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Estimation of Rotation Angle in the Full Length Helical Dipole Based on Data in the Half-Length Prototype HRC001

A. K. Jain

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Collider Accelerator Department

Brookhaven National Laboratory

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Alternating Gradient Synchrotron Department
Relativistic Heavy Ion Collider Project
BROOKHAVEN NATIONAL LABORATORY
Upton, New York 11973

Spin Note

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Animesh K. Jain

AGS Department, Brookhaven National Laboratory, NY 11973

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Animesh K. Jain
RHIC Project, Brookhaven National Laboratory, Upton, NY 11973

1. Introduction

In the helical dipoles, the net deflection of beam should be kept negligible. In an oversimplified model, assuming a constant field strength over the magnetic length and zero field outside, the rotation angle of the field should be 360 degrees. In practice, there are end regions where the field strength reduces gradually and the phase angle of the field changes very little. These end regions contribute to a net integrated dipole field, which must be compensated by an equal and opposite integrated dipole field from the body of the magnet. Due to lack of well defined boundaries, it is difficult to specify unambiguously a rotation angle. However, an effective rotation angle can be defined as the rotation angle in a simplified, "no ends" magnet, which will produce the same horizontal and vertical integrated dipole field as the real magnet. Thus, a magnet with zero integrated dipole field will be said to have an effective rotation angle of 360 degrees.

If B_0 is the dipole field strength at the center of the magnet and θ_R is the effective rotation angle, then the integrated y and x components of the dipole field in the transverse direction are given by

$$\int_{-\infty}^{\infty} B_y(z)\,dz = \left(\frac{2B_0}{d\alpha / dz}\right)\cos\left(\alpha_0 + \frac{\theta_R}{2}\right)\sin\left(\frac{\theta_R}{2}\right) \tag{1}$$

$$\int_{-\infty}^{\infty} B_x(z)\,dz = \left(\frac{2B_0}{d\alpha / dz}\right)\sin\left(\alpha_0 + \frac{\theta_R}{2}\right)\sin\left(\frac{\theta_R}{2}\right) \tag{2}$$

where $(d\alpha/dz)$ is the rate of change of phase angle in the body of the magnet. The quantity $(\alpha_0 + \theta_R/2)$ depends on the choice of reference frame, and can be interpreted as the phase angle at the center of the magnet, assuming symmetric ends. It is easy to see that for a rotation angle of 360 degrees, the integrals of both the components vanish. One can eliminate the dependence on the reference frame by examining the total amplitude of the integrated field, C_1, given by

$$C_1 = \left[\left(\int_{-\infty}^{\infty} B_y\,dz\right)^2 + \left(\int_{-\infty}^{\infty} B_x\,dz\right)^2\right]^{1/2} = \left(\frac{2B_0}{d\alpha / dz}\right)\sin\left(\frac{\theta_R}{2}\right) \tag{3}$$

Using Eq. (3), one can estimate the effective rotation angle from the integrated dipole amplitude. It is clear from Eq. (3) that the integrated field is zero for a 360 degrees rotation, and has a maximum for 180 degrees. A consequence of this fact is that the integrated field is least sensitive to the rotation angle in a magnet with 180 degrees rotation. A small error in the measurement of the integral field will result in a rather large error in estimating the effective rotation angle for such a magnet.

2. Experimental Data in the Half-length Magnet HRC001

A half-length prototype, HRC001, has been built and cold tested. The field quality in this magnet has been measured using a rotating system consisting of two Hall-probes. Axial scans of the field were carried out at two currents – 105A and 220A. These currents correspond to approximately 1.43 and 3.0 Tesla respectively in the body of the magnet. The axial scan at 105A was done over several days, two cool downs and with *Z*-positions in no particular order. Discrepancies were noticed between data taken at the same nominal *Z*-position on different days, indicating a shift in the origin of the transporter used to move the probe. As a result, the data at 105A are not expected to give a good measurement of the integral field in this magnet. The data at 220A are much more reliable for the purpose of estimating the integral field, and hence the effective rotation angle. Fig. 1 shows the measured dipole field amplitude and the phase angle as a function of axial position at 220A, measured by the Hall probe-2, located at a radius of 3.56 cm.

