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Numerical Study of Spin Depolarization in RHIC Due to Beam-Beam Collision

Y. Batygin

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Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

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Alternating Gradient Synchrotron Department
Relativistic Heavy Ion Collider Project
BROOKHAVEN NATIONAL LABORATORY
Upton, New York 11973

Spin Note

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Y. Batygin and T. Katayama

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1. Beam parameters

Effect of beam-beam interaction on spin in RHIC was studied with beam parameters, presented in Table 1.

Table 1. Parameters of the interacted beams

Colliding particles	proton
Particle energy	250 GeV
Normalized emittance, 95%	1 π cm mrad
Rms beam size at interaction point (IP), σ	0.08 mm
Collision angle	0
Number of IP's	2
Beam-beam tune shift per collision ξ	0.0125
Betatron tune per turn	$Q_x = 28.19$
	$Q_y = 29.18$

Accelerator ring contains two interaction points (IP). In simulation, ring is represented as a combination of arc lattice and two short nonlinear lenses, which describe interaction points (see Fig. 1).

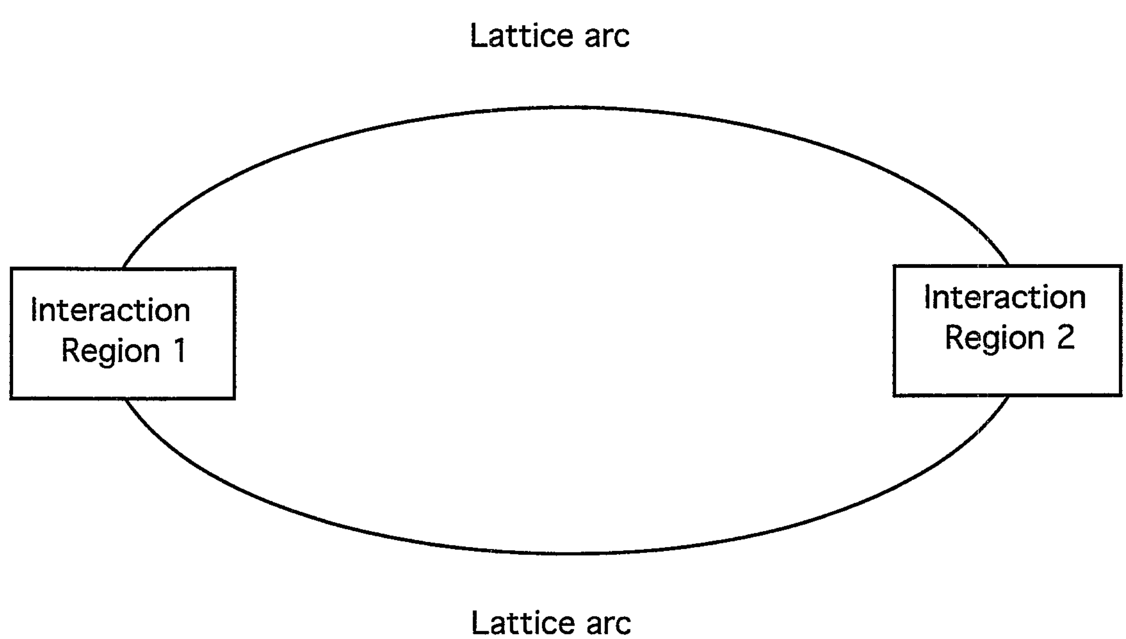


Fig. 1. Accelerator ring with 2 interaction points.

2. Numerical model

We use a two-dimensional particle model in coordinates $(x, p_x = \beta^* \frac{dx}{dz})$, $(y,\ p_y = \beta^* \frac{dy}{dz})$, where β^* is a beta-function of the ring. The ring is supposed to have two colliding points, therefore the tune shift between sequent beam-beam interactions is equal to half of their values $Q_x^* = \frac{Q_x}{2} = 14.095$; $Q_y^* = \frac{Q_y}{2} = 14.59$. Particle motion between subsequent collisions combines linear matrix with betatron angle $\theta_x = 2\pi Q_x^*$ and matrix of beam-beam interaction, treated as a thin nonlinear lens

$$\begin{pmatrix} x_{n+1} \\ p_{x,\,n+1} \end{pmatrix} = \begin{pmatrix} \cos\theta_x & \sin\theta_x \\ -\sin\theta_x & \cos\theta_x \end{pmatrix} \begin{pmatrix} x_n \\ p_{x,\,n} + \Delta p_{x,\,n} \end{pmatrix}, \tag{1}$$

analogously for $(y,\ p_y)$. Beam-beam kick Δp_x is expressed as a result of interaction of test particle with opposite beam with Gaussian distribution function

$$\Delta p_x = 4\pi\xi x \frac{1 - \exp(-\frac{r^2}{2\sigma^2})}{(\frac{r^2}{2\sigma^2})}, \tag{2}$$

and similar for Δp_y. Parameter ξ is a beam-beam parameter, which characterizes strength of interaction :

$$\xi = \frac{N r_o \beta^*}{4\pi \sigma^2 \gamma}, \tag{3}$$

where N is a number of particles per bunch, $r_o = \frac{q^2}{4\pi\varepsilon_o mc^2}$ is a classical particle radius, σ is a transverse standard deviation and γ is a particle energy.

Advance of spin vector $\vec{S} = (S_x, S_y, S_z)$ is described by subsequent matrix transformation in lattice arc and in interaction point [1]. Matrix of spin advance in the arc between collisions is described by matrix of dipole magnet with bending angle $\delta\theta = \pi$:

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix} = \begin{vmatrix} \cos(\omega\delta s) & 0 & \sin(\omega\delta s) \\ 0 & 1 & 0 \\ -\sin(\omega\delta s) & 0 & \cos(\omega\delta s) \end{vmatrix} \begin{vmatrix} S_{x,o} \\ S_{y,o} \\ S_{z,o} \end{vmatrix},$$

$$\omega\delta s = (1+G\gamma)\,\delta\theta, \qquad G = 1.7928. \tag{4}$$

Spin advance after crossing the interaction point is described as follow:

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix} = \begin{vmatrix} 1- a(B^2+C^2) & ABa + Cb & ACa + Bb \\ ABa - Cb & 1- a(A^2+C^2) & BCa - Ab \\ ACa - Bb & BCa + Ab & 1- a(A^2+B^2) \end{vmatrix} \begin{vmatrix} S_{x,o} \\ S_{y,o} \\ S_{z,o} \end{vmatrix} , \qquad (5)$$

$$A = \frac{P_x}{P_o}, \quad B = \frac{P_y}{P_o}, \quad C = \frac{P_z}{P_o}, \quad P_o = \sqrt{P_x^2 + P_y^2 + P_z^2}, \qquad (6)$$

$$a = 1 - \cos (P_o\, \delta z), \quad b = \sin (P_o\, \delta z), \qquad (7)$$

where δz is an interaction distance, defined below. Vector $\vec{P} = (P_X, P_Y, P_Z)$ in case of head-on beam-beam collision is as follow:

$$P_x = \frac{1}{B\rho} \left[(1+G\gamma)B_x + (G\gamma + \frac{\gamma}{1+\gamma}) \frac{\beta E_y}{c}\right] ,$$

$$P_y = \frac{1}{B\rho} \left[(1+G\gamma)B_y - (G\gamma + \frac{\gamma}{1+\gamma}) \frac{\beta E_x}{c}\right] , \qquad (8)$$

$$P_z = 0 \quad ,$$

where $B\rho$ is a rigidity of particles, $\vec{E} = (E_x, E_y, 0)$ is an electrical field and $\vec{B} = (B_x, B_y, 0)$ is a magnetic field of the opposite bunch. Due to Lorentz transformations, components of electromagnetic field of the opposite bunch are connected via relationships

$$B_x = \beta \frac{E_y}{c}, \qquad B_y = - \beta \frac{E_x}{c} \quad . \qquad (9)$$

Taking into account, that interacted particles are ultra relativistic $\beta \approx 1$, $\gamma >> 1$ and particle rigidity is $B\rho = m_o c\beta\gamma/q$, the vector $\vec{P}$ is simplified:

$$P_x = \frac{q E_y}{m_o c^2 \gamma} \left[(1+G\gamma) + (G\gamma + \frac{\gamma}{1+\gamma})\right] \approx 2\, G \frac{q E_y}{m_o c^2} ,$$

$$P_y = \frac{q E_x}{m_o c^2 \gamma} \left[- (1+G\gamma) - (G\gamma + \frac{\gamma}{1+\gamma})\right] \approx - 2\, G \frac{q E_x}{m_o c^2} \quad . \qquad (10)$$

