\beta^*} = 9 \cdot 10^{-3}, \quad \tilde{\theta} = 2\pi (Q - 0.006) . \qquad (26)$$

Operational point in simulations was chosen far enough from low order resonances. Particle trajectories in phase space were slightly deformed ellipses due to beam-beam collisions (see Fig. 1). Initial particle distribution in phase space was chosen to be Gaussian:

$$f = f_o \exp - \left(\frac{p_x^2 + p_y^2}{2p_o^2} + \frac{x^2 + y^2}{2\sigma^2}\right) . \qquad (27)$$

In the absence of beam-beam instability beam envelopes and beam emittances were stable (see Fig. 2).

During simulations, the average and rms values of spin distribution were calculated according to formulas:

$$\bar{S}_x = \frac{1}{N} \sum_{i=1}^{N} S_x(i); \quad \bar{S}_y = \frac{1}{N} \sum_{i=1}^{N} S_y(i); \quad \bar{S}_z = \frac{1}{N} \sum_{i=1}^{N} S_z(i), \qquad (28)$$

$$\sqrt{<S_x^2>} = \sqrt{\sum_{i=1}^{N} \frac{1}{N} [S_x(i) - \bar{S}_x]^2} , \qquad (29)$$

$$\sqrt{<S_y^2>} = \sqrt{\sum_{i=1}^{N} \frac{1}{N} [S_y(i) - \bar{S}_y]^2} , \qquad (30)$$

$$\sqrt{<S_z^2>} = \sqrt{\sum_{i=1}^{N} \frac{1}{N} [S_z(i) - \bar{S}_z]^2} . \qquad (31)$$

Initial spin vector for all particles was supposed to be $\vec{S} = (0, 1, 0)$ (see Fig. 3).

Results of numerical simulations indicate that in presence of Siberian Snakes spin depolarization is suppressed. Depolarization is expressed as spin components distribution (see Fig. 4). Distribution of S_x component is much narrow than that of S_z component. It also follows from eqs. (24), (25), where $<S_x^2>$ is proportional to φ^4, while $<S_z^2>$ is proportional to φ^2. Average value of S_y oscillates around stable point close to unit (see Fig. 5). Average values of S_x and S_z oscillate around zero. Rms values of spin distribution are also stable.

5. Spin Depolarization in a Ring without Siberian Snakes

To estimate effect of Siberian Snakes on spin depolarization in presence of beam-beam interaction, consider a ring without Snakes. Derivation of a spin matrix rotation after arbitrary number of turns results in awkward expressions, so we have to rely on computer simulations. In Figs. 6, 7 spin distribution as well as average and rms values of spin component are presented. As shown, without Snakes beam-beam collisions result in monotonous spin depolarization.

In special cases, when the following condition is achieved:

$$(1+G\gamma)\, \upsilon = 2\pi k, \text{ or } \gamma = \frac{1}{G}\left(\frac{2\pi k}{\upsilon} - 1\right), \quad k = 1, 2, 3, ... \qquad (32)$$

the matrix of spin rotation in lattice arc is the unit matrix:

$$D_\upsilon = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} . \qquad (33)$$

We assume, that spin resonances are avoided in a ring. In this case matrix of spin advance after arbitrary number of turns can be calculated analytically, as product of matrixes (18), where turn number is changed from 1 to n:

$$\begin{vmatrix} 1 - \frac{\varphi^2}{2} [\sum_{i=0}^{n} \cos(i\theta+\Psi)]^2 & -\frac{\varphi^2}{4} \sum_{i=0}^{n} \sin 2(i\theta+\Psi) + .. & -\varphi \sum_{i=0}^{n+1} \cos(i\theta+\Psi) \\ -\frac{\varphi^2}{4} \sum_{i=0}^{n} \sin 2(i\theta+\Psi) + .. & 1 - \frac{\varphi^2}{2} [\sum_{i=0}^{n} \sin(i\theta+\Psi)]^2 & -\varphi \sum_{i=0}^{n} \sin(i\theta+\Psi) \\ \varphi \sum_{i=0}^{n} \cos(i\theta+\Psi) & \varphi \sum_{i=0}^{n} \sin(i\theta+\Psi) & [1 - \frac{n+1}{2} \varphi^2 + ..] \end{vmatrix} . \qquad (34)$$

From attained matrix (34) it is clear, that in this case spin depolarization is suppressed. Assuming as in section 3, that initial spin vector is $\vec{S}(0) = (0, 1, 0)$, spin components after (n-1) turns are as follow:

$$S_x = -\frac{\varphi^2}{4} \sin [2\Psi + (n-1)\theta] \frac{\sin (n\theta)}{\sin \theta} , \qquad (35)$$

$$S_y = 1 - \frac{\varphi^2}{2} \sin^2(\Psi + \frac{n-1}{2}\theta) \frac{\sin^2(\frac{n}{2}\theta)}{\sin^2(\frac{\theta}{2})} , \quad (36)$$

$$S_z = \varphi \sin(\Psi + \frac{n-1}{2}\theta) \frac{\sin(\frac{n}{2}\theta)}{\sin(\frac{\theta}{2})} . \quad (37)$$

Average and rms values of spin component distributions are calculated similar to that of Section 3:

$$\bar{S}_x = 0 \ , \ \bar{S}_z = 0 , \quad (38)$$

$$\bar{S}_y = 1 - \frac{\varphi^2}{4} \frac{\sin^2(\frac{n\theta}{2})}{(\sin\frac{\theta}{2})^2} , \quad (39)$$

$$<S_x^2> = \frac{\varphi^4}{16} [\frac{\sin^2(n\theta)}{2(\sin\theta)^2} + ...] , \quad (40)$$

$$<S_y^2> = \frac{\varphi^4}{32} \frac{\sin^4(\frac{n\theta}{2})}{(\sin\frac{\theta}{2})^4} , \quad (41)$$

$$<S_z^2> = \frac{\varphi^2}{2} \frac{\sin^2(\frac{n\theta}{2})}{(\sin\frac{\theta}{2})^2} . \quad (42)$$

