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Beam-Beam Simulation at RHIC

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Abstract

Numerical and analytical study of beam-beam effects in RHIC, employing strong-weak approach, has been done. Numerical model, which combines linear matrix of particle motion between subsequent collisions and thin lens nonlinear matrix of beam-beam interaction, has been developed. It is demonstrated, that without noise in opposite beam size, particle motion is stable in wide range of variation of beam-beam parameter. On the contrary, even small noise in opposite beam size creates instability. Analytical expression for diffusion coefficient under noise regime has been derived. It is shown, that to provide long-term stability under value of beam-beam parameter per collision ξ =0.005, noise amplitude in opposing beam size has to be in the limit of 0.3% .

1. Beam-beam collision

Let us consider strong-weak model of interaction of an ellipsoidal bunch (1) with test particle of the counteracted beam (2), see Fig.1. Number of particles in bunch is N_1, bunch charge is Q_1, bunch velocity is $-\beta_1 c$.

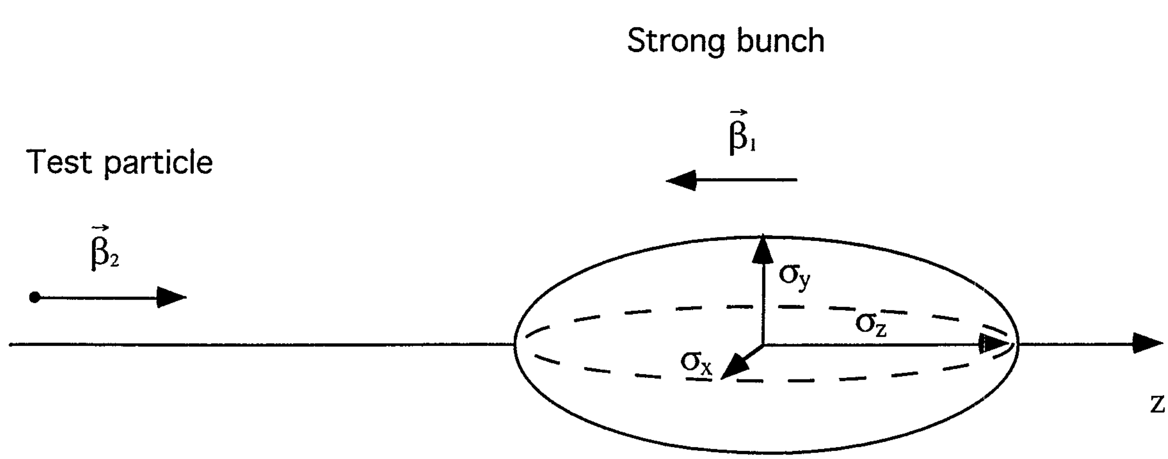


Fig. 1. Interaction of test particle with ellipsoidal bunch.

In self system of coordinate rms bunch sizes are $\sigma_{s,x}$, $\sigma_{s,y}$, $\sigma_{s,z}$. Space charge density of the bunch in self frame is supposed to be Gaussian:

$$\rho_s = \frac{Q_1}{(2\pi)^{3/2}\ \sigma_{s,x}\,\sigma_{s,y}\,\sigma_{s,z}} \exp\left(-\frac{x_s^2}{2\,\sigma_{s,x}^2} - \frac{y_s^2}{2\,\sigma_{s,y}^2} - \frac{z_s^2}{2\,\sigma_{s,z}^2}\right) \qquad (1)$$

Electrostatic potential of the space charge field created by the bunch is [1]:

$$U_s = \frac{Q_1}{4\,\pi^{3/2}\,\varepsilon_o} \int_o^\infty \frac{\exp\left(-\frac{x_s^2}{2\,\sigma_{s,x}^2 + w} - \frac{y_s^2}{2\,\sigma_{s,y}^2 + w} - \frac{z_s^2}{2\,\sigma_{s,z}^2 + w}\right)}{\sqrt{2\,\sigma_{s,x}^2 + w}\,\sqrt{2\,\sigma_{s,y}^2 + w}\,\sqrt{2\,\sigma_{s,z}^2 + w}}\,dw \qquad (2)$$

Components of space charge electrostatic field are:

$$E_{s,x} = \frac{Q_1}{2\,\pi^{3/2}\,\varepsilon_o}\,x \int_o^\infty \frac{\exp\left(-\frac{x_s^2}{2\,\sigma_{s,x}^2 + w} - \frac{y_s^2}{2\,\sigma_{s,y}^2 + w} - \frac{z_s^2}{2\,\sigma_{s,z}^2 + w}\right)}{(2\,\sigma_{s,x}^2 + w)^{3/2}\,\sqrt{2\,\sigma_{s,y}^2 + w}\,\sqrt{2\,\sigma_{s,z}^2 + w}}\,dw, \qquad (3)$$

analogously for $E_{s,y}$, $E_{s,z}$. Let us make transformation from the self system of coordinates to laboratory frame, where bunch is moving with velocity $-\beta_1 c$. The rms semi-axis of ellipsoidal bunch and coordinate transformations in the laboratory frame are:

$$\begin{aligned} \sigma_x &= \sigma_{s,x}; & x &= x_s, & & \\ \sigma_y &= \sigma_{s,y}; & y &= y_s, & & \\ \sigma_z &= \frac{\sigma_{s,z}}{\gamma_1}, & z &= \frac{z_s}{\gamma_1}, & \gamma_1 &= (1-\beta_1^2)^{-1/2}\ . \end{aligned} \qquad (4)$$