Let us express matrix parameters ($P_x\,\delta z$), ($P_y\,\delta z$) via beam-beam parameter ξ. Electrostatic field of the opposite bunch in the laboratory frame can be approximated as a round Gaussian bunch with length l and peak current $I = \frac{q\,N\,\beta c}{l}$:

$$E_r = \frac{q_1 N}{2\pi\varepsilon_o\, l\, r}\,[1 - \exp(-\frac{r^2}{2\sigma^2})] = \frac{I}{2\pi\varepsilon_o\,\beta c\, r}\,[1 - \exp(-\frac{r^2}{2\sigma^2})]; \qquad (11)$$

$$E_x = E_r\frac{x}{r}\,, \qquad E_y = E_r\frac{y}{r}\,. \qquad (12)$$

Substitution of the expression of electrostatic field into eq. (10) gives expression for vector $\vec{P}$

$$P_x = 4G\frac{I}{I_c}\frac{y}{r^2}\,[1 - \exp(-\frac{r^2}{2\sigma^2})]\,,$$
$$P_y = -\,4G\frac{I}{I_c}\frac{x}{r^2}\,[1 - \exp(-\frac{r^2}{2\sigma^2})]\,, \qquad (13)$$

where $I_c = 4\pi\varepsilon_o\, m_o\, c^3/q = (A/Z)\cdot 3.13\cdot 10^7$ Amp is a characteristic value of the beam current. Beam-beam parameter ξ in eq. (3) can be rewritten as follow:

$$\xi = \frac{\beta^*}{4\pi}\frac{I}{I_c}\frac{l}{\gamma\,\sigma^2}\,. \qquad (14)$$

To define the interaction distance δz, let us suppose, that at the time moment $t = 0$ test particle enters the opposite bunch (Fig. 2a). Equation of motion of test particle is $z_1 = v_1 t$. Equation of motion of the right edge of the bunch is $z_2 = l - v_2 t$. Test particle will leave opposite bunch, when $z_1 = z_2$, or after time interval $t = \frac{l}{v_1 + v_2}$ (Fig. 2 b). Therefore, the coordinate of test particle at this moment $z_1 = v_1 t$, which is equal to the interaction distance δz, will be

$$\delta z = v_1 t = l\,\frac{v_1}{v_1 + v_2} = \frac{l}{2}\,. \qquad (15)$$

Taking into account eqs.(14), (15), the parameters of spin matrix $P_x\,\delta z$, $P_y\,\delta z$ can be expressed as follow

$$P_x\,\delta z = P_x\frac{l}{2} = 4\pi\,G\,\gamma\,\xi\,\frac{y}{\beta^*}\,[\frac{1 - \exp(-\frac{r^2}{2\sigma^2})}{(\frac{r^2}{2\sigma^2})}]\,,$$
$$P_y\,\delta z = P_y\frac{l}{2} = -\,4\pi\,G\,\gamma\,\xi\,\frac{x}{\beta^*}\,[\frac{1 - \exp(-\frac{r^2}{2\sigma^2})}{(\frac{r^2}{2\sigma^2})}]\,. \qquad (16)$$

3. Results of simulations

Numerical simulation of spin depolarization in presence of beam-beam interaction was performed for two regimes:

(i) without noise in beam-beam kick (see Figs. 4-7) ,

(ii) with 1% noise in parameter σ, eq. (2) (see Figs. 8–11).

In previous note [2] it was demonstrated, that incorporation of noise in beam-beam kick results in slow diffusion of beam sizes and expansion of beam emittances.

Beam was represented as a combination of 5000 modeling particles. Particle distribution in real space (see Fig.3) as well as in phase space was Gaussian. Initial conditions for spin were chosen as follow:

$$S_x = 0, \qquad S_y = 1, \qquad S_z = 0 \ . \tag{17}$$

The given value of normalized beam emittance $\varepsilon = 1\ \pi$ cm mrad contains 95% of particles, which is supposed to be 6 times larger, than rms beam emittance

$$\varepsilon = 6\sqrt{<x^2><p_x^2> - <xp_x>^2} \ . \tag{18}$$

Therefore, the value of beta -function is defined via beam parameters as follow

$$\beta^* = \frac{6\,\sigma^2\,\gamma}{\varepsilon} = 1.02 \text{ m} \ . \tag{19}$$

In Fig. 4 stable particle motion in phase space in presence of beam-beam interaction without noise is shown. Particles move along tilted ellipses in phase space and trajectories remain closed, which results in stable values of beam envelopes and beam emittances (Fig.5). Spin distribution quickly stabilized and remains approximately the same (Figs. 6 a-d). Meanwhile, the tail of distribution of S_y component become longer (not visible in Figs. 6 a-d). It is confirmed by calculation of average values of spin components (Fig. 7). The average value of S_y component slowly decreases, while average values of S_x, S_z remain close to zero.

Introduction of noise in beam-beam kick results in beam-beam instability. Particle trajectories in phase space become unclosed (Fig. 8). Beam envelopes and beam emittances increase with time (Fig. 9). Spin distribution becomes wider, than in the case of absence of noise (Figs. 10 a-d). Decreasing of average of S_y component is faster and oscillation of average values of S_x, S_z around zero are larger (Fig. 11). But in both regimes the spin distribution after long-term interaction ($0.5 \cdot 10^6$ turns) is close to the initial spin distribution. Finally one can conclude, that beam-beam interaction has a little effect on spin depolarization.

References

1. T.Katayama, "Basic Theory of Spin Dynamics in RHIC", RIKEN Internal Note.
2. Y.Batygin and T.Katayama, "Beam-beam simulation at RHIC". Spin Note AGS/RHIC/SN No. 052, February 24, 1997

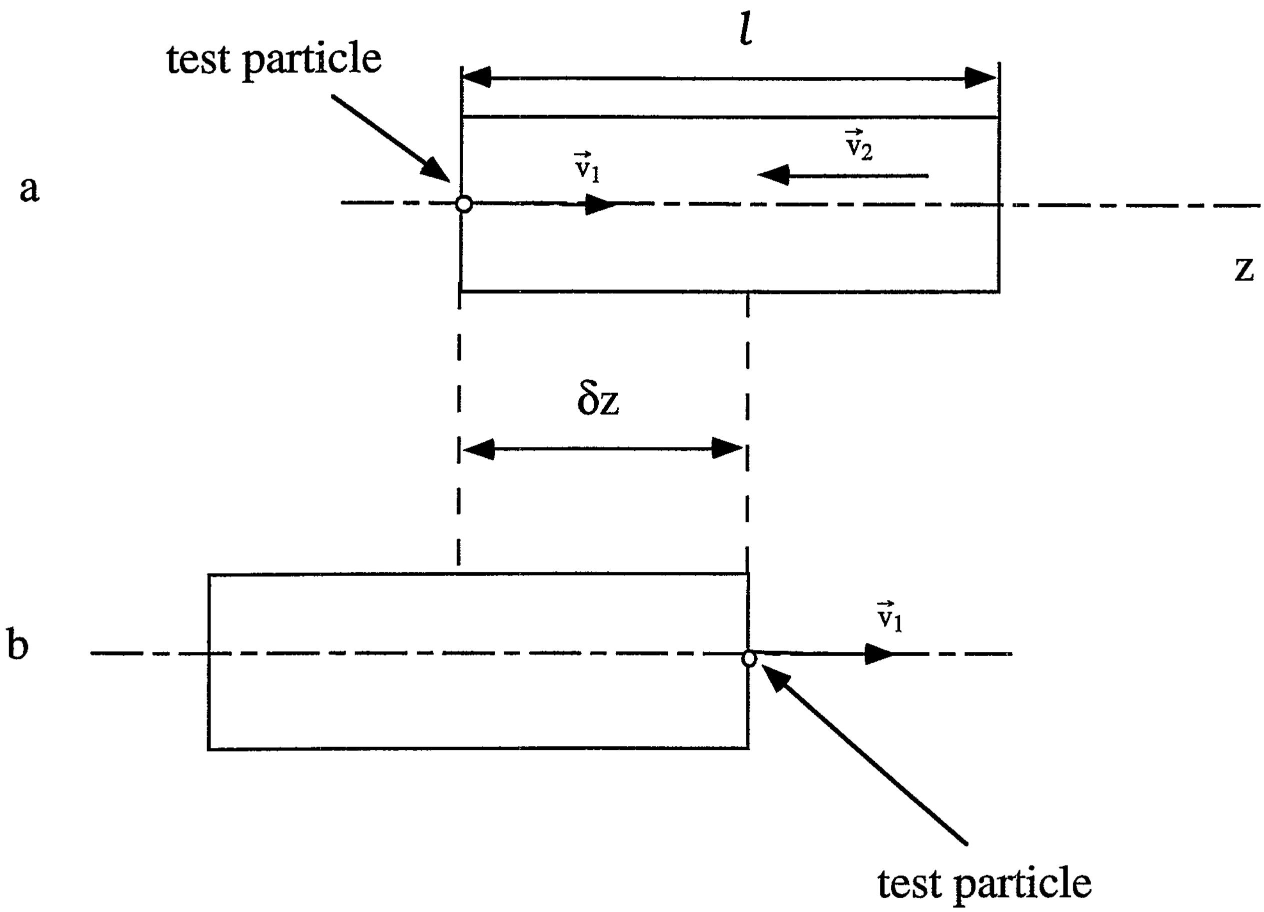
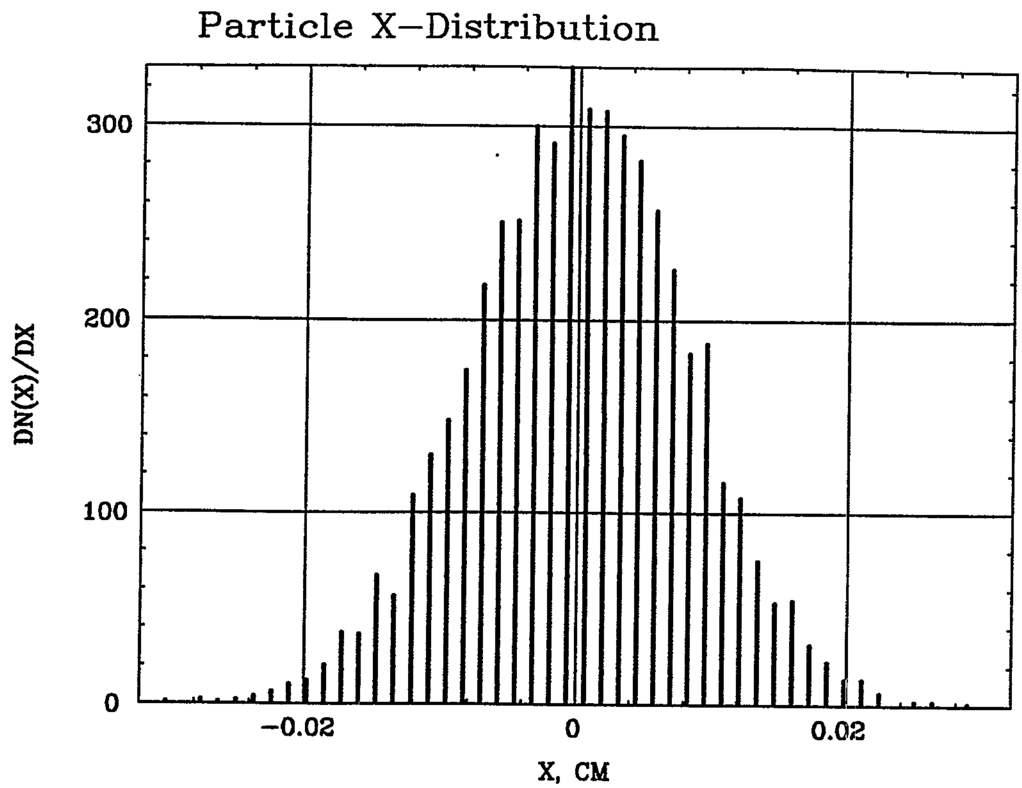