Averaging over turn number gives the final values of averaged and rms values of spin components:

$$\bar{\bar{S}}_x = 0, \quad \bar{\bar{S}}_y = 1 - \frac{\tilde{\varphi}^2}{8(\sin\frac{\tilde{\theta}}{2})^2}, \quad \bar{\bar{S}}_z = 0 , \quad (43)$$

$$\overline{<S_x^2>} = \frac{\tilde{\varphi}^4}{16} [\frac{1}{4(\sin\tilde{\theta})^2} + ...] , \overline{<S_y^2>} = \frac{3\tilde{\varphi}^4}{256(\sin\frac{\tilde{\theta}}{2})^4} , \quad (44)$$

$$\overline{<S_z^2>} = \frac{\tilde{\varphi}^2}{4(\sin\frac{\tilde{\theta}}{2})^2} . \quad (45)$$

From Eqs. (43) - (45) it follows, that the most strong depolarization can be observed if $\sin\frac{\tilde{\theta}}{2} \to 0$, or if betatron tune is close to integer numbers. In other cases depolarization is suppressed. In Fig. 8 results of suppressed spin depolarization for $\gamma = 260.4$ are presented.

6. Spin depolarization in presence of noisy beam-beam instability

Up to now we have considered stable particle motion in presence of beam-beam interaction. There are several mechanisms, which lead to particle instability under beam-beam collisions. Excitation of nonlinear resonances and unstable stochastic particle motion due to overlapping of resonance islands are the most fundamental phenomena in beam-beam interaction. Another mechanism of unstable particle motion is a diffusion created by random fluctuations in distribution of the opposite beam. In Ref. [4] the noise beam-beam instability was studied for the case of random fluctuations in opposite beam size

$$\sigma_n = \sigma_o \ (1 \pm \frac{u \cdot u_n}{2}), \quad (46)$$

where u is a noise amplitude and u_n is a uniform random function with unit amplitude. It was shown, that in the presence of noise, beam emittance is increased with turn number as

$$\frac{\varepsilon_n}{\varepsilon_o} = \sqrt{1 + D n} . \quad (47)$$

Diffusion coefficient D is a function of beam-beam parameter ξ, noise amplitude u and ratio of beam size, a with respect to opposite beam size $2\sigma_o$:

$$D = \pi^2 (\xi u)^2 (\frac{a}{2\sigma_o})^4 . \quad (48)$$

Noise in the beam-beam collision always induces instability if beam-beam kick is a nonlinear function of the coordinate. Due to diffusion character, noise beam-beam instability does not have a threshold character and can exist at any value of beam-beam tune shift. Increase of beam emittance is accompanied with increase of beam size.

In Figs. 9, 10 results of beam dynamics and spin depolarization in presence of noisy beam-beam instability are presented. Particle ring was supposed to have one interaction point and two Siberian Snakes. Parameters of the process were chosen the same as for stable beam-beam interaction without noise, presented in Figs. 2-5. The value of noise amplitude u = 0.025 was chosen arbitrary, to demonstrate the main features of diffusion beam-beam instability. As seen, increase of beam sizes due to beam-beam collisions results in spin depolarization. It is also expected from analytical formulas (22) - (25), where average and rms beam parameters are proportional to the powers of small parameter φ, which, is proportional to beam size according to eq. (23). Therefore, if beam is subjected to beam-beam instability, it provides spin depolarization.

7. Conclusions

Effect of beam-beam interaction on spin depolarization in ion particle collider has been studied. Employed method is based on matrix formalism for spin advance and for perturbed betatron particle motion in a ring. Analytical calculations were done for a collider with one interaction point and two installed Siberian Snakes in each ring. Matrix for spin advance after arbitrary number of turns is accomplished. Performed study indicates, that spin depolarization due to beam-beam collisions is suppressed. Depolarization depends on collider operation point. The most significant depolarization is observed if the value of betatron tune is close to half integer. For other values of collider betatron tune spin depolarization is much smaller. Numerical and analytical estimations of spin depolarization are close to each other. Absence of Snakes as well as unstable particle motion in noisy beam-beam instability provide steady depolarization due to beam-beam collisions.

8. References

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[4] Y.Batygin and T.Katayama "Noise instability in nonlinear beam-beam interaction", RIKEN-AF-AC-3, March 1997, to be appear in NIM-A.

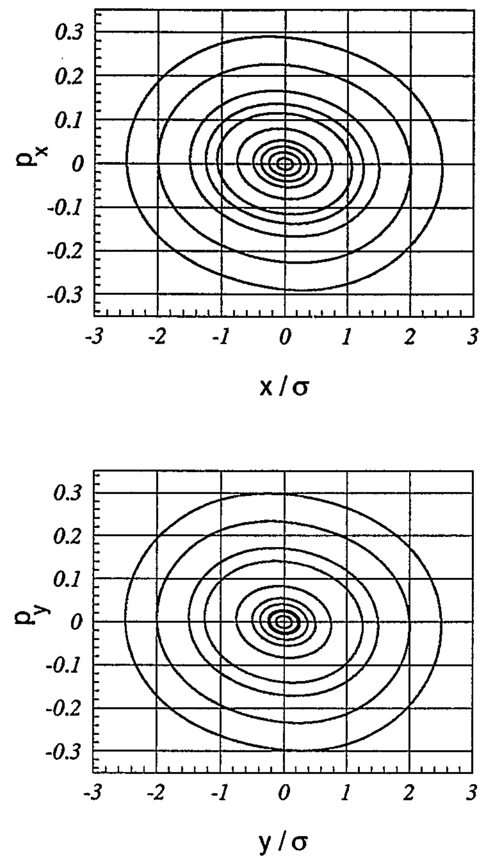

Fig.1 Stable particle trajectories in phase space in presence of beam-beam interaction without noise