According to Lorentz transformation, the transverse electric field of the bunch in laboratory frame is:

$$E_x = E_{s,x}\cdot\gamma = \gamma\,\frac{Q_1}{2\,\pi^{3/2}\,\varepsilon_o}\,x \int_o^\infty \frac{\exp\left(-\frac{x_s^2}{2\sigma_{s,x}^2 + w} - \frac{y_s^2}{2\sigma_{s,y}^2 + w} - \frac{z_s^2}{2\sigma_{s,z}^2 + w}\right)}{(2\sigma_{s,x}^2 + w)^{3/2}\,\sqrt{2\sigma_{s,y}^2 + w}\,\sqrt{2\sigma_{s,z}^2 + w}}\,dw =$$

$$= \frac{Q_1}{2\,\pi^{3/2}\,\varepsilon_o}\,x \int_o^\infty \frac{\exp\left(-\frac{x^2}{2\sigma_x^2 + w} - \frac{y^2}{2\sigma_y^2 + w} - \frac{(z + v_1 t)^2}{2\sigma_z^2 + \frac{w}{\gamma^2}}\right)}{(2\sigma_x^2 + w)^{3/2}\,\sqrt{2\sigma_y^2 + w}\,\sqrt{2\sigma_z^2 + \frac{w}{\gamma^2}}}\,dw\ . \qquad (5)$$

analogously for E_y. Axis z coincides with the vector of test particle with velocity $\beta_2 c$. Since velocity of the strong bunch is negative $-\beta_1 c$, the Lorentz force of the strong bunch acting at the test particle is:

$$F_x = q_2 (E_x - v_2 B_y) = q_2 E_x (1+\beta_1\beta_2)$$
$$F_y = q_2 (E_y + v_2 B_x) = q_2 E_y (1+\beta_1\beta_2) \qquad (6)$$

Change of slope of particle trajectory after crossing the strong bunch is

$$\Delta \frac{dx}{dz} = \frac{1}{m_2\gamma_2 (\beta_2 c)^2} \int_{-\infty}^{\infty} F_x \, dz = \frac{q_2 (1+\beta_1\beta_2)}{m_2\gamma_2 (\beta_2 c)^2} \int_{-\infty}^{\infty} E_x \, dz \qquad (7)$$

Calculation of the integral gives:

$$\int_{-\infty}^{\infty} E_x \, dz = \frac{Q_1}{2\pi^{3/2}\varepsilon_o} x \int_o^{\infty} dw \frac{\exp\left(-\frac{x^2}{2\sigma_x^2+w} - \frac{y^2}{2\sigma_y^2+w}\right)}{(2\sigma_x^2+w)^{3/2}\sqrt{2\sigma_y^2+w}} \frac{1}{\sqrt{2\sigma_z^2+\frac{w}{\gamma^2}}} \int_{-\infty}^{\infty} \exp\left(-\frac{(z+v_1 t)^2}{2\sigma_z^2+\frac{w}{\gamma^2}}\right) dz$$

$$= \frac{Q_1}{2\pi\varepsilon_o} x \int_o^{\infty} dw \frac{\exp\left(-\frac{x^2}{2\sigma_x^2+w} - \frac{y^2}{2\sigma_y^2+w}\right)}{(2\sigma_x^2+w)^{3/2}\sqrt{2\sigma_y^2+w}} . \qquad (8)$$

Let us suppose that $\sigma_x = \sigma_y = \sigma$, i.e. we consider round Gaussian beam. Then the calculation of the integral is simplified:

$$\int_{-\infty}^{\infty} E_x \, dz = \frac{Q_1}{2\pi\varepsilon_o} x \int_o^{\infty} \frac{\exp\left(-\frac{r^2}{2\sigma^2+w}\right)}{(2\sigma^2+w)^2} dw = \frac{Q_1}{2\pi\varepsilon_o} x \int_{2\sigma^2}^{\infty} \frac{\exp\left(-\frac{r^2}{\eta}\right)}{\eta^2} d\eta =$$

$$= \frac{Q_1}{2\pi\varepsilon_o} \frac{x}{r^2} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right] \qquad (9)$$

Then the change of slope of particle trajectory after crossing the strong round bunch is

$$\Delta \frac{dx}{dz} = \frac{4\pi\xi}{\beta^*} x \left\{ \frac{1 - \exp[-r^2/(2\sigma^2)]}{r^2/(2\sigma^2)} \right\}, \qquad (10)$$

where β^* is a value of beta-function of the ring and beam-beam parameter ξ is defined as follow:

$$\xi = \frac{\beta^*}{4\pi} \frac{q_1 q_2 N_1 (1 + \beta_1 \beta_2)}{(\beta_2 c)^2 2\pi\varepsilon_o m_2 \gamma_2 2\sigma_1^2} . \tag{11}$$

Limitation in ξ results in constraints of luminosity L, which is defined as follows:

$$L = \frac{f N_1 N_2}{4 \pi \sigma^2} N_{bunch} , \tag{12}$$

where f is a particle revolution frequency in a ring, and N_{bunch} is a number of bunches per beam.

2. Numerical model

We consider a two-dimensional particle model in coordinates $(x, p_x = \beta^* \frac{dx}{dz})$, $(y, p_y = \beta^* \frac{dy}{dz})$. Particle motion between subsequent collisions combines linear matrix with betatron angle $\theta_x = 2\pi Q_x$ and matrix of beam-beam interaction, treated as a thin lens with nonlinear focusing function f(x) :

$$\begin{pmatrix} x_{n+1} \\ p_{x,\,n+1} \end{pmatrix} = \begin{pmatrix} \cos\theta_x & \sin\theta_x \\ -\sin\theta_x & \cos\theta_x \end{pmatrix} \begin{pmatrix} 1 & 0 \\ f(x_n) & 1 \end{pmatrix} \begin{pmatrix} x_n \\ p_{x,\,n} \end{pmatrix} . \tag{13}$$

analogously for (y, p_y). Equation (13) can be rewritten as follows:

$$\begin{pmatrix} x_{n+1} \\ p_{x,\,n+1} \end{pmatrix} = \begin{pmatrix} \cos\theta_x & \sin\theta_x \\ -\sin\theta_x & \cos\theta_x \end{pmatrix} \begin{pmatrix} x_n \\ p_{x,\,n} + \Delta p_{x,\,n} \end{pmatrix} , \tag{14}$$

where beam-beam kick $\Delta p_{x,\,n} = x_n f(x_n)$ is expressed as

$$\Delta p_x = 4\pi\xi x \frac{1 - \exp(-\frac{r^2}{2\sigma^2})}{(\frac{r^2}{2\sigma^2})} , \tag{15}$$

and similar for Δp_y.