Fig. 2. Position of test particle with respect to opposite bunch:

(a) before interaction;

(b) after interaction.



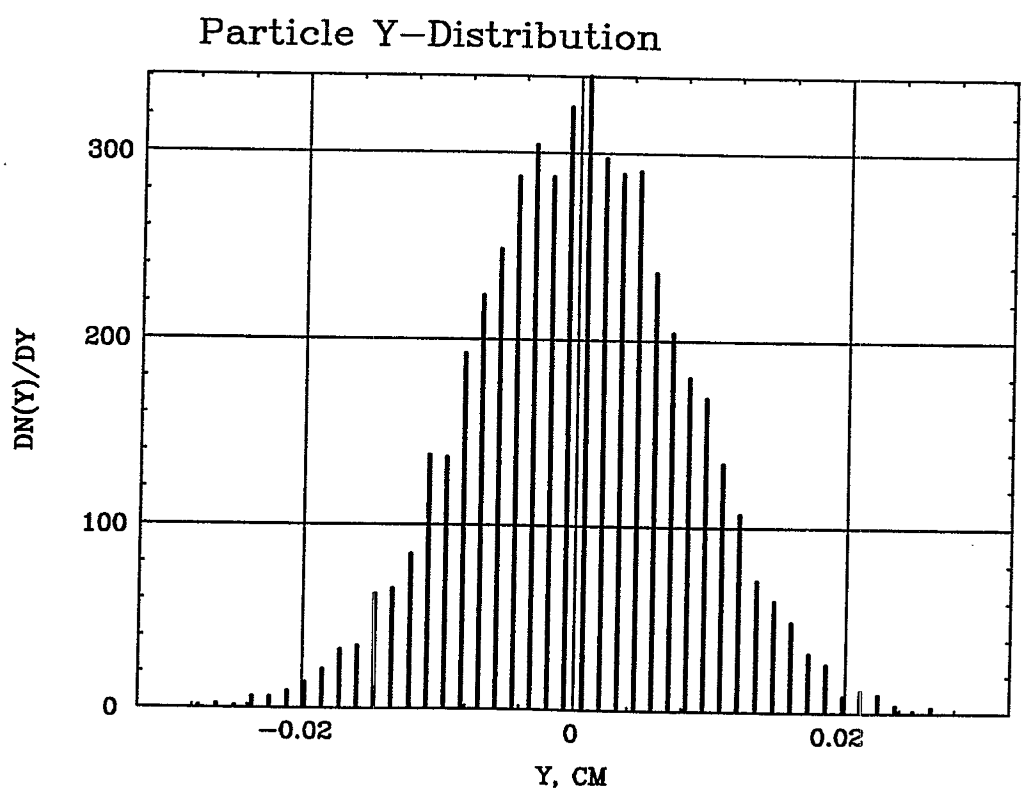
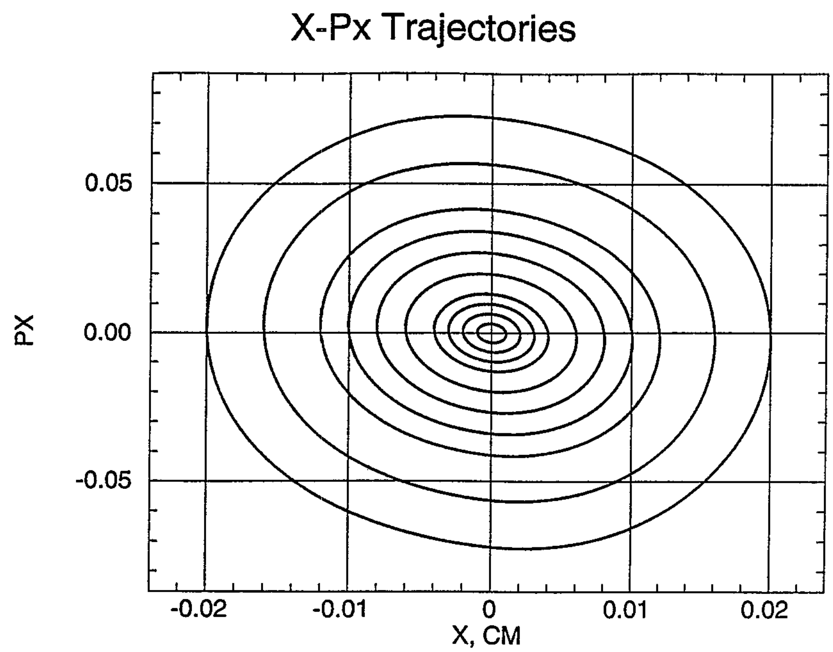


Fig. 3. Gaussian particle distribution in real space



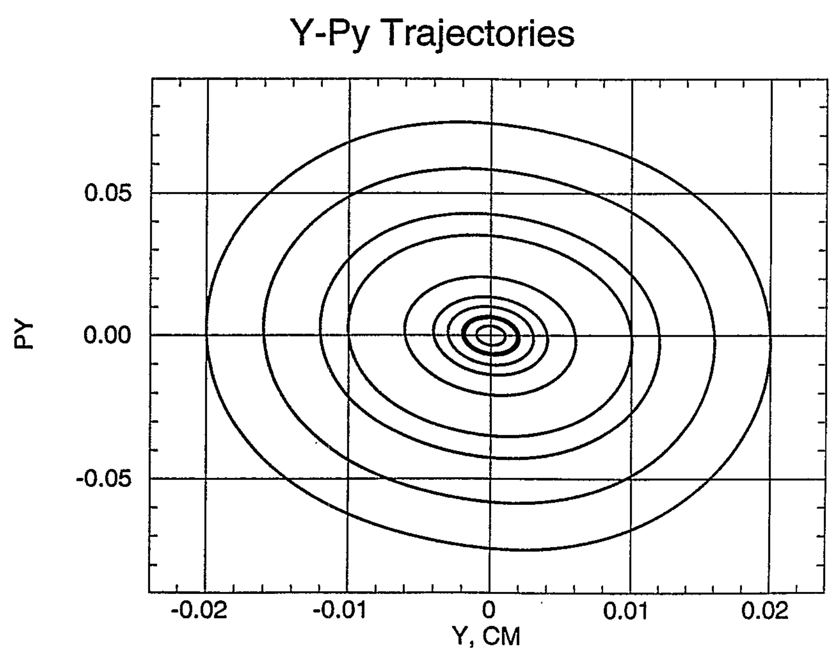


Fig. 4. Stable particle trajectories in phase space in presence of beam-beam interaction without noise.