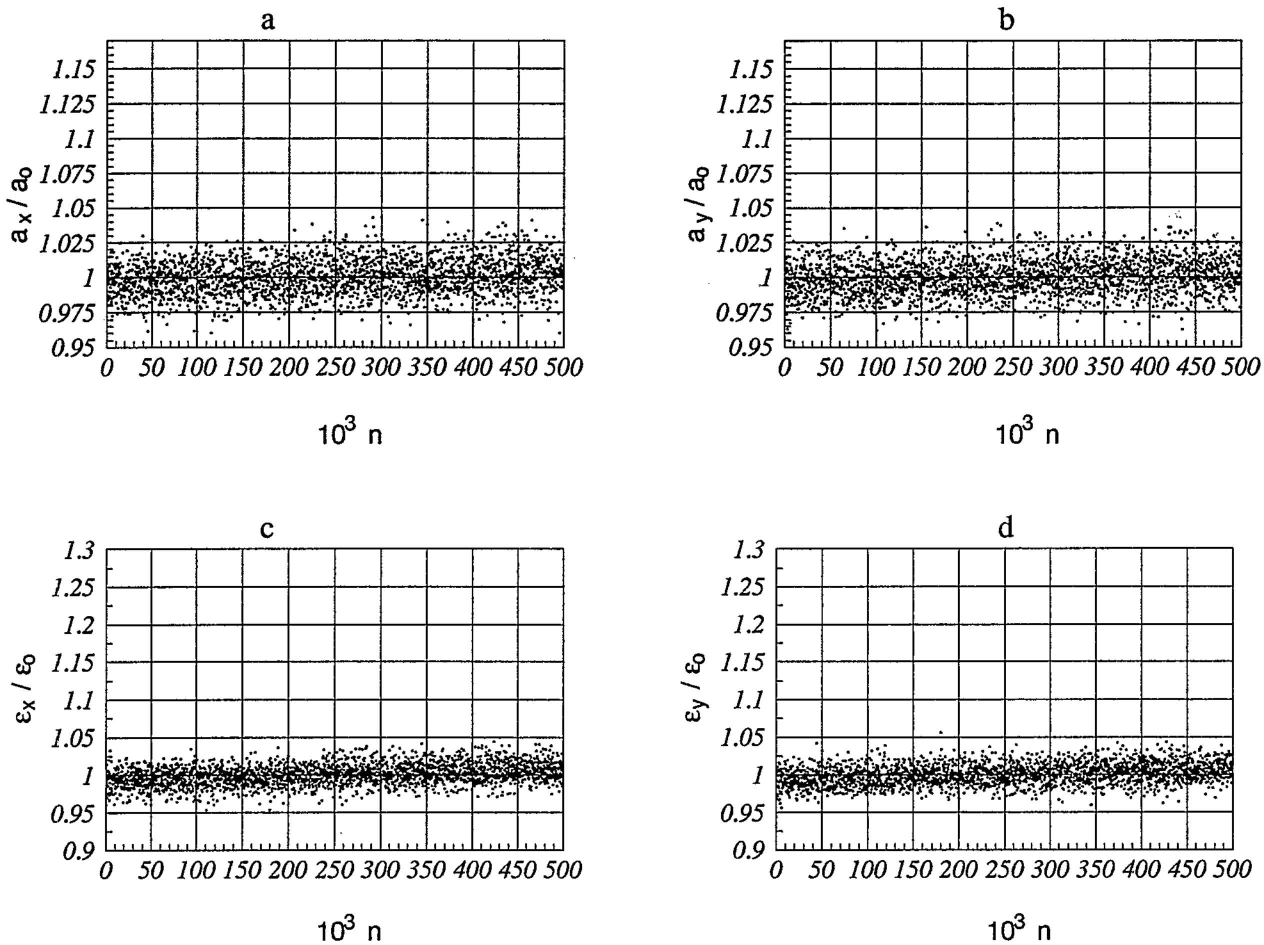

Fig.2. Envelopes (a), (b) and beam emittances (c), (d) in presence of beam-beam interaction without noise.