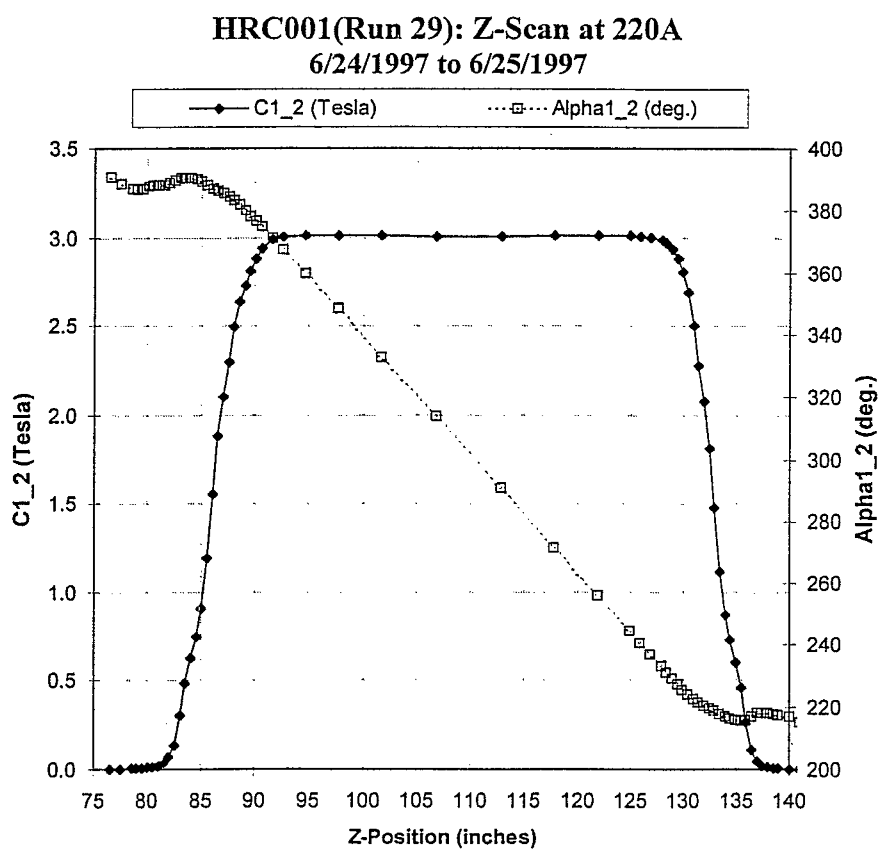

Fig. 1 Measured dipole amplitude and phase at 220A in HRC001

The Z-positions are from the reference point of the probe transporter, which is at an arbitrary position outside the magnet. Also, the Z-positions increase from the lead end to the non-lead end of the magnet, which is inconsistent with a right handed coordinate system. The phase angle is in a reference frame decided by the orientation of the probe. This orientation is also arbitrary with respect to the yoke. The dipole amplitude and phase are obtained from a Fourier analysis of the measured radial component of the field at 64 angular positions. The measured data have been shown to agree very well with a three dimensional numerical calculation using TOSCA by Okamura [1].

3. Rate of Twist in the Body of the Magnet

The determination of the effective rotation angle requires a knowledge of the twist rate, $d\alpha/dz$, in the body of the magnet [see Eq. (3)]. This quantity can be easily calculated from the data shown in Fig. 1. The phase angle changes almost linearly in the central region. A straight line fit to the data in the central region gives a rate of twist of 3.819 degrees/inch or 1.5035 degrees/cm [2]. The design value of the twist rate is 1.5 degrees/cm at room temperature. Assuming a 0.38% reduction in the length on cool down to liquid helium temperature, the expected twist rate is 1.5057 degrees/cm. The measured value is in good agreement with the value expected from the design. For calculating the effective rotation angle, the design value at liquid helium temperature is used in this note.

4. Rotation Angle in the Half-length Magnet

In principle, one could calculate the integral of the field components, B_x and B_y, from the data shown in Fig. 1. These integrated values can be used to obtain the total integrated dipole amplitude, C_1, and the effective rotation angle, θ_R. Such a calculation yields a value of the integrated dipole amplitude of 2.286 T.m at a central field of 3.0083 Tesla. The effective rotation angle calculated from this integrated field [see Eq. (3)] is 173.8 degrees. However, as pointed out in Sec. 1, the amplitude of the integrated field is not very sensitive to the effective rotation angle in a half-length magnet. Even small measurement errors could lead to a very large error in the value of the effective rotation angle. For example, the integral dipole field expected for a 180 degrees effective rotation angle instead of 173.8 degrees is 2.289 T.m, which differs from the measured value by only 0.13%, well within the experimental errors.