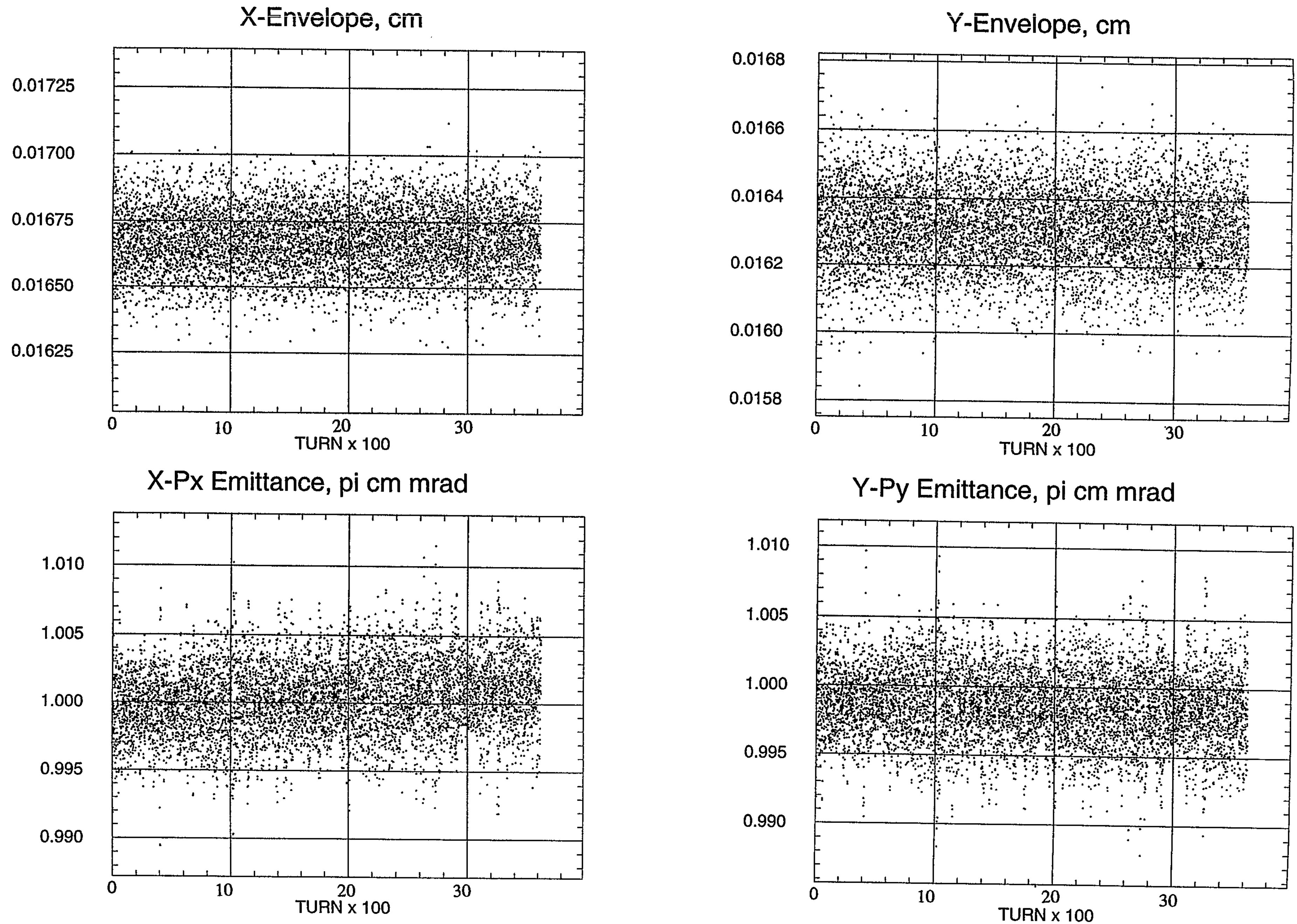


Fig. 5. Envelopes and beam emittances in presence of beam-beam interaction without noise.

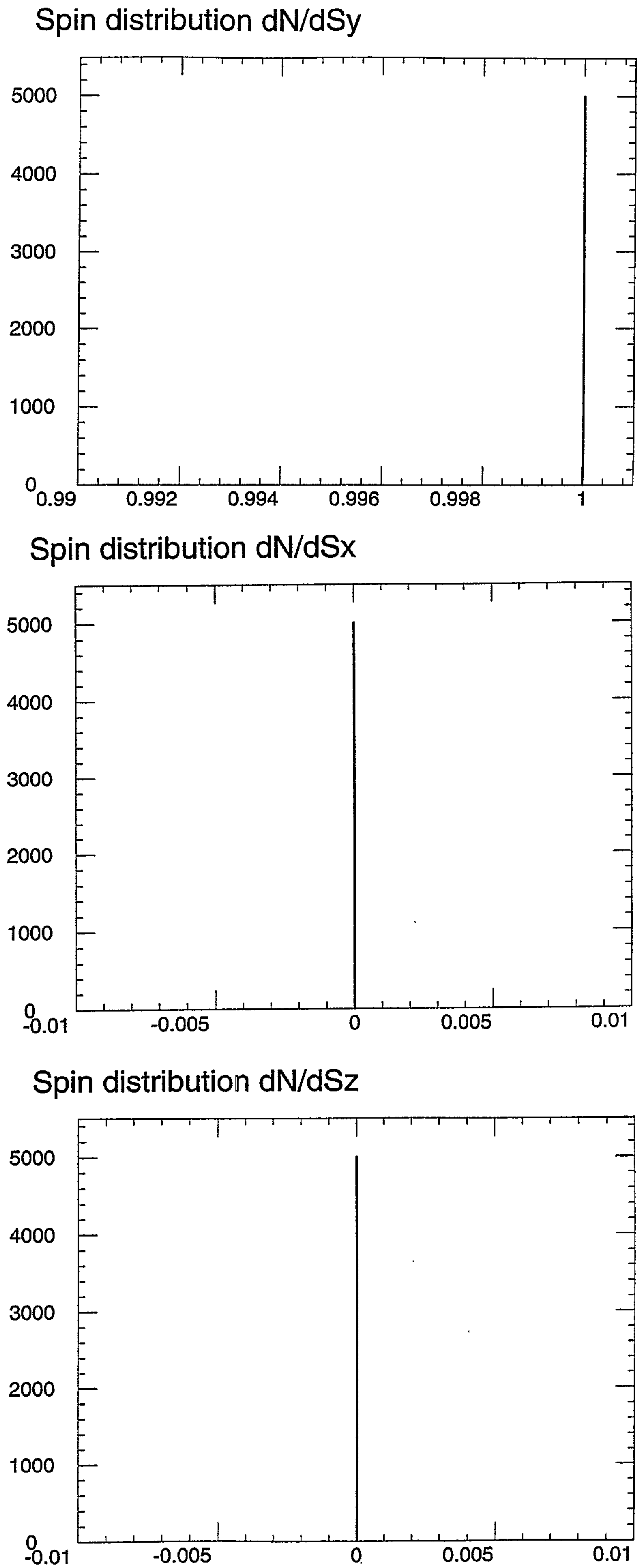


Fig. 6a. Initial spin distribution.

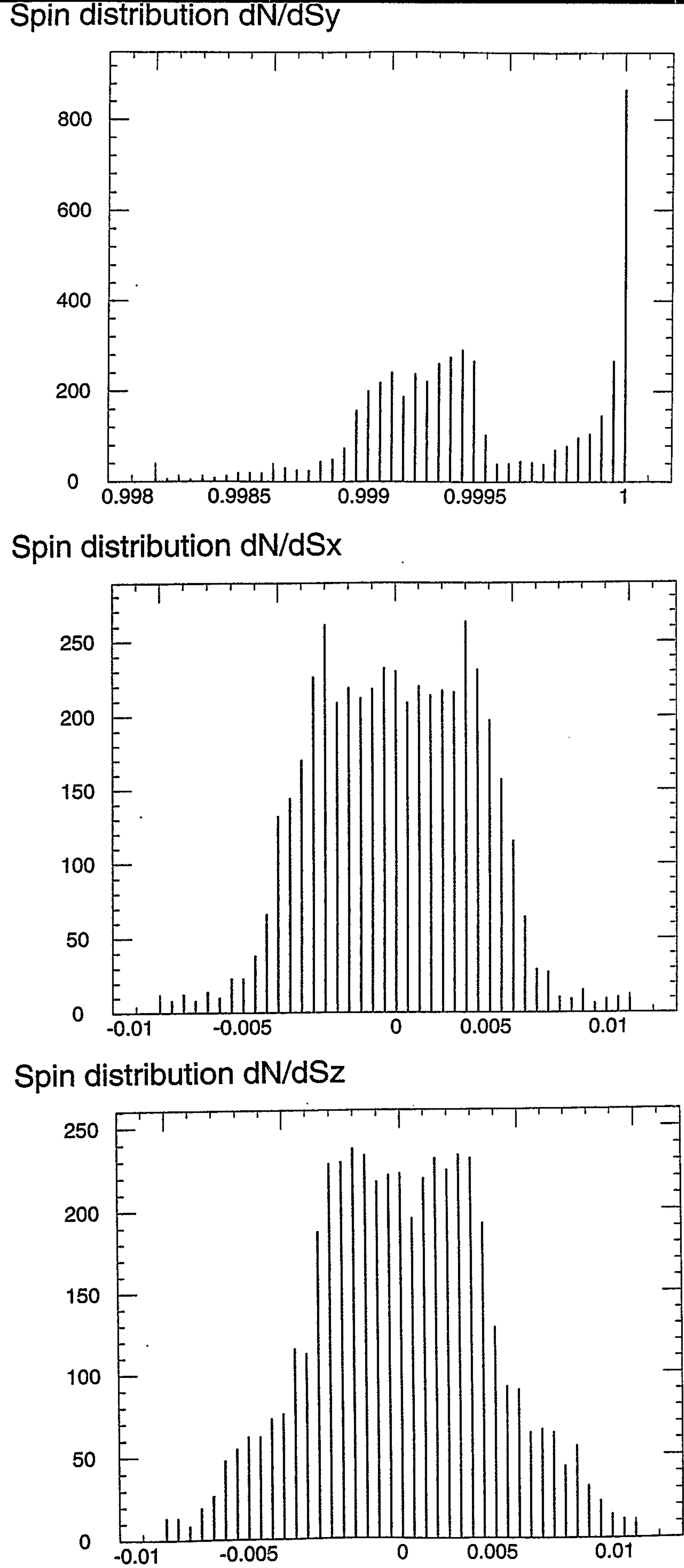


Fig. 6b. Spin distribution after 125 000 turns in presence of beam-beam interaction without noise.