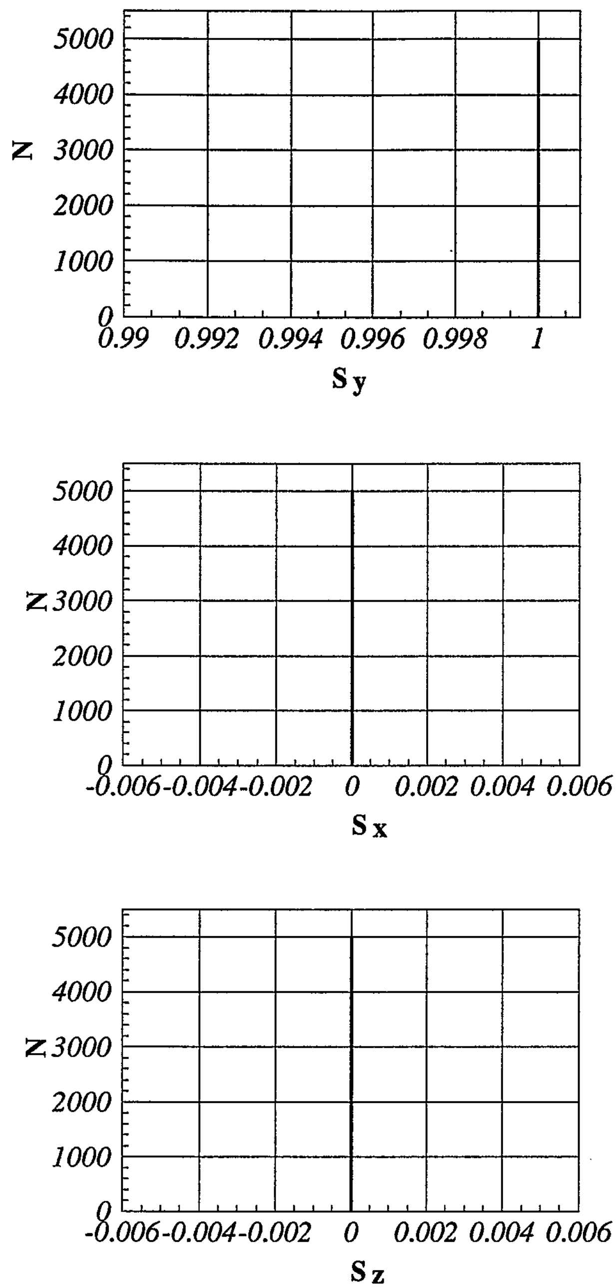

Fig.3 Initial spin distribution

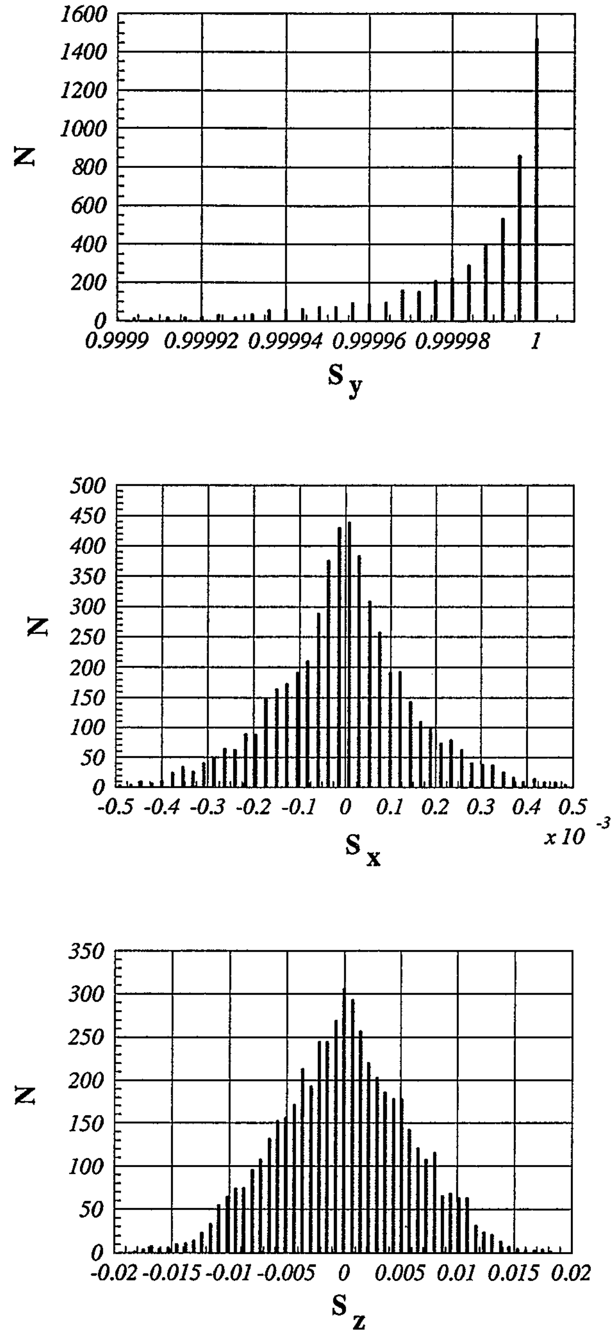

Fig. 4. Spin distribution after 500000 turns in a ring with two Siberian Snakes, $\overline{\theta}_x = \overline{\theta}_y = 14.249$.