3. Results of simulation

Simulation of beam-beam effects in RHIC were performed with parameters, presented at Table 1. The ring is supposed to have two colliding points, therefore the tune shifts between subsequent beam-beam interaction are equal to half of their values $Q_x/2 = 14.095$; $Q_y/2 = 14.59$.

Table 1. Parameters of the interacted beams

Particle energy	250 GeV
RMS normalized emittance	1 π cm mrad
Rms beam size at IP, σ	0.08 mm
Collision angle	0
Number of IP's	2
Beam-beam tune shift/collision ξ	0 - 0.05
Betatron tune/ring	$Q_x = 28.19$
	$Q_y = 29.18$

In the considered model particles without beam-beam perturbation move along elliptical trajectories at phase planes. Working point of the collider (14.095; 14.59) is chosen far from the lowest order resonances. For colliding beams with the same sign of charge, effective betatron tune is reduced due to beam-beam collisions. Nonlinear resonances are not excited in the large range of variation of parameter beam-beam ξ. Phase space trajectories in the presence of beam-beam collision are deformed tilted ellipses. Example of that trajectories for case of $\xi = 0.025$ is presented in Fig.2.

Beam-beam perturbation results in expansion of beam envelopes and beam emittances. In Figs. 3 a-d results of beam dynamics for different values of $\xi = 0$; 0.025; 0.05 are presented. Number of modeling particles N was varied between 5000, 25000, 50000. During the simulations, the values of rms (root-mean-square) beam envelope a_n as well as rms beam emittance ε_n were calculated as functions of turn number n:

$$a_n = 2\sqrt{\frac{1}{N}\sum_{k=1}^{N} x_n^2(k)} \quad , \tag{16}$$

$$\varepsilon_n = 4\sqrt{\frac{1}{N^2}\left[\sum_{k=1}^{N} x_n^2(k)\right]\left[\sum_{k=1}^{N} p_n^2(k)\right] - \left[\frac{1}{N}\sum_{k=1}^{N} x_n(k)\, p_n(k)\right]^2} \quad . \tag{17}$$

Performed calculations indicate, that beam-beam interaction results in increase of amplitude and expanding of beam emittance, but particle motion is stable. As seen in Fig. 3a, with increase of parameter ξ, both beam envelopes and beam emittances perform oscillations at the first 30-50 turns, after that their values are stabilized. In Figs. 3b-d results of beam-beam interaction with different number of modeling particles are presented. The results are identical, which means that results do not depend on number of macroparticles.

Another situation is observed if parameter σ of the beam-beam kick in Eq. (15) is subjected to noise. In the calculations, presented in Fig. 4, standard deviation σ in the beam-beam kick of Eq. (15) was changed from turn to turn according to the expression

$$\sigma_n = \sigma_o (1 \pm 0.025\eta_n) , \tag{18}$$

where η_n is a uniform random function within the interval (0,1). Simulations in Fig. 4 correspond to the noise in the size of the opposite beam. This noise naturally exists due to even small beam mismatching with the channel. In contrast with Fig. 3, both beam envelopes and beam emittances expand with time. Therefore, even weak noise in the opposite beam size creates instability [2]. An important feature of the noise regime is that noise beam-beam instability can appear apart from excitation of nonlinear resonances.

4. Diffusion rate

In computer simulation the noise instability was observed under the fluctuation in opposite beam size $\sigma_n = \sigma_o (1 \pm \frac{u_n}{2})$, where $|u_n| \leq u$ is a uniform random function. Using approximation

$$\frac{1}{\sigma_n^m} = \frac{1}{\sigma_o^m (1 \pm \frac{u_n}{2})^m} \approx \frac{1}{\sigma_o^m} (1 \pm m \frac{u_n}{2}) , \tag{19}$$

the noisy beam-beam kick can be presented as follow:

$$\Delta p = 4\pi\xi \sum_{k=1}^{\infty} \frac{(-1)^k}{k!\,(2\sigma^2)^{k-1}} x^{2k-1} = \Delta p_o \pm 4\pi\xi u_n [\frac{x^3}{4\sigma_o^2} + ...+ \frac{(-1)^k (k-1)}{k!\,(2\sigma_o^2)^{k-1}} x^{2k-1}], \tag{20}$$

where $\Delta p_o = 4\pi\xi \sum_{k=1}^{\infty} \frac{(-1)^k}{k!\,(2\sigma_o^2)^{k-1}} x^{2k-1}$ is an unperturbed beam-beam kick and the rest of the function (20) is a noise perturbation. In Ref. [3] noise beam-beam instability was studied analytically for perturbation of cubic term in momentum kick

$$\Delta p_n = 4\ \delta_n x^3 , \tag{21}$$

where $|\delta_n| \le \delta$ is a uniform random function. The beam emittance growth was found to be

$$\frac{\varepsilon_n}{\varepsilon_o} = \sqrt{1 + \delta^2 a^4 n} = \sqrt{1 + D\,n} \approx 1 + \frac{Dn}{2} , \tag{22}$$

where a is an amplitude of unperturbed trajectory $x = a\cos(n\theta)$ and D is a diffusion coefficient. Comparison of the term, proportional to x^3 in eq. (20) with that in eq. (21), gives:

$$\delta = \frac{\pi \xi u}{4\sigma_o^2} . \tag{23}$$

Taking into account, that $a = 2\sigma_o$, the diffusion coefficient is

$$D = \pi^2 (\xi u)^2 . \tag{24}$$

In Table 2 and in Figs. 5, 6 results of numerical evaluation of diffusion coefficients for different values of beam-beam parameter ξ and amplitude of noise u are presented. As it follows from Figs. 5, 6, numerical values of the diffusion coefficient are close to the analytical estimation of Eq. (24). Therefore, eq. (24) can be used for evaluation of stability region.

In RHIC revolution frequency is $f = 7.9 \cdot 10^4$ Hz. Let us take the number of turns required for long-term stability as $n_{max} = 10^9$, which corresponds to 3.5 hours of operation. To prevent doubling in emittance growth during that time, the product of diffusion coefficient on maximum turn number has to be restricted as $Dn_{max} < 3$, or $D < 3 \cdot 10^{-9}$ (see eq. 22). It imposes limitation on product (ξu):

$$\xi u < \frac{1}{\pi}\sqrt{\frac{3}{n_{max}}} = 1.7 \cdot 10^{-5} . \tag{25}$$

Typical value of ξ, obtained in the experiment, is 0.005 per collision. Therefore, the noise in opposing beam size has to be in the limit of 0.35% to provide long-term beam-beam stability.