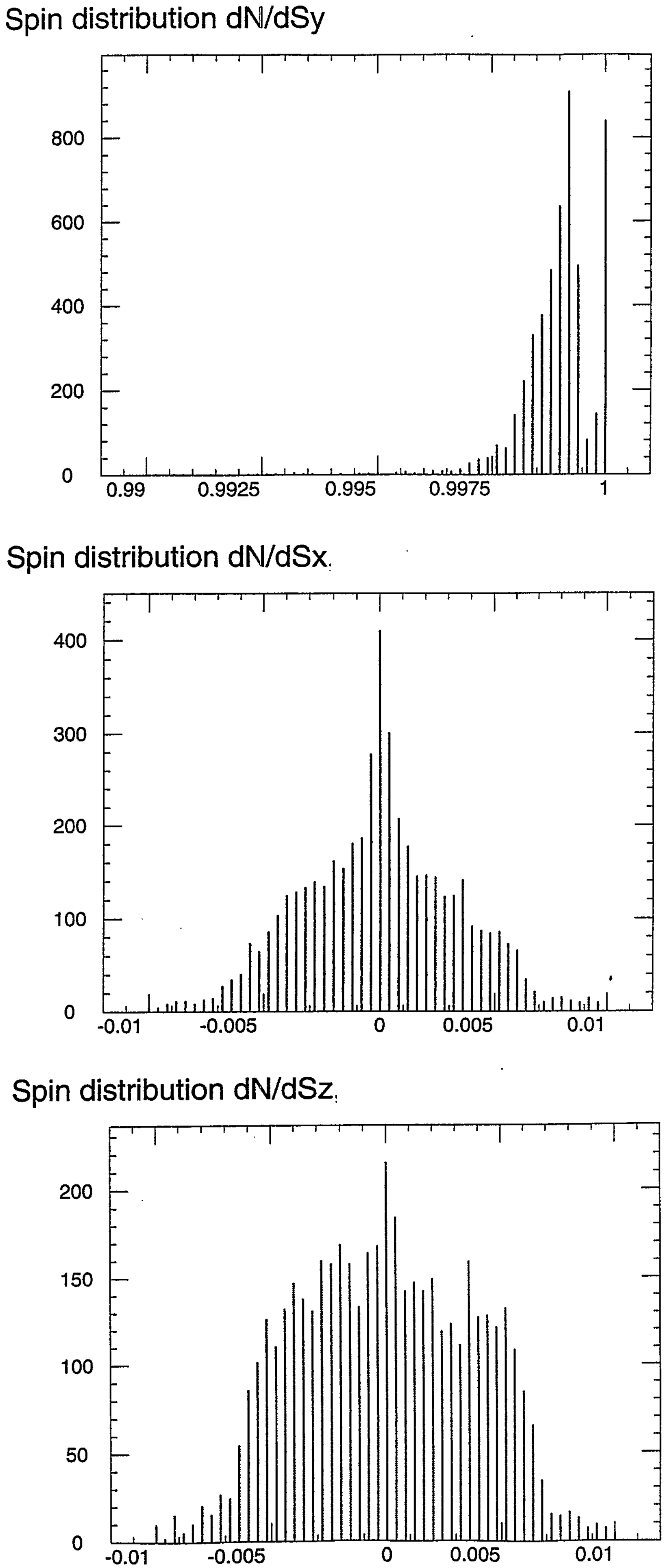


Fig. 6c. Spin distribution after 250 000 turns in presence of beam-beam interaction without noise.

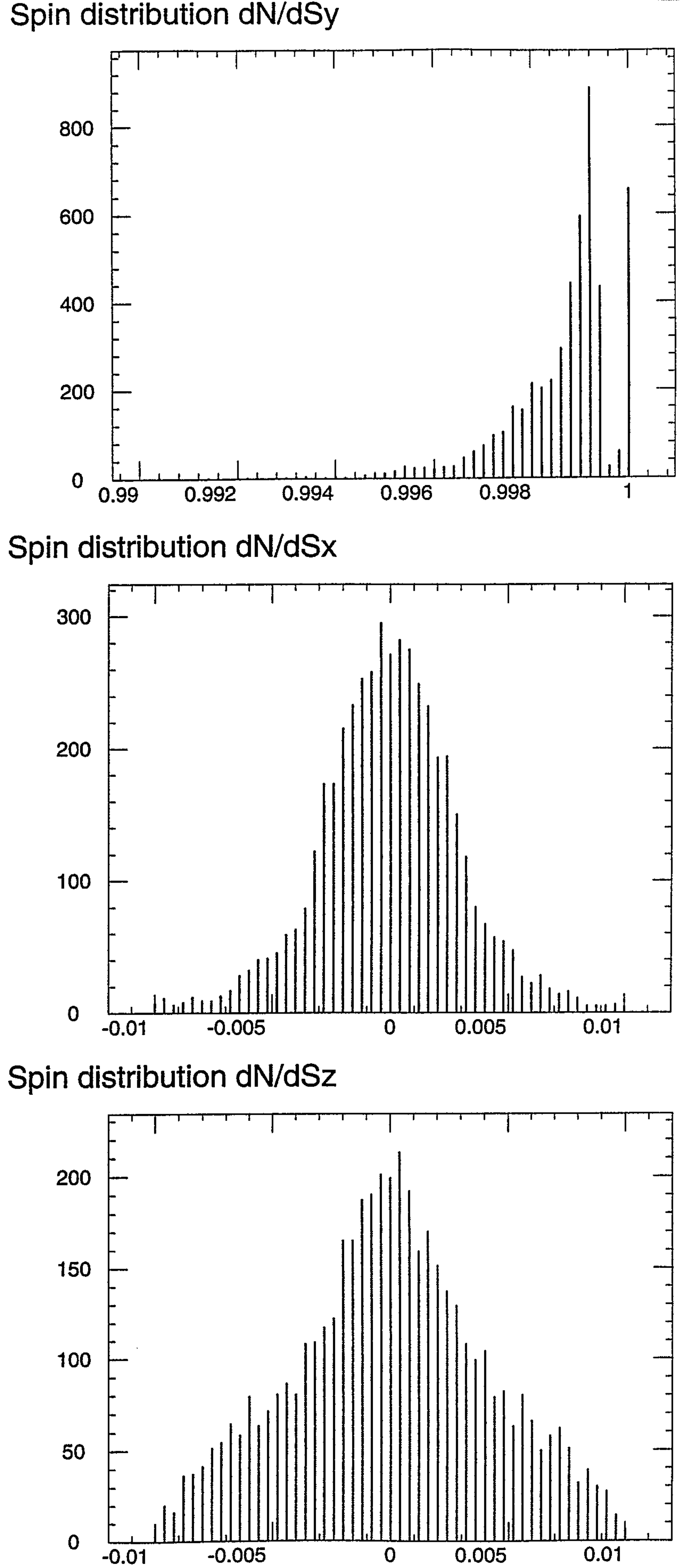


Fig. 6d. Spin distribution after 500 000 turns in presence of beam-beam interaction without noise