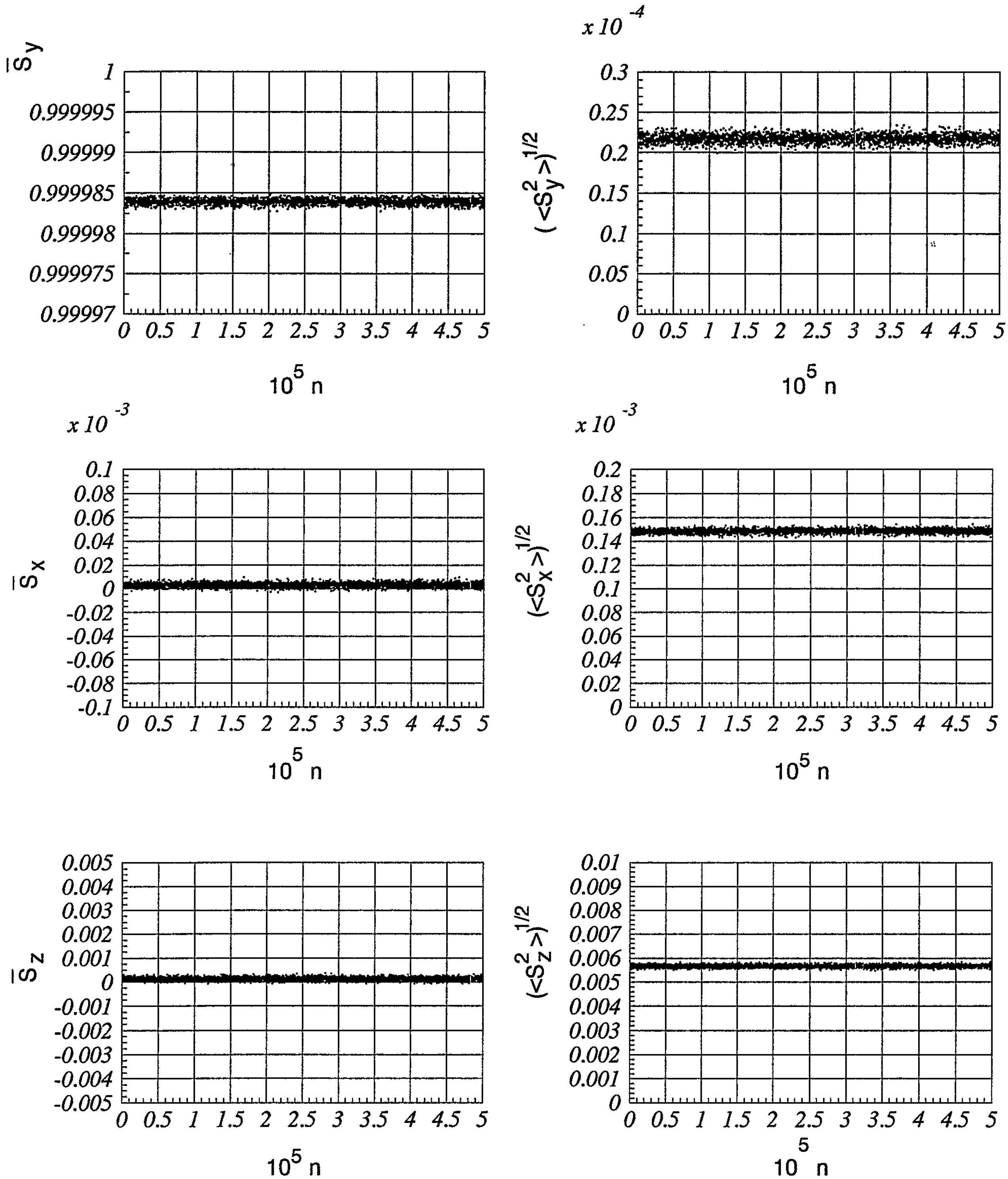

Fig.5. Average and rms values of spin components as functions of turn number for a ring with two Siberian Snakes, $\overline{\theta}_x = \overline{\theta}_y = 14.249$.

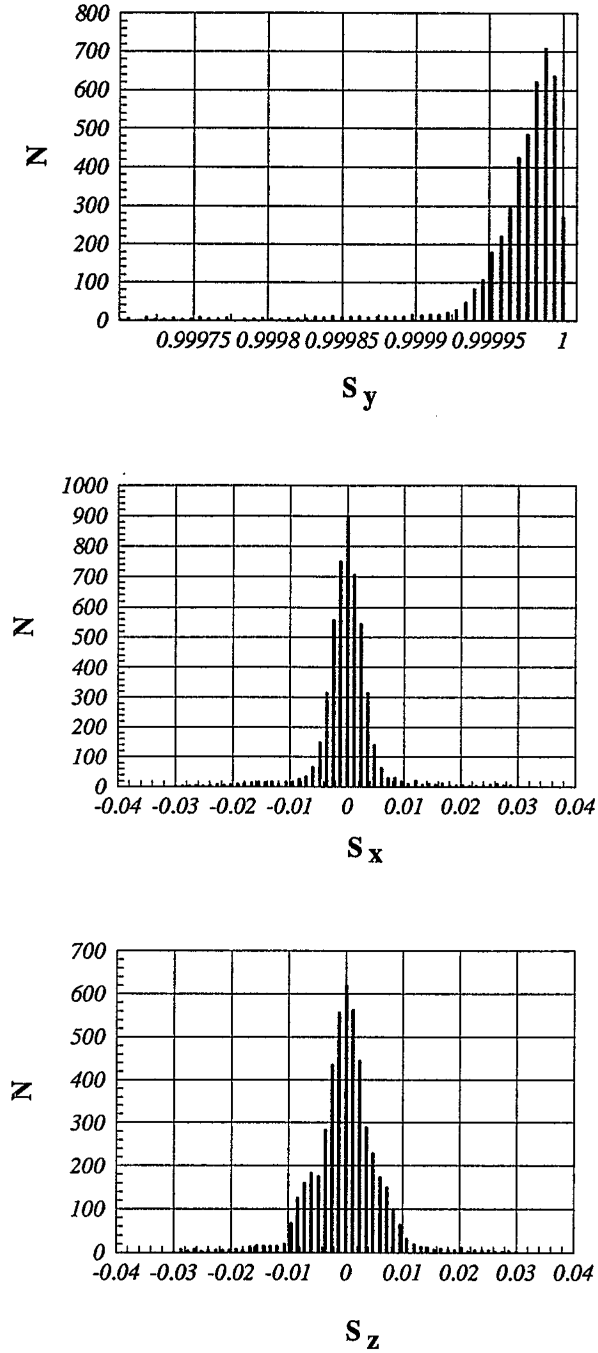

Fig.6. Spin distribution after 1 000 000 turns in a ring without Siberian Snakes, $\bar{\theta}_x$= 14.095, $\bar{\theta}_y$ = 14.59.