Table 2. Effect of noise on amplitude and emittance growth.

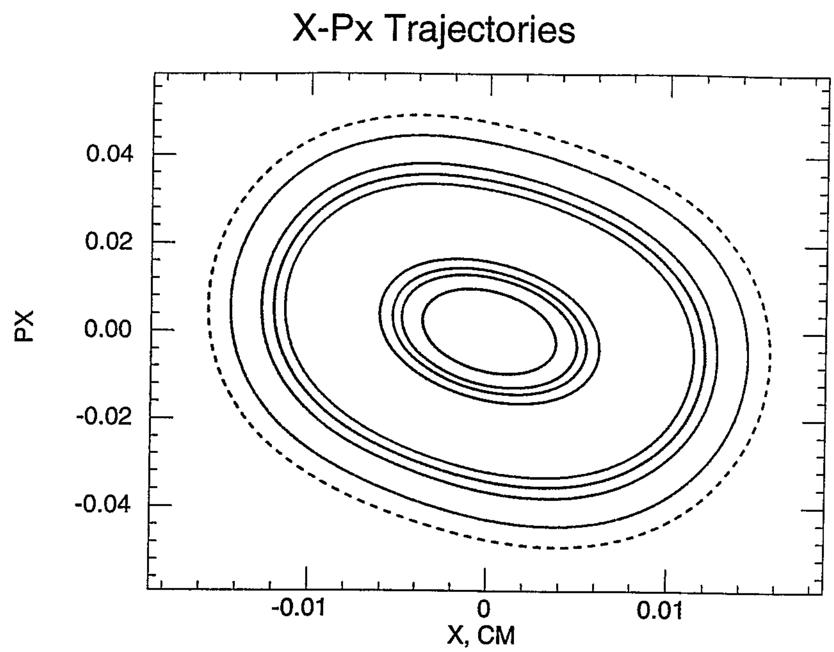
Number of turns	ξ	Noise in σ	Amplitude Growth $\frac{a_x}{a_o}$	$\frac{a_y}{a_o}$	$\frac{\varepsilon_x}{\varepsilon_o}$	$\frac{\varepsilon_y}{\varepsilon_o}$	Diffusion coefficients D_{a_x}	D_{a_y}	D_{ε_x}	D_{ε_y}
$2.4 \cdot 10^5$	0.005	0.025	1.015	1.006	1.029	1.015	$1.25 \cdot 10^{-7}$	$5 \cdot 10^{-8}$	$2.4 \cdot 10^{-7}$	$1.25 \cdot 10^{-7}$
$3.2 \cdot 10^5$	0.005	0.05	1.05	1.05	1.11	1.08	$3.1 \cdot 10^{-7}$	$3.1 \cdot 10^{-7}$	$6.8 \cdot 10^{-7}$	$5 \cdot 10^{-7}$
$2.3 \cdot 10^5$	0.005	0.075	1.06	1.07	1.2	1.15	$5.2 \cdot 10^{-7}$	$6 \cdot 10^{-7}$	$1.7 \cdot 10^{-6}$	$1.3 \cdot 10^{-6}$
$3.12 \cdot 10^5$	0.005	0.1	1.18	1.15	1.42	1.32	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$	$2.3 \cdot 10^{-6}$	$2 \cdot 10^{-6}$
$3.12 \cdot 10^5$	0.005	0.125	1.25	1.2	1.6	1.46	$1.6 \cdot 10^{-6}$	$1.3 \cdot 10^{-6}$	$3.8 \cdot 10^{-6}$	$3 \cdot 10^{-6}$
$3.12 \cdot 10^5$	0.0075	0.05	1.12	1.08	1.26	1.17	$7.7 \cdot 10^{-7}$	$5 \cdot 10^{-7}$	$1.6 \cdot 10^{-6}$	$1 \cdot 10^{-6}$
$2.3 \cdot 10^5$	0.01	0.05	1.15	1.1	1.35	1.23	$1.3 \cdot 10^{-6}$	$8.7 \cdot 10^{-7}$	$3. \cdot 10^{-6}$	$2 \cdot 10^{-6}$
$3.12 \cdot 10^5$	0.0125	0.04	1.19	1.12	1.45	1.28	$1.2 \cdot 10^{-6}$	$7.7 \cdot 10^{-7}$	$3. \cdot 10^{-6}$	$1.8 \cdot 10^{-6}$
$3.12 \cdot 10^5$	0.0125	0.05	1.28	1.17	1.7	1.4	$1.8 \cdot 10^{-6}$	$1 \cdot 10^{-6}$	$4.5 \cdot 10^{-6}$	$2.5 \cdot 10^{-6}$
$3.12 \cdot 10^5$	0.0125	0.075	1.47	1.37	2.35	1.8	$3. \cdot 10^{-6}$	$2.3 \cdot 10^{-6}$	$2.2 \cdot 10^{-6}$	$5 \cdot 10^{-6}$

5. Conclusion

Our previous study indicated, that beam-beam configuration is always unstable, if two conditions are valid: (i) beam-beam kick is a nonlinear function of coordinate and (ii) beam-beam kick is subject to noise. It creates diffusion instability of a beam induced by random fluctuations in the size of an opposite colliding beam. In this note beam-beam effects in RHIC have been studied utilizing model in 4-dimensional phase space. It was found, that diffusion coefficient is proportional to $(\xi u)^2$, where ξ is a beam-beam parameter and u is a reduced amplitude of noise in beam size. Considered phenomena imposes limitation on amplitude of noise $u < 0.003$ to provide long-term beam stability under typical value of beam-beam parameter of $\xi = 0.005$.

References

1. K.Takayama, Lettere al Nuovo Cimento, Vol. 34, No.7 , 190.
2. Y.Batygin and T.Katayama, Proceedings of the EPAC96, Barcelona, 1996, p.p.1170-1172, 1173-1175.
3. Y.Batygin and T.Katayama, "Noise Instability in Nonlinear Beam-Beam Interaction", submitted to Physical Review E.