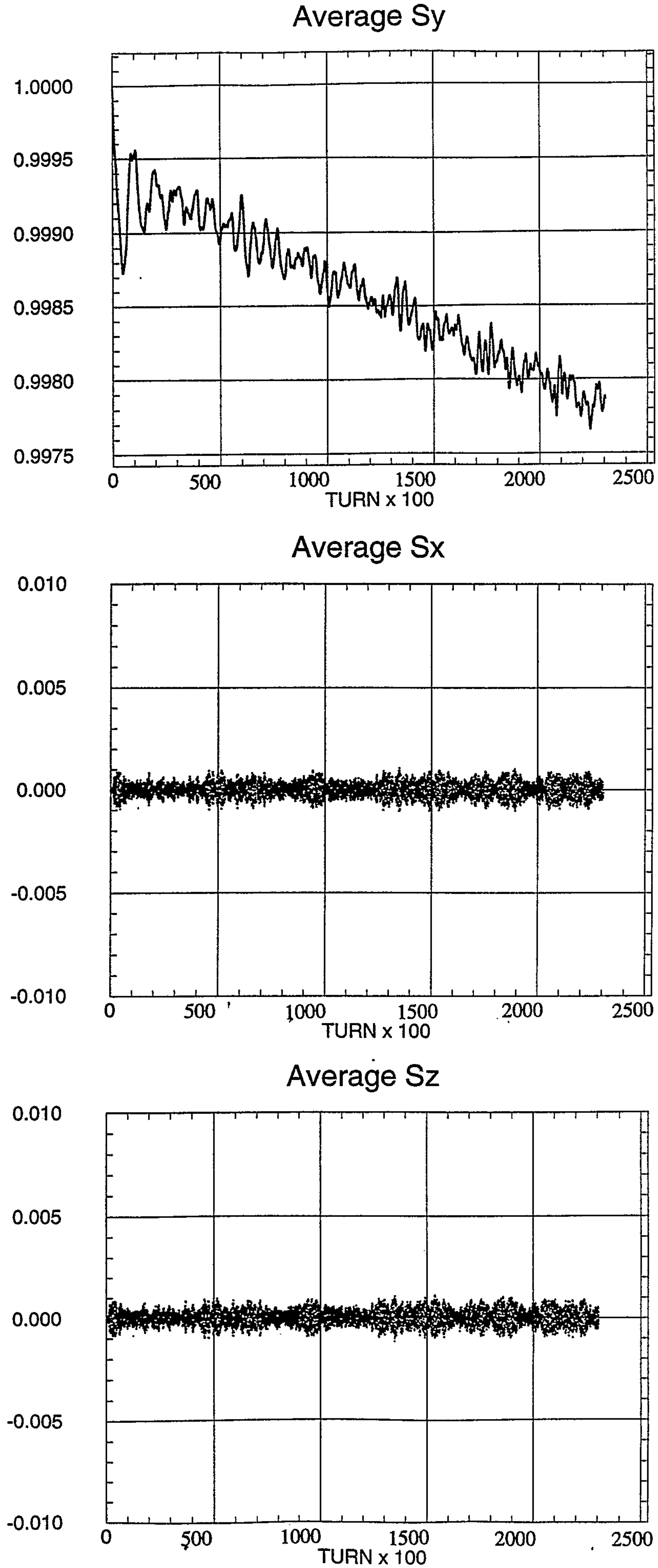
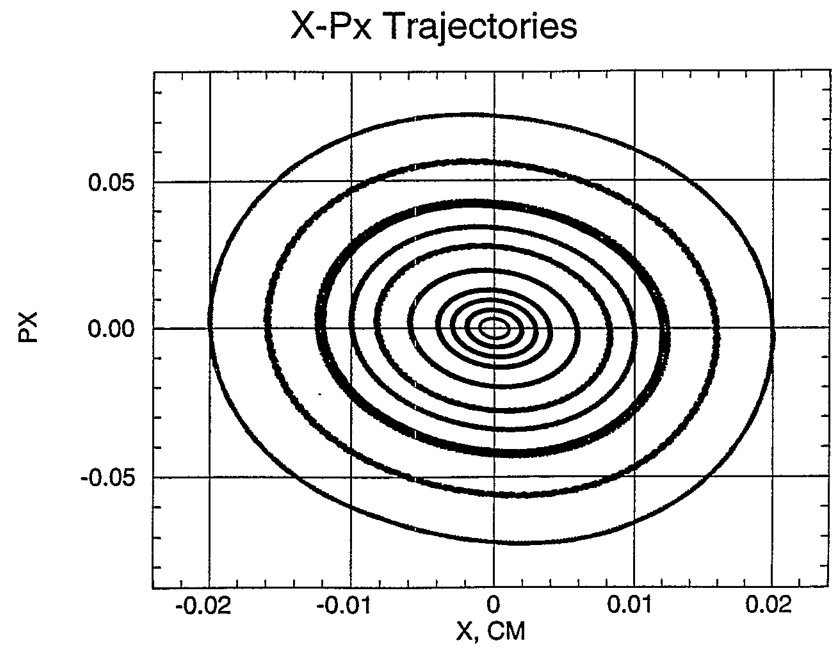


Fig. 7. Average values of spin components S_x, S_y, S_z as functions of turn number in presence of beam-beam interaction without noise .



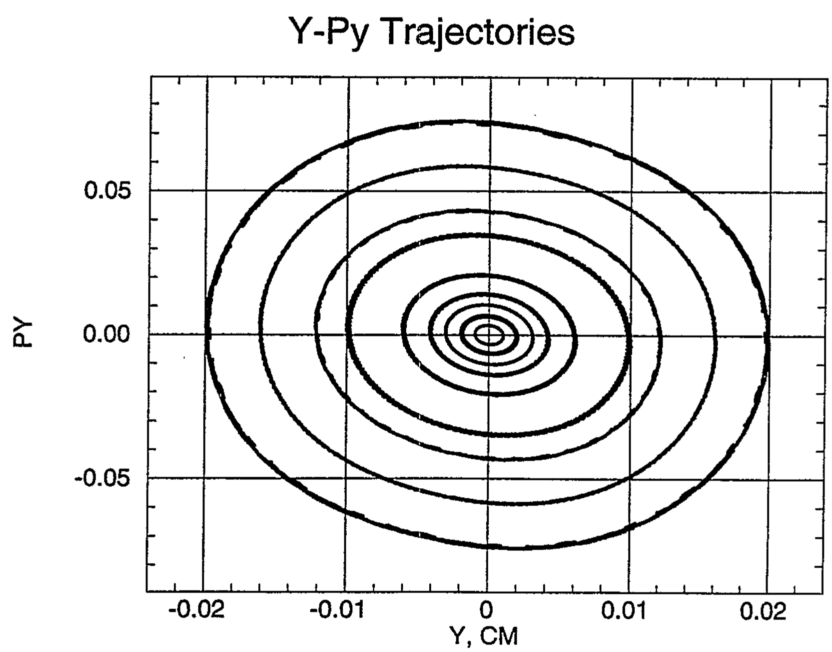


Fig. 8. Unstable particle trajectories in presence of 1% noise in parameter σ in beam-beam kick, eq. (2).

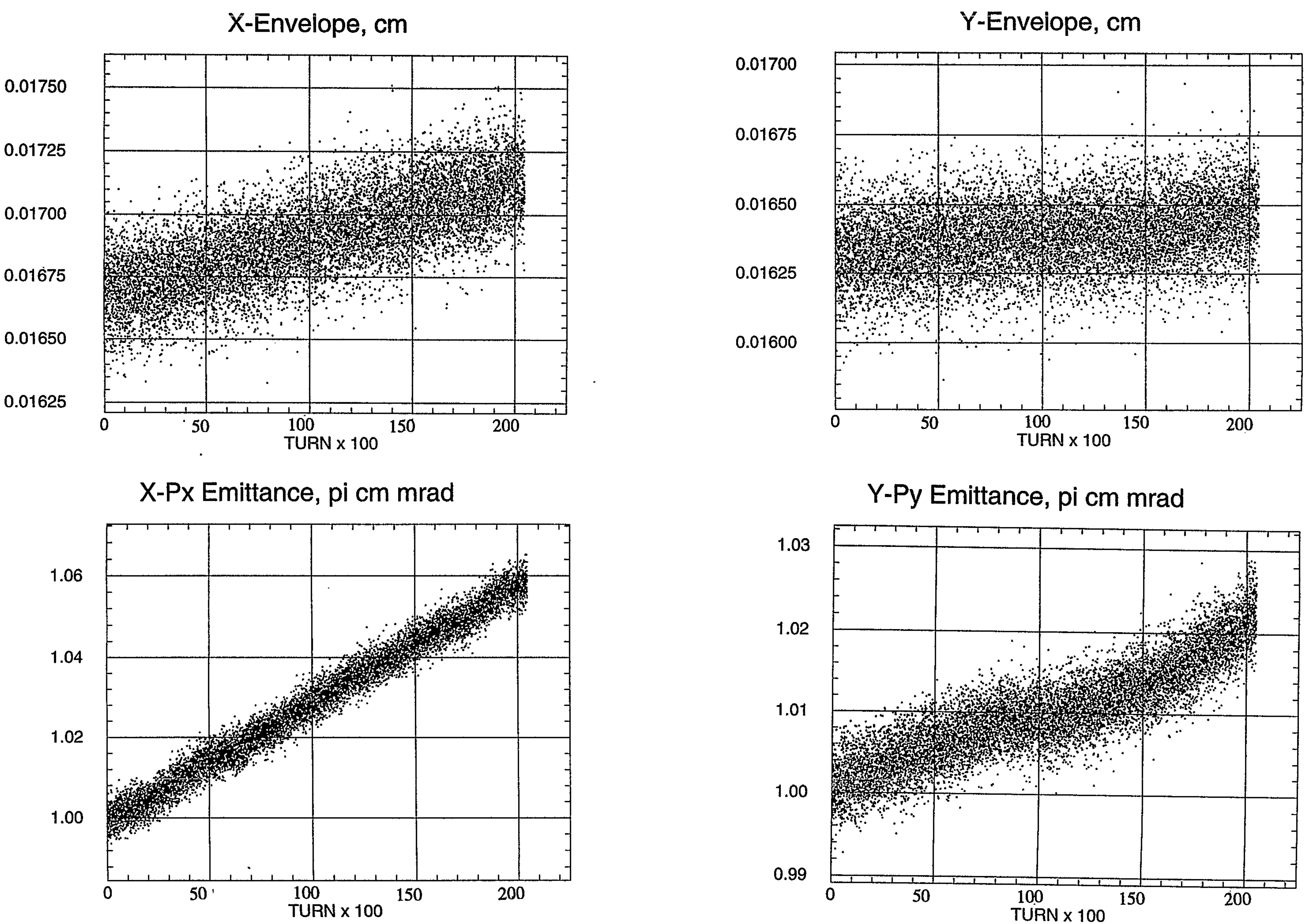


Fig. 9. Envelopes and beam emittances in presence of 1% noise in parameter σ in beam-beam kick, eq. (2).