5. Simulating "Experimental" Data in the Full-length Magnet

As pointed out above, the experimental data in the half-length magnet does not provide us with a reliable estimate of the effective rotation angle. It is desirable, therefore, to have experimental data in a full length magnet. However, by the time such data will be available, it will perhaps be too late to make any changes, if needed, to the length of the magnet. It is thus desirable to obtain a reliable estimate of the effective rotation angle in the full length magnet from the data in the half length magnet, HRC001. Such an estimate could also be compared to the numerical analysis by Okamura [3].

A reasonable simulation of the actual full length magnet can be made by assuming that the ends of the magnet are identical to those in HRC001, but the straight section is longer by one half-period. The "experimental" data in such a full length magnet can be generated by dividing the actual data in HRC001 into two halves at any location in the body of the magnet, and then sandwiching a straight section with a uniform rate of twist and a constant dipole field amplitude between the two halves. The field amplitude of this extra section is given by the average amplitude over the central region of the prototype HRC001. The phase angle at various points in this region can be calculated from the nearest data point in HRC001 and the known rate of twist. With this extra half length sandwiched between the two halves of the measured data, the phase angles of the second half will be shifted by 180 degrees from the actually measured values and the *Z*-positions will change by half a wavelength. In this way, one can generate a very good approximation to the experimental data that would have been recorded, had the magnet been of a full length.

Fig. 2 shows the simulated data at 220A for a full length magnet, generated by a procedure as described above. In order to be consistent with the reference frame used in numerical simulations, the origin in the experimental data has been shifted to bring the

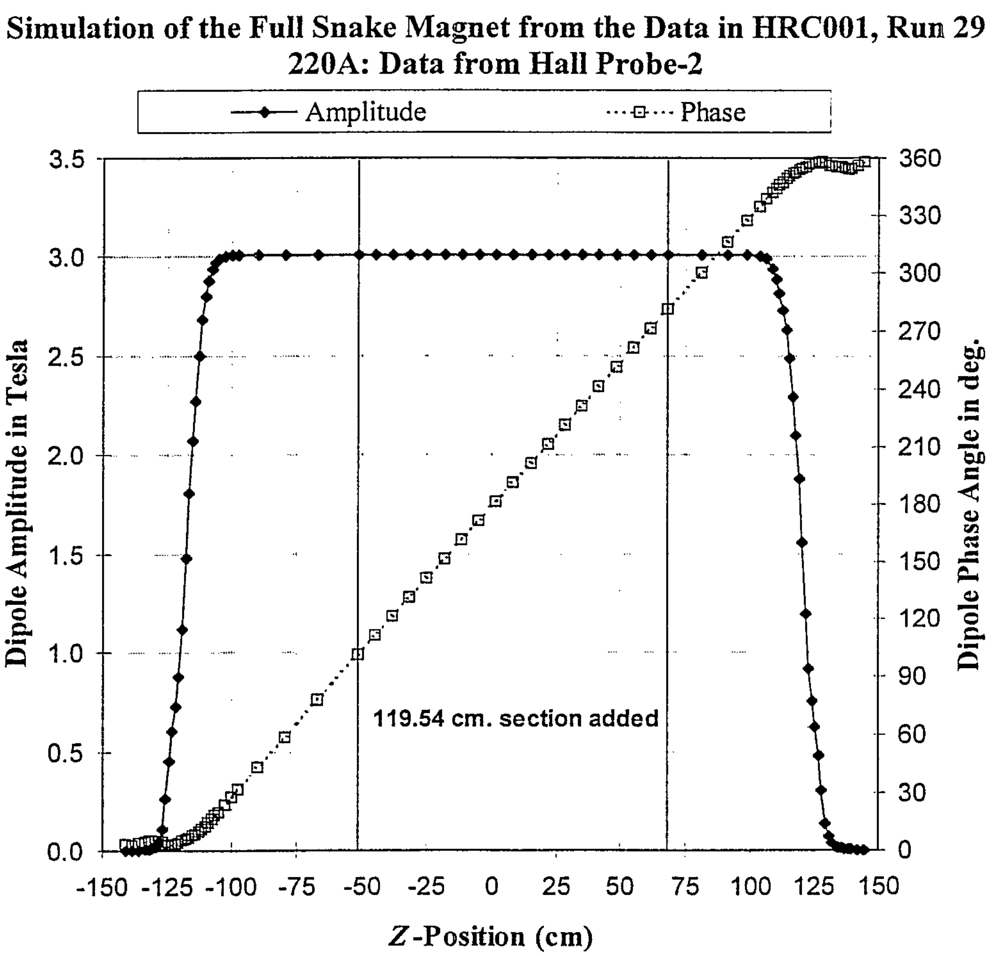

Fig. 2 Simulated data at 220A in a full length helical magnet. The *Z*-axis is inverted from the raw data in Fig. 1 to obtain a right handed coordinate system.