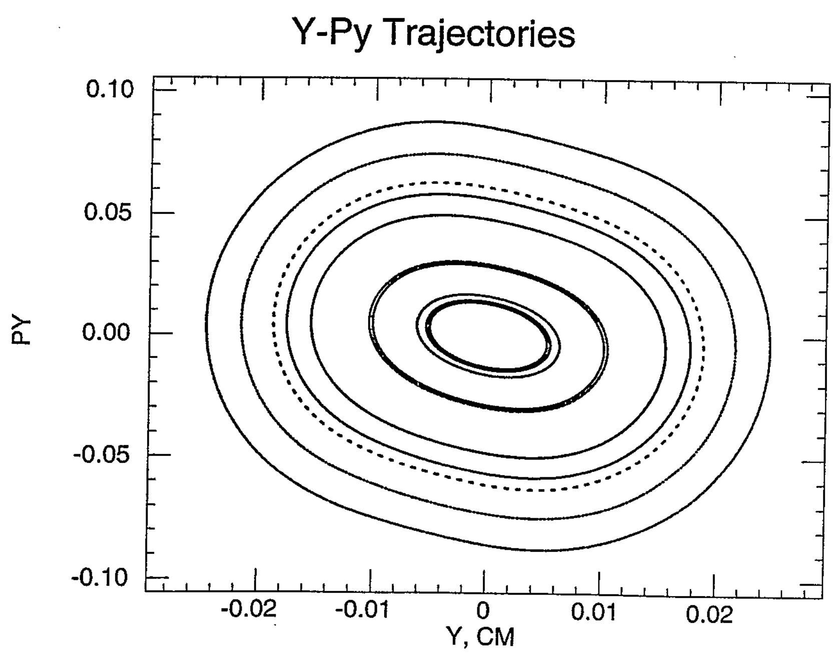


Fig. 2. Phase space trajectories for betatron tunes $Q_x = 14.095$; $Q_y = 14.59$, and beam-beam tune shift $\xi = 0.025$.

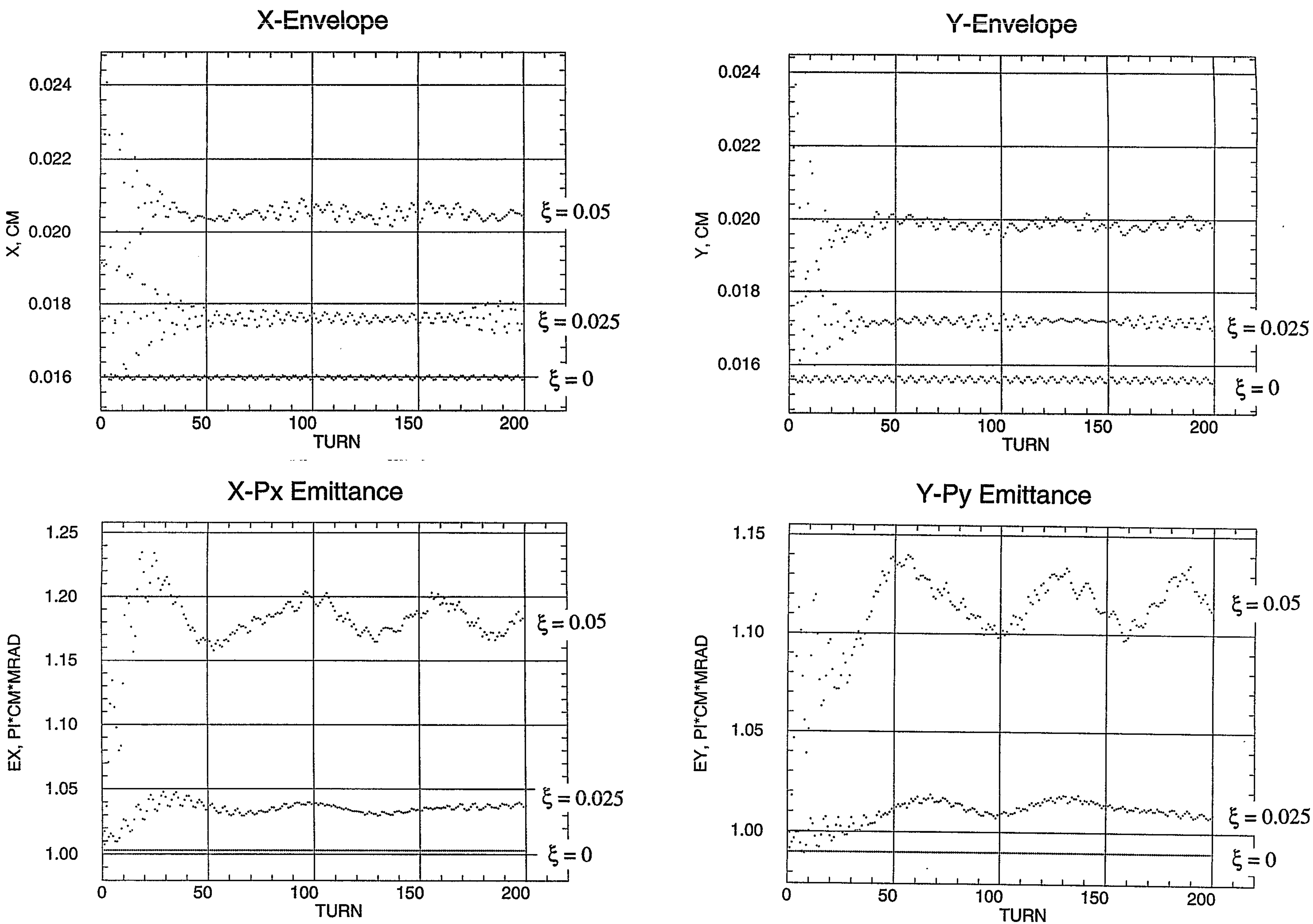


Fig. 3a. Initial stage of setting of beam envelopes and beam emittances for betatron tunes Q_x = 14.095; Q_y = 14.59, number of modeling particles N = 5000.