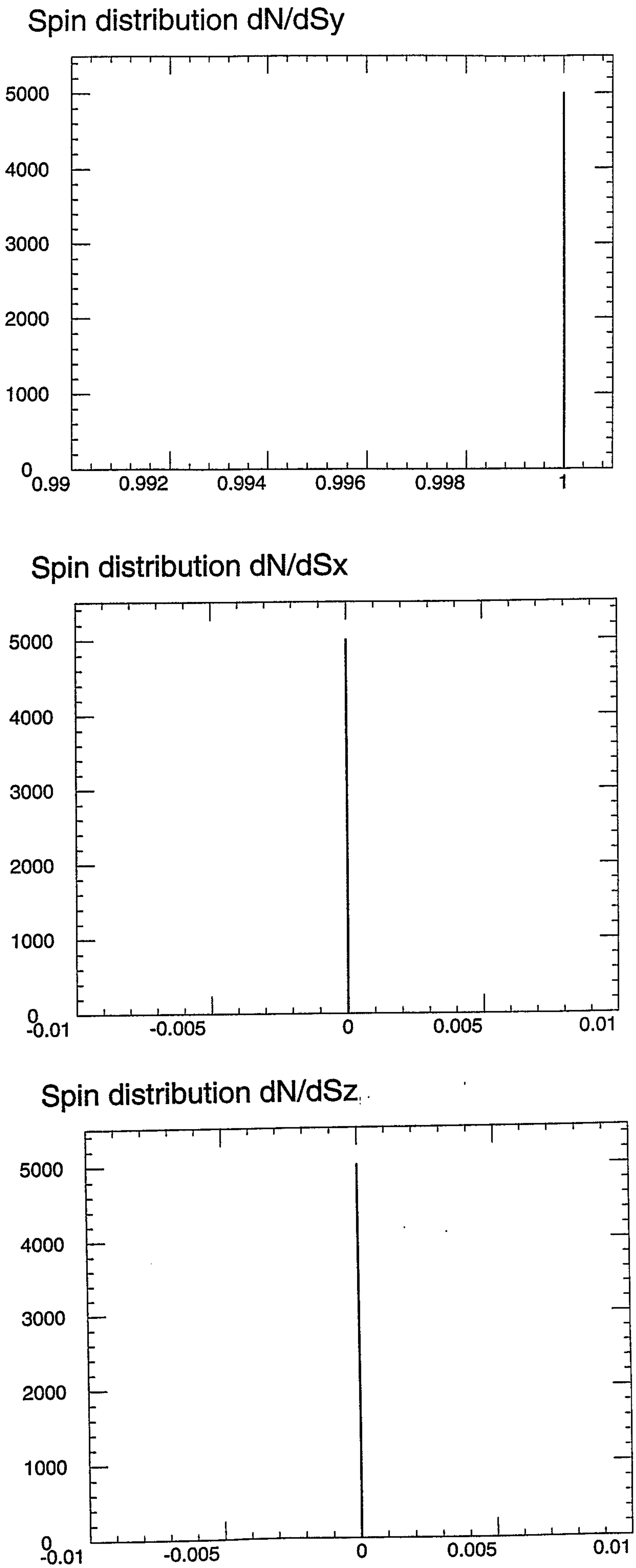


Fig. 10a. Initial spin distribution.

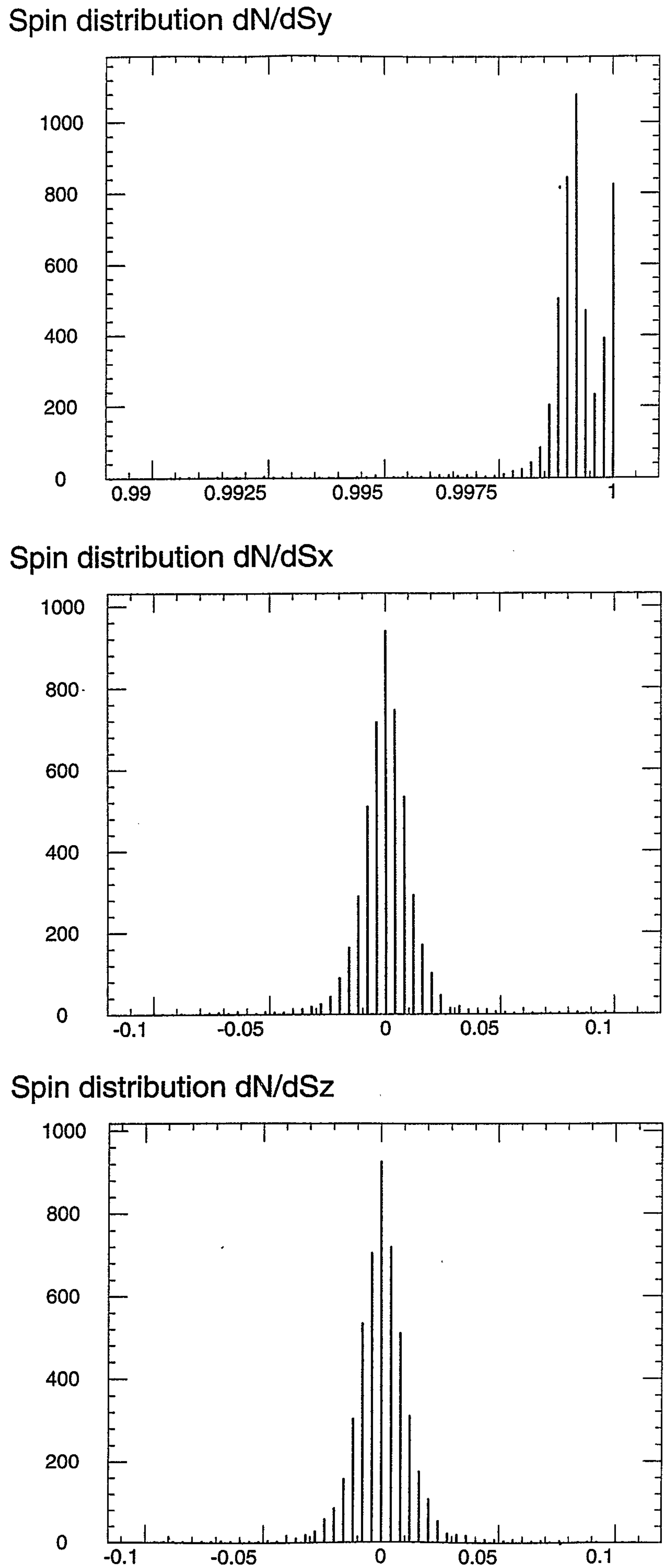


Fig. 10b. Spin distribution after 125 000 turns in presence of 1% noise in parameter σ in beam-beam kick, eq. (2).