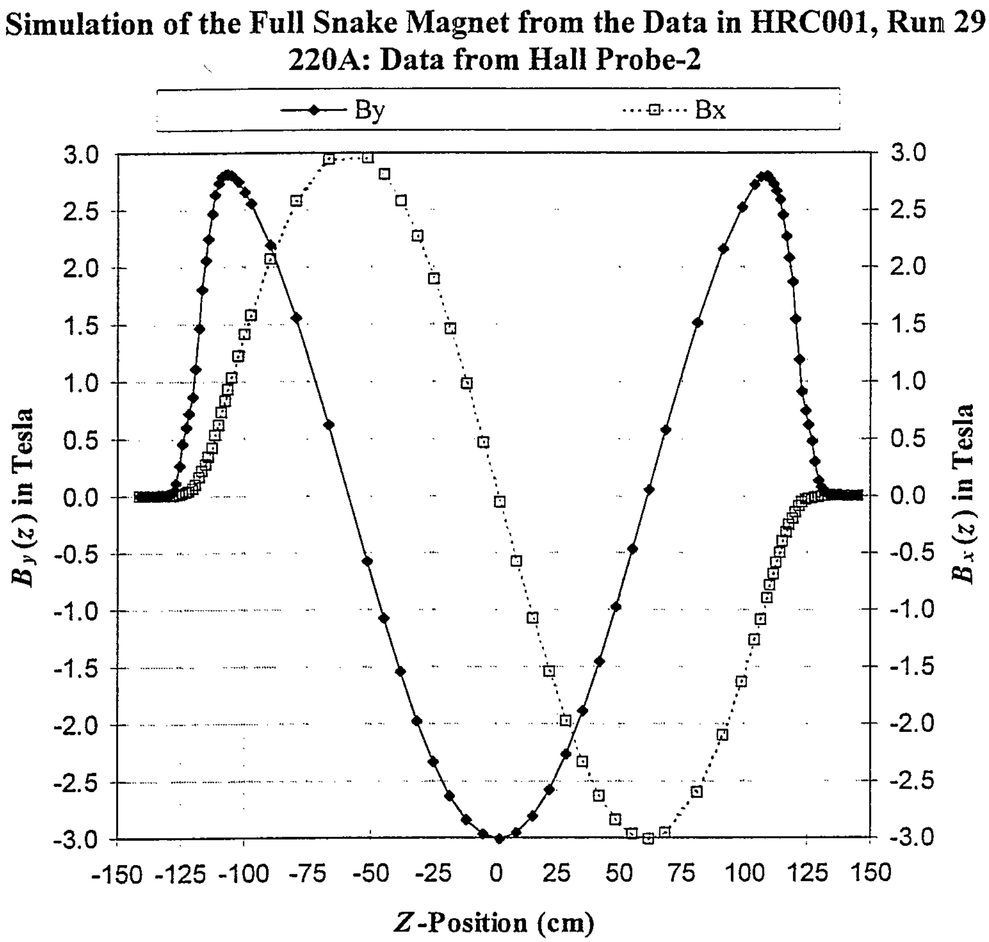

Fig. 3 Simulated data for $B_x(z)$ and $B_y(z)$ at 220A in a full length helical magnet.

magnet center at Z=0. Also, the reference frame is rotated to make the integrated B_x component zero. Furthermore, the Z-axis is inverted to make the coordinate system right handed. The two transverse components of the dipole field are shown in Fig. 3. It is seen that with this choice of a reference frame, the dipole field is in the positive y-direction near the ends of the magnet [$\alpha_0 \approx 0$ in Eqs.(1) and (2)]. The length of the section added is 1.2 m at room temperature, or 1.1954 m at liquid helium temperature. A twist rate of 1.5057 degrees/cm gives a rotation of exactly 180 degrees over this extra section.