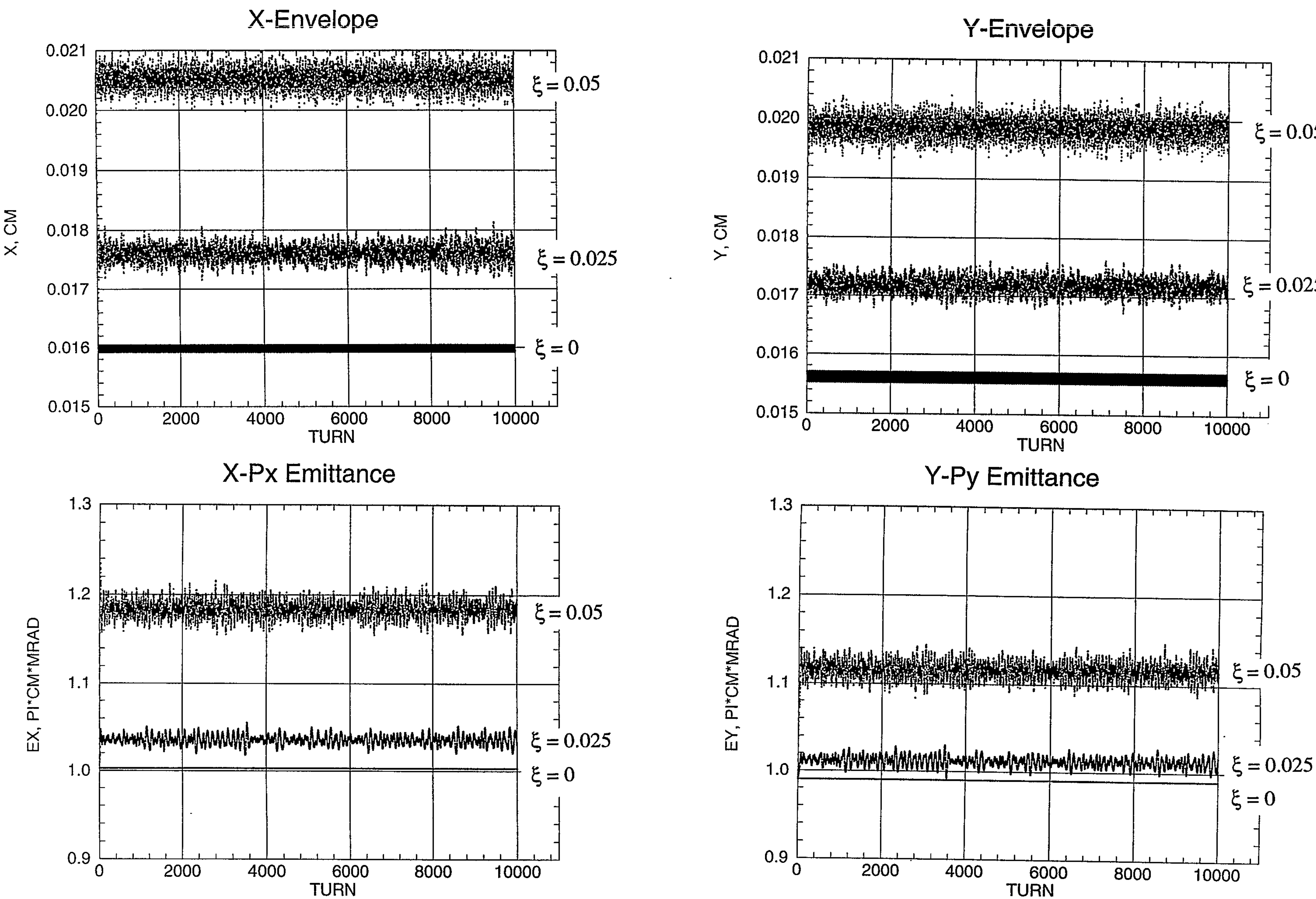


Fig. 3b. Beam envelopes and beam emittances as functions of turn number for betatron tunes $Q_X = 14.095$; $Q_y = 14.59$, number of modeling particles $N = 5000$.