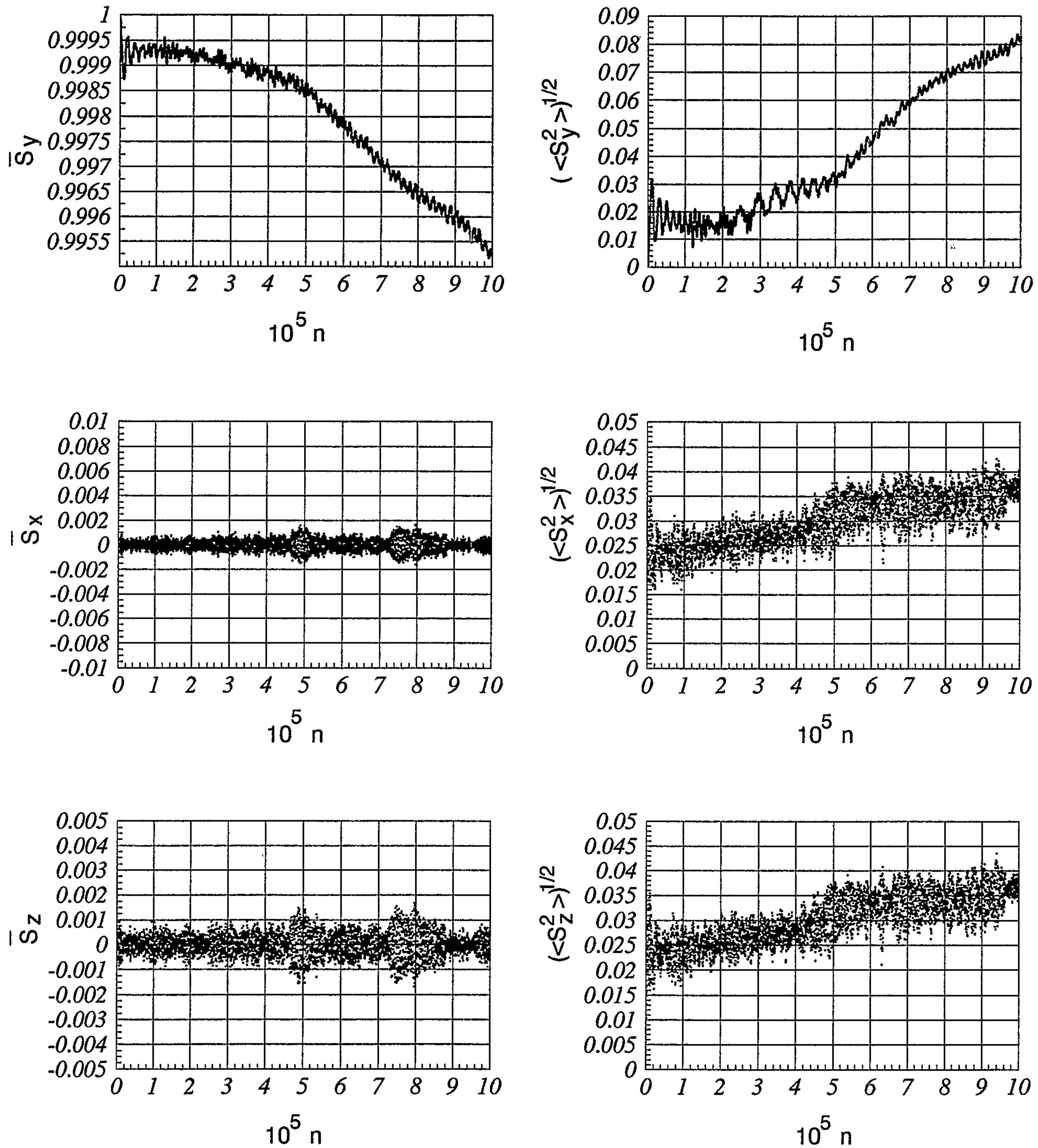

Fig. 7. Average and rms values of spin component as function of turn number for a ring without Siberian Snakes, $\bar{\theta}_x$ = 14.095, $\bar{\theta}_y$ =14.59.