The values of $\int B_y(z)dz$ and $\int B_x(z)dz$ are obtained by numerically integrating the data in Fig. 3. This integration was carried out in three separate regions – the two "end" regions made up of the two halves of the actually measured data, and the "central" region consisting of the additional half length in the middle of the magnet. The integrated field in the "central" region can also be calculated analytically using Eqs. (1) and (2). The analytical result was used to check the accuracy of the numerical integration procedure. Since the measured data points are at non-uniform Z-intervals, it was convenient to use a piecewise linear approximation to the actual curve. Since the final result of integration is expected to be a small number, even small errors in integration can lead to large errors. Based on a comparison with the analytical value in the central region, a linear approximation between data points was found to be unsatisfactory. A 3-point integration formula for arbitrary increments was developed for a more precise integration. If the three distinct points are given by x_1, x_2 and x_3, then the integration of any function from x_1 to x_3 can be shown to be given by:

$$\int_{x_1}^{x_3} f(x)\,dx \approx a_1 f(x_1) + a_2 f(x_2) + a_3 f(x_3) \tag{4}$$

where,

$$a_1 = \frac{(x_3 - x_1)}{6(x_2 - x_1)}[2(x_2 - x_1) - (x_3 - x_2)] \tag{5}$$

$$a_2 = \frac{(x_3 - x_1)^3}{6(x_2 - x_1)(x_3 - x_2)} \tag{6}$$

$$a_3 = \frac{(x_3 - x_1)}{6(x_3 - x_2)}[2(x_3 - x_2) - (x_2 - x_1)] \tag{7}$$

Eq. (4) gives an exact integration for functions up to the second order in x. This integration formula gave the correct integral field for the central region. The end regions were integrated numerically using Eq. (4), but the analytical value was used for the central region, even though a numerical integration also gave the same result.

6. Results

The results of calculations from various sets of data are summarized in Table I. The integration of simulated data in Fig. 3, as described in the previous section, gave an integrated dipole field, C_1, of 193.96 Gauss.m at a central field of 3 Tesla, corresponding to an effective rotation angle of $\theta_R = 360.97$ deg. It should be recalled that the integrated x-component is made zero by an appropriate choice of reference frame. It is interesting to

Table I. Summary of rotation angles calculated for the full length magnet

Current (A)	**Hall Probe**	**Central Field B_0 (T)**	$\int B_y(z)dz$ **(Gauss.m)**	$\int B_x(z)dz$ **(Gauss.m)**	**Integrated dipole Ampl. C_1 (Gauss.m)**	**Rotation Angle θ_R (deg.)**
105	1	1.438	0.707	93.908	93.911	360.98
	2	1.441	12.267	92.716	93.524	360.98
220	1	3.007	208.06	0.00	208.06	361.04
	2	3.008	193.96	0.00	193.96	360.97

estimate the uncertainty in the calculated value of the effective rotation angle. Assuming a ±0.2% error in the calculation of the integrated field strength (based on the 0.13% discrepancy seen in the half length magnet data), the maximum error in the effective rotation angle is only ±0.002 degree. Even if a rather large error of ±10% in the integral field is assumed, the error in the calculation of the effective rotation angle is only ±0.10 degree. This is in sharp contrast with the case of the half length magnet where even a 0.1% uncertainty leads to an error of several degrees.

A similar analysis has been performed on the data at 220A from Hall probe-1, located at a radius of 0.66 cm. The value of effective rotation angle calculated from this probe is 361.04 degrees. This value is in agreement with the previous result from probe-2. The small difference could be due to slightly different calibrations of the two probes.

Even though the data at 105A had its own problems, an analysis was nevertheless performed to obtain the effective rotation angle at this current also. A value of 360.98 degrees was obtained with the data from either of the two probes. These values are consistent with those obtained at 220A. However, in the case of the 105A data, the integrated x-component does not vanish. In fact, with a reasonable choice of reference frame, most of the integrated field is in the x-direction. This result is not as expected, and points towards errors in the Z-position and/or the phase angles in data taken at 105A on different days.

A full three dimensional analysis of the field in the helical magnet has been carried out by Okamura [3] to optimize the rotation angle. His analysis suggests that the effective rotation angle of the full length magnet with the present end design should be about 360.75 degrees, which is close to the 360.97 degrees estimated here from the simulated data. His preliminary analysis with the modified ends shows that the effective rotation angle in the as-built full length magnet will be smaller by at least 0.25 degree. This should significantly reduce the value of the integrated dipole field from the present estimate. Even the value of 194 Gauss.meter for the integral field at a central field of 3.0 Tesla is well within the tolerance of 500 Gauss.meter at 4.0 Tesla proposed by Syphers [4]. Based on the present analysis, it is believed that no change in the length of the production magnets is necessary at this stage.

References

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