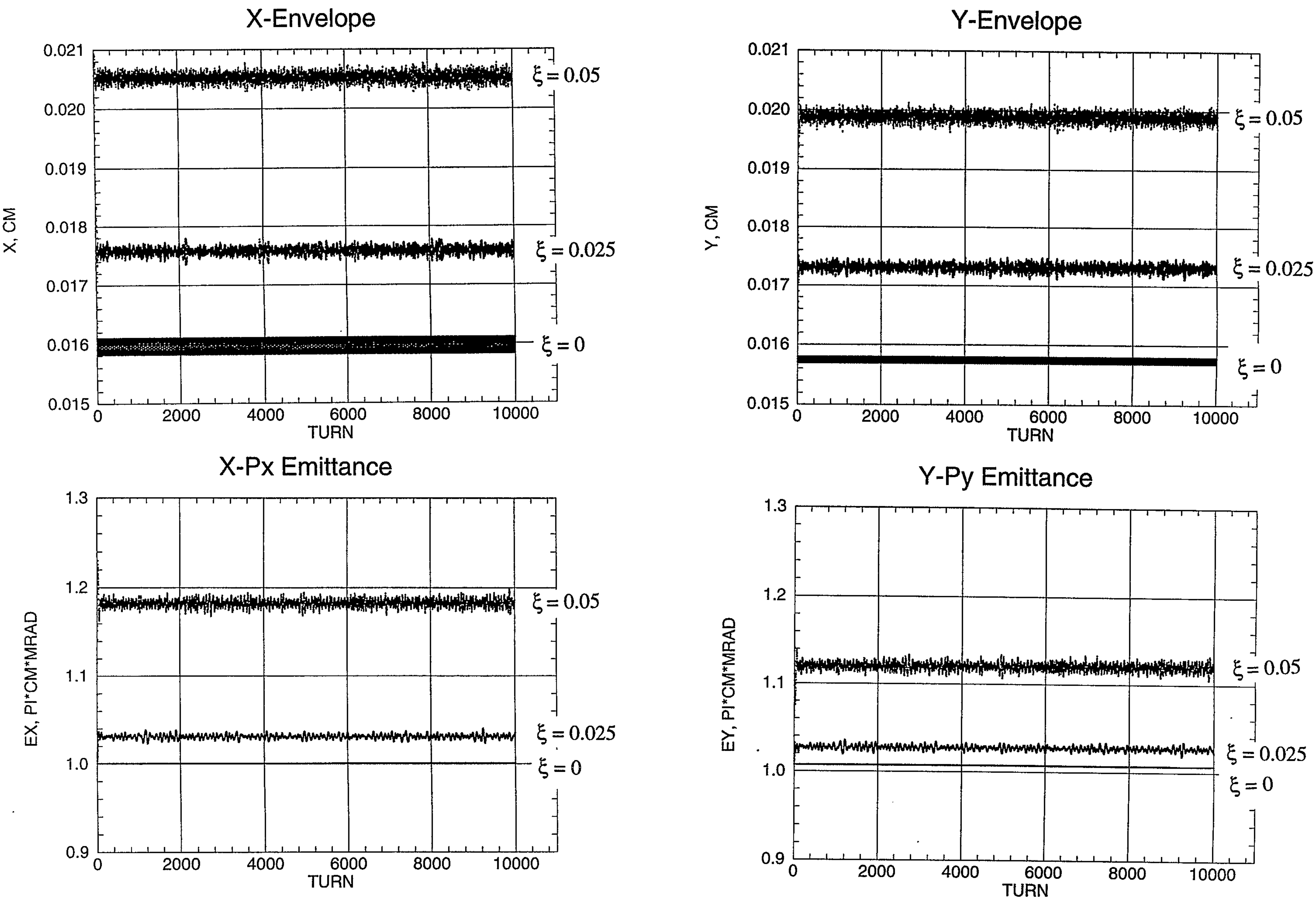


Fig. 3c. Beam envelopes and beam emittances as functions of turn number for betatron tunes $Q_x = 14.095$; $Q_y = 14.59$, number of modeling particles N = 25000.

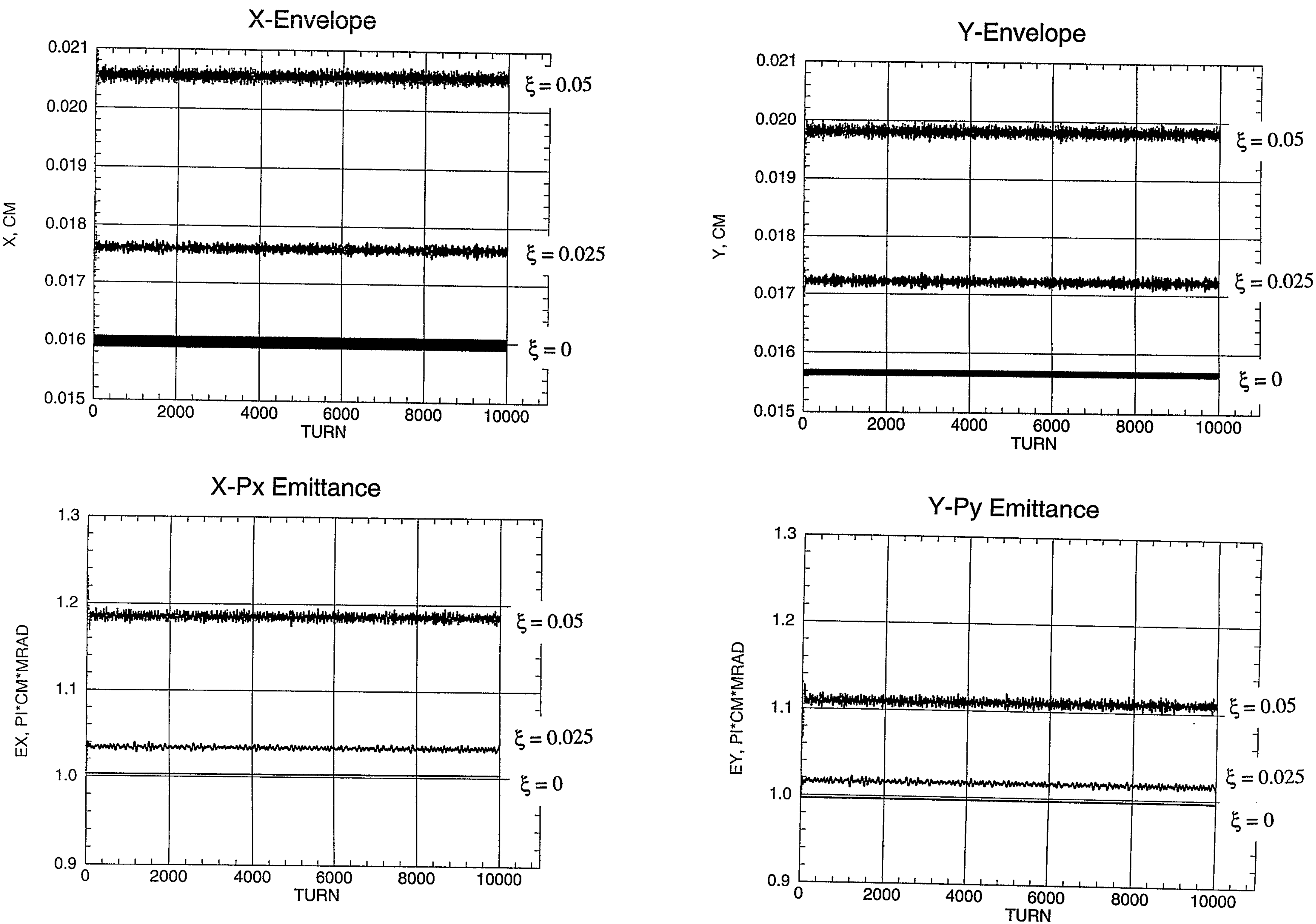


Fig. 3d. Beam envelopes and beam emittances as functions of turn number for betatron tunes $Q_x = 14.095$; $Q_y = 14.59$, number of modeling particles $N = 50000$.