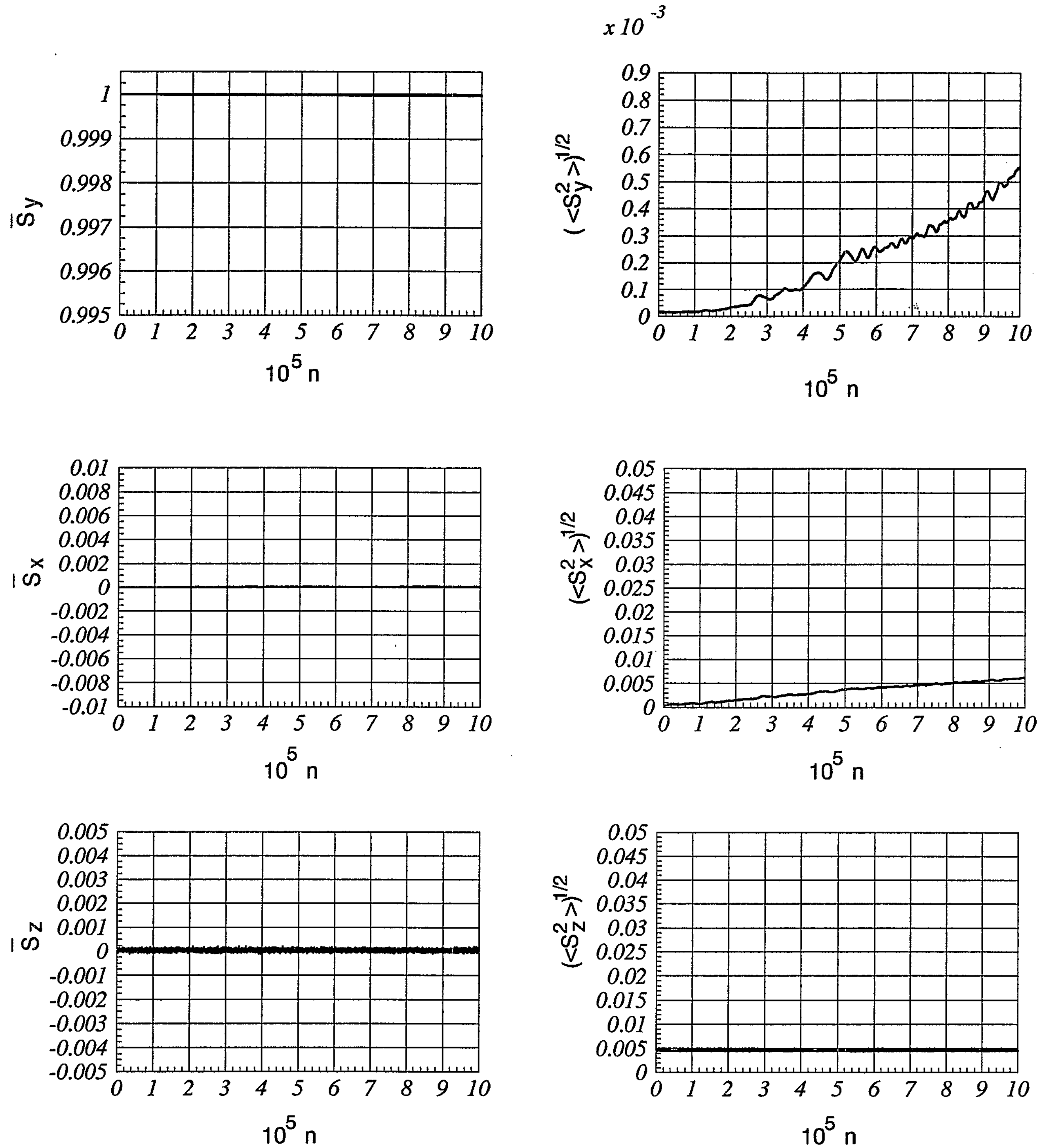

Fig. 8. Average and rms values of spin components as functions of turn number for a ring without Siberian Snakes for $\gamma = 260.4$; $\bar{\theta}_x = 14.095$, $\bar{\theta}_y = 14.59$.

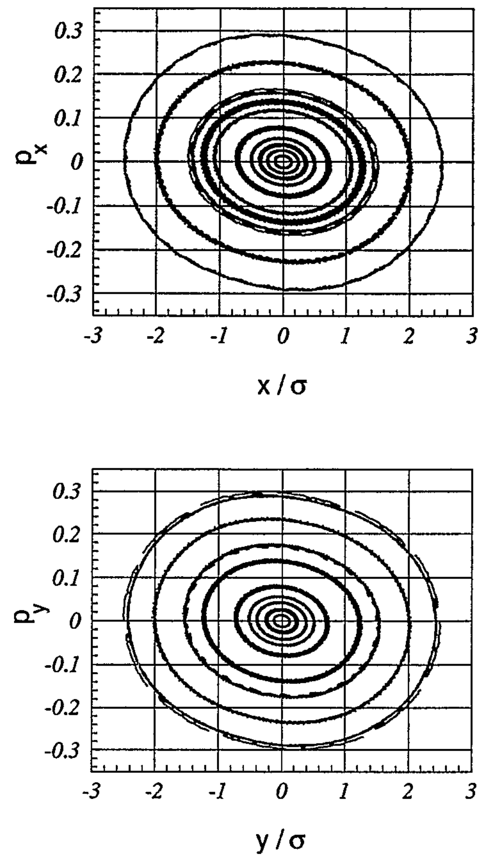

Fig. 9. Distorted particle trajectories in presence of 2.5 % noise in parameter σ in beam-beam kick