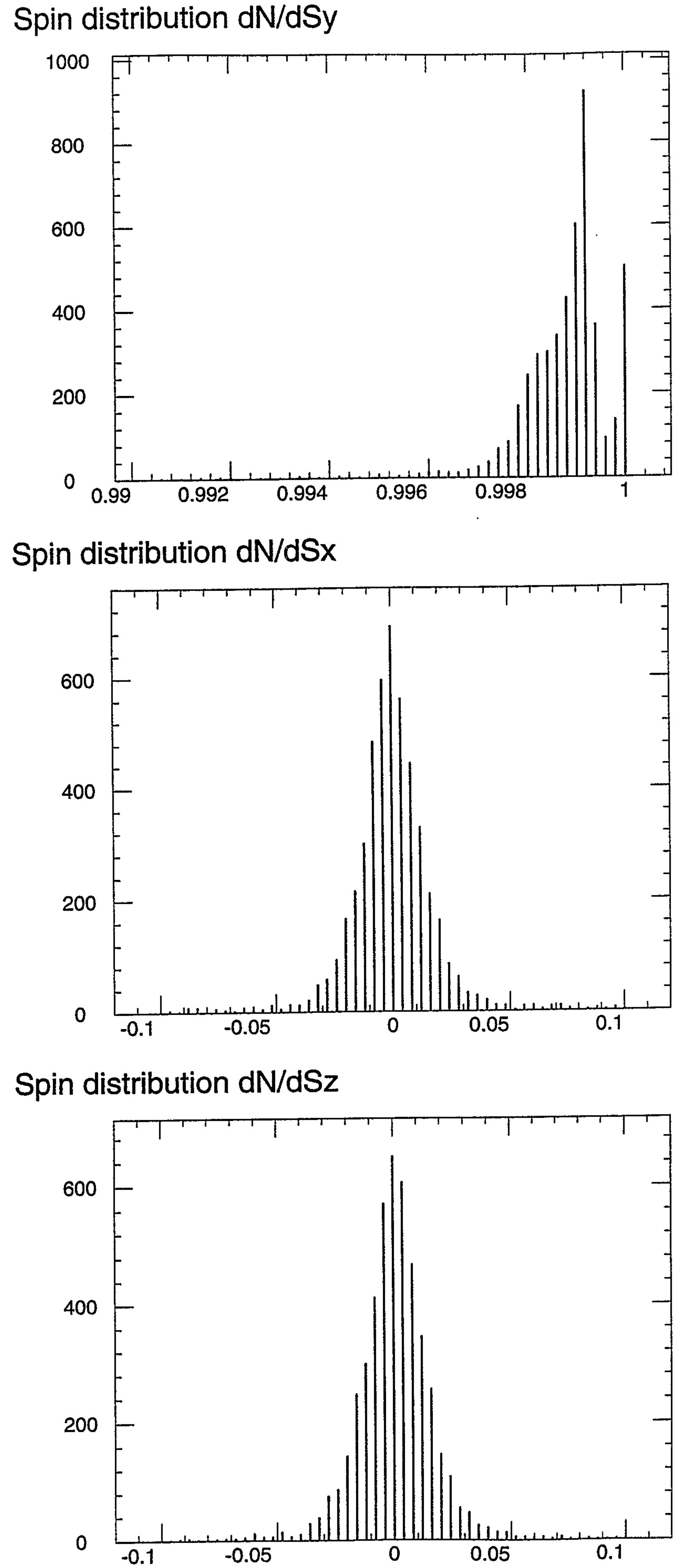


Fig. 10c. Spin distribution after 250 000 turns in presence of 1% noise in parameter σ in beam-beam kick, eq. (2).

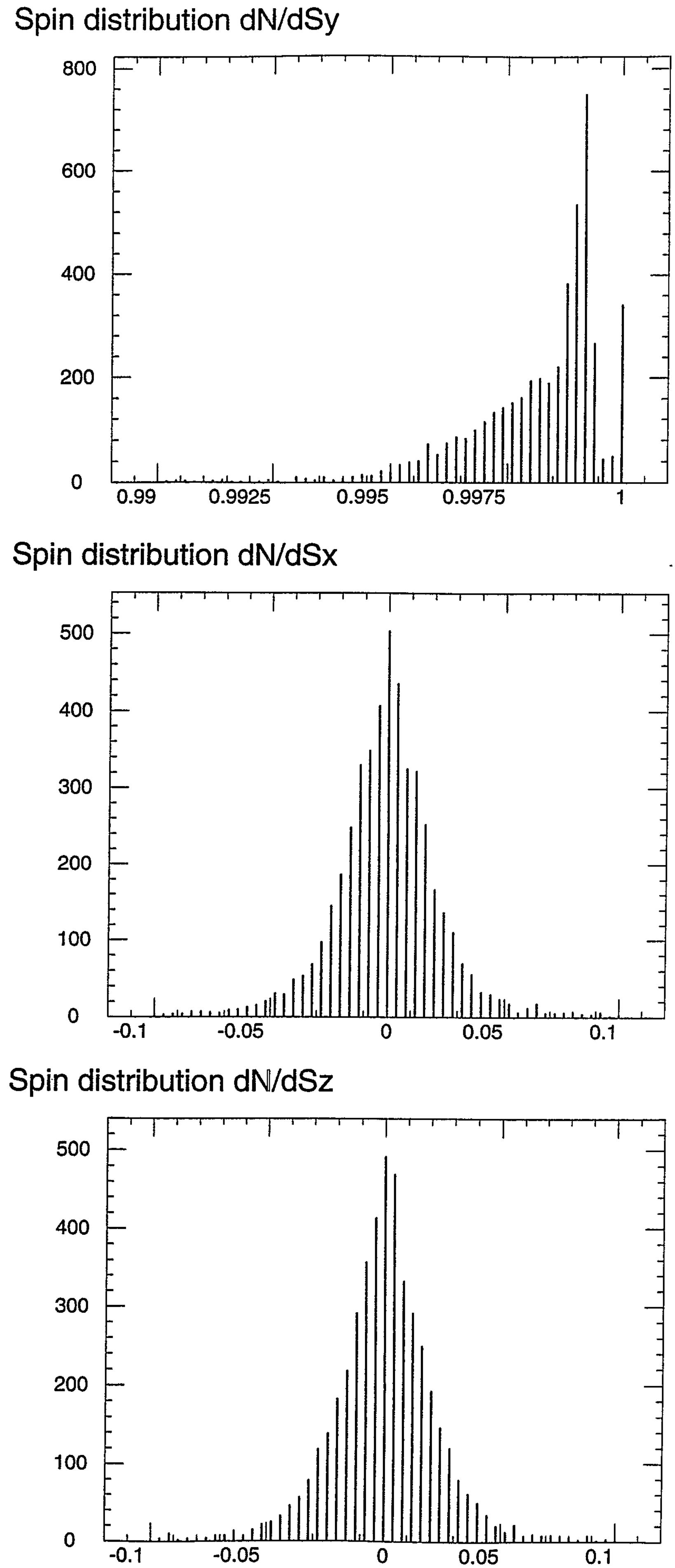


Fig. 10d. Spin distribution after 500 000 turns in presence of 1% noise in parameter σ in beam-beam kick, eq. (2).

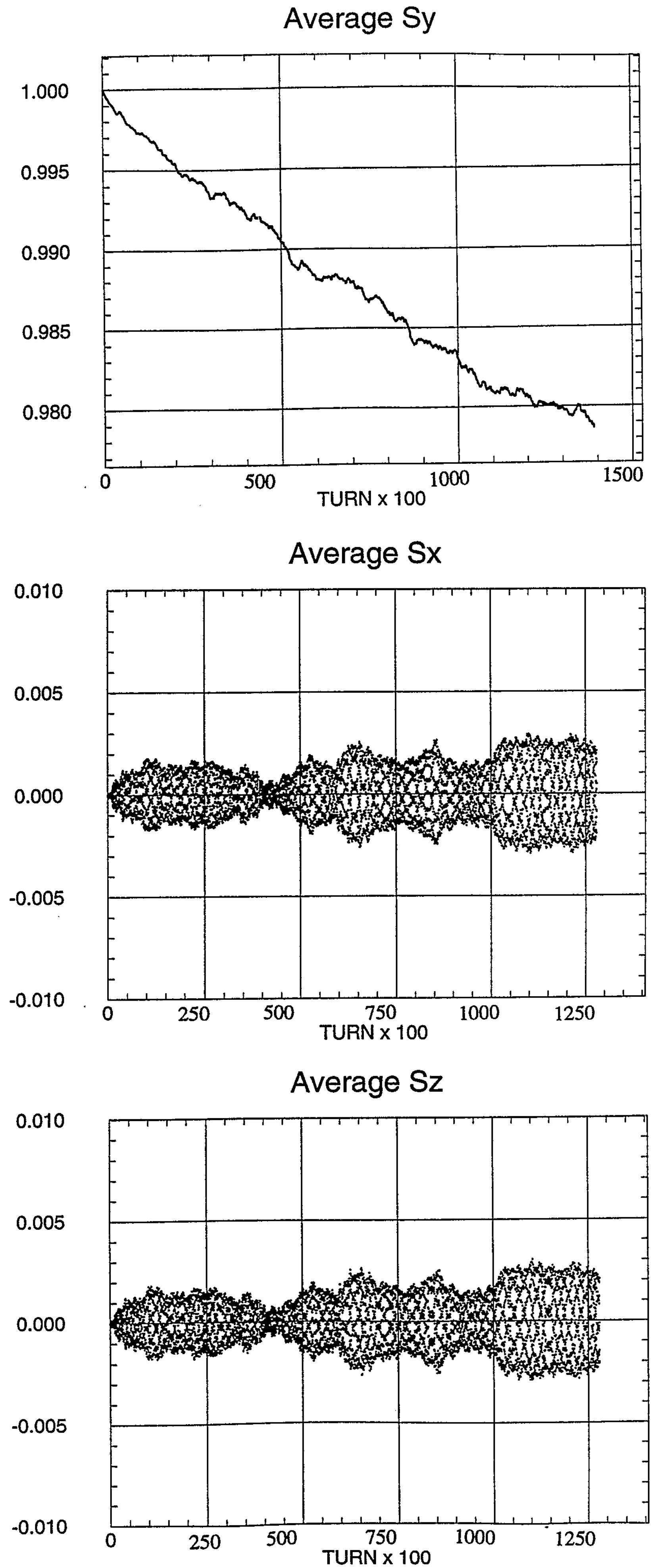


Fig. 11. Average values of spin components S_X, S_Y, S_Z as functions of turn number in presence of 1% noise in parameter σ in beam-beam kick, eq. (2).