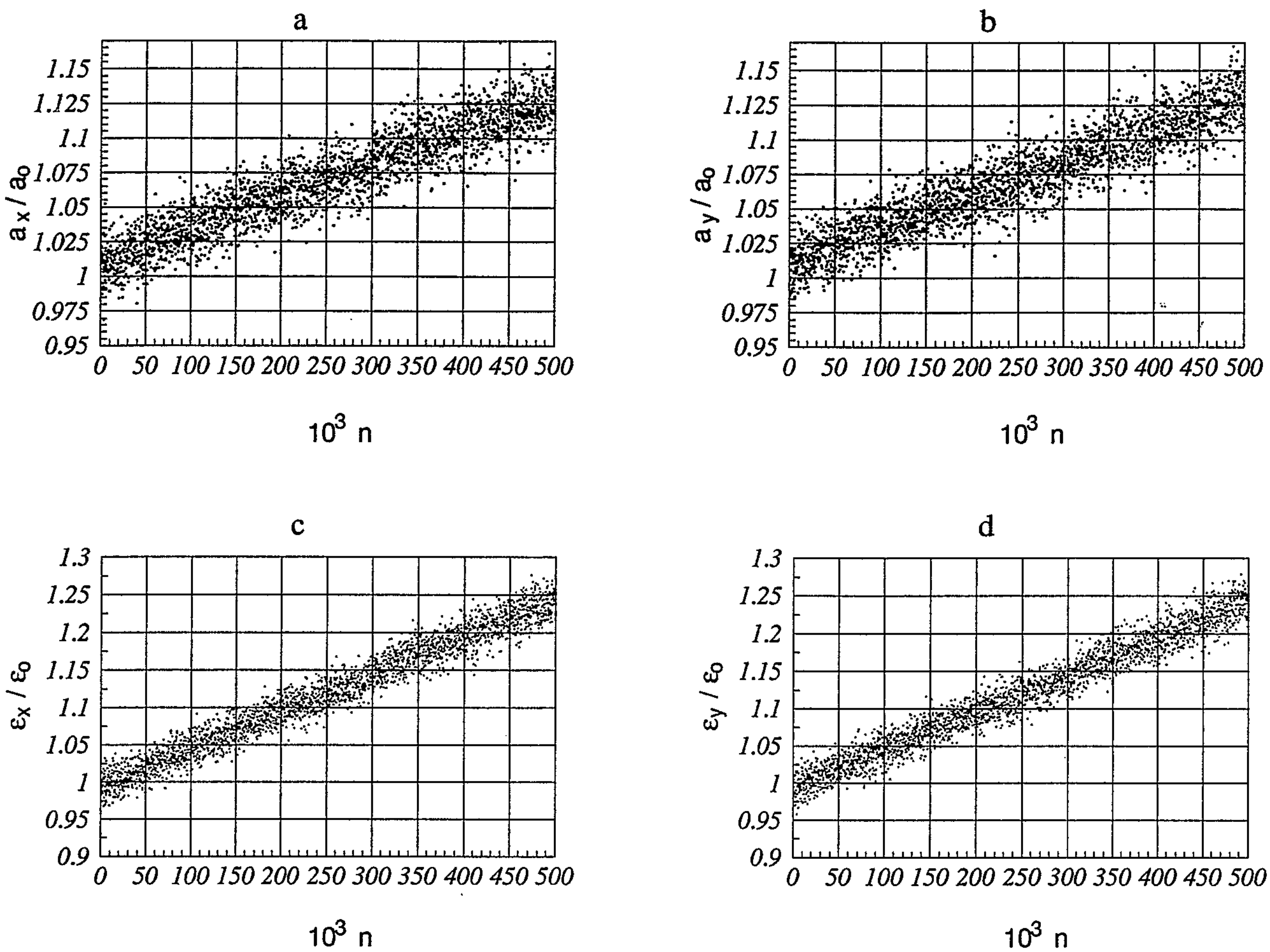

Fig. 10. Envelopes (a), (b) and beam emittances (c), (d) in presence of beam-beam interaction with 2.5% noise in parameter σ of beam-beam kick

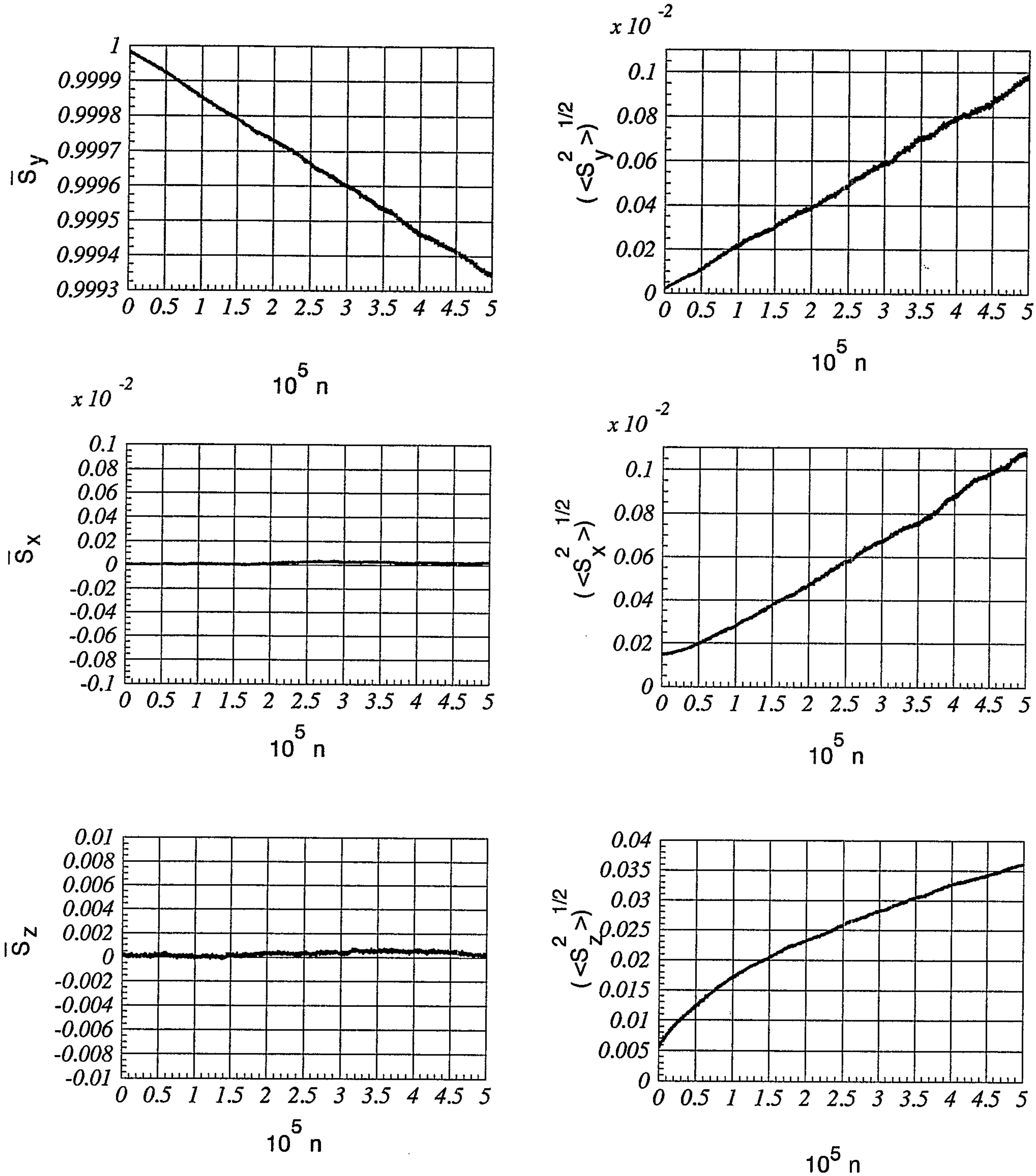

Fig. 11. Average and rms values of spin components as functions of turn number for a ring with two Siberian Snakes and 2.5 % noise in parameter σ; $\bar{\theta}_x = \bar{\theta}_y = 14.249$.