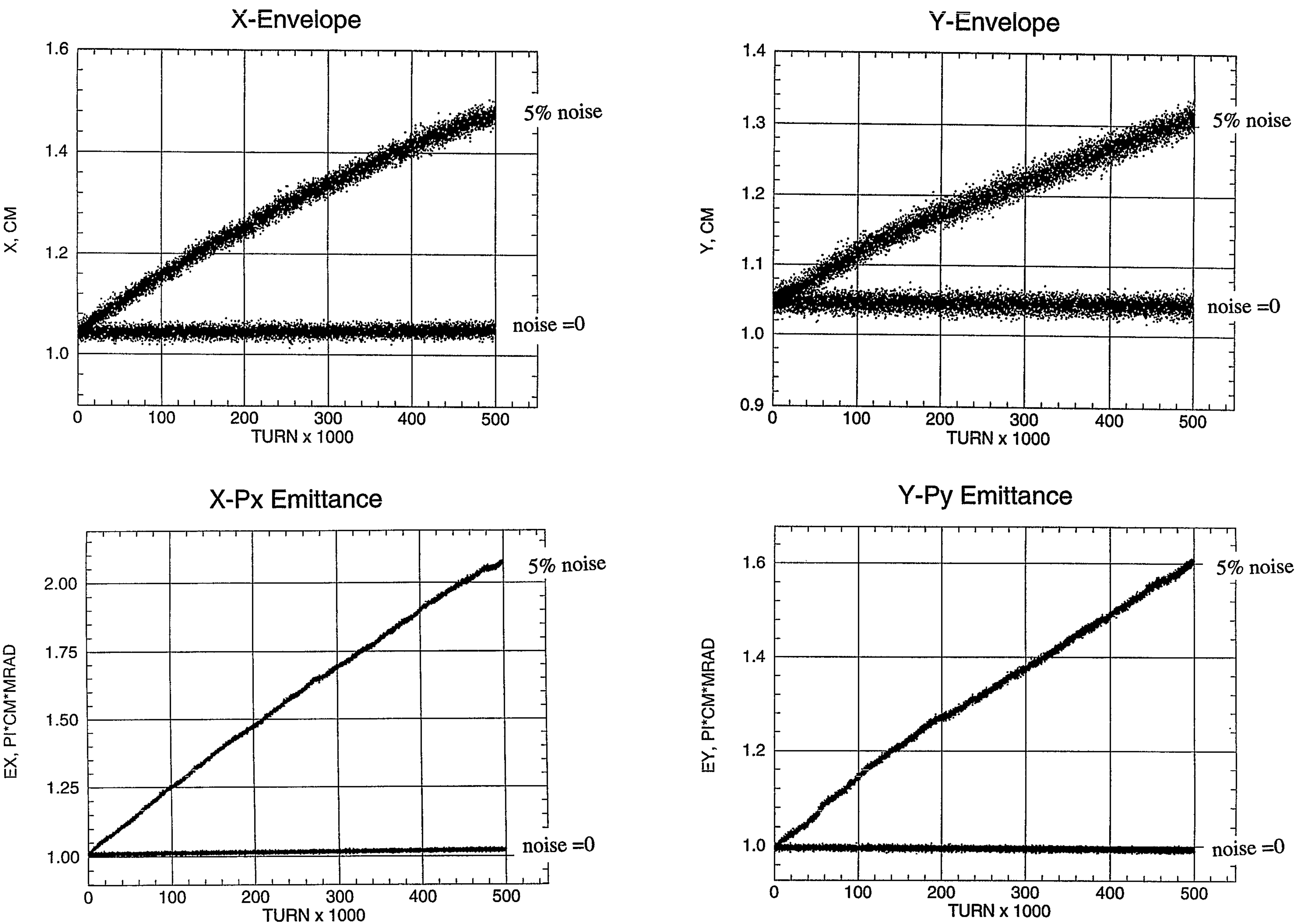


Fig.4. Effect of noise in σ on beam envelopes and beam emittances as functions of turn number for betatron tunes $Q_x = 14.095$; $Q_y = 14.59$ and beam-beam tune shift $\xi = 0.0125$

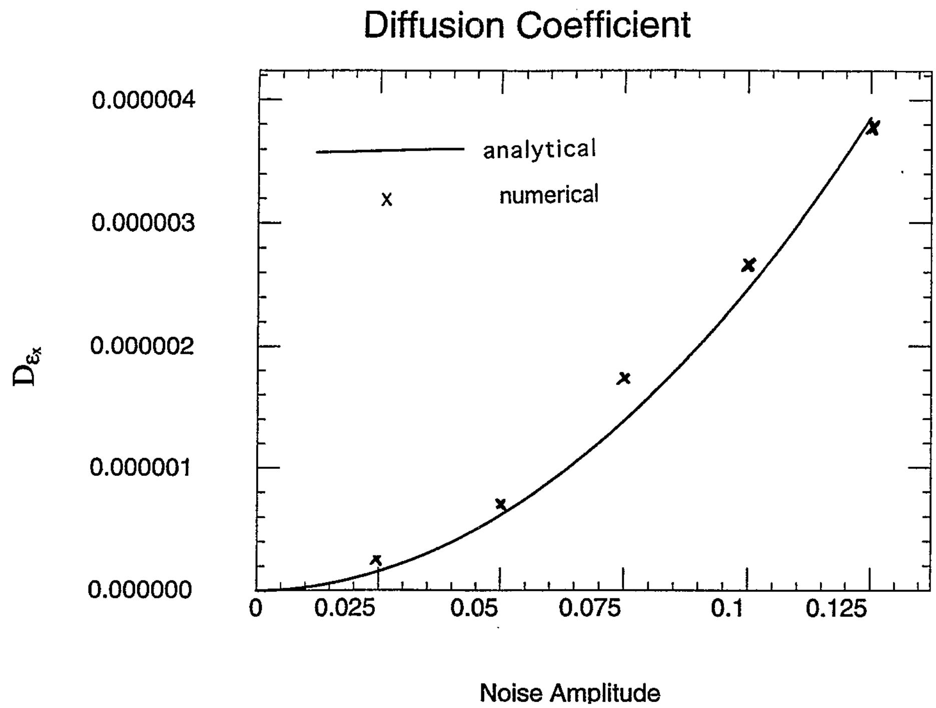


Fig.5. Diffusion coefficient D_{ε_x} as a function of noise amplitude in parameter σ for ξ =0.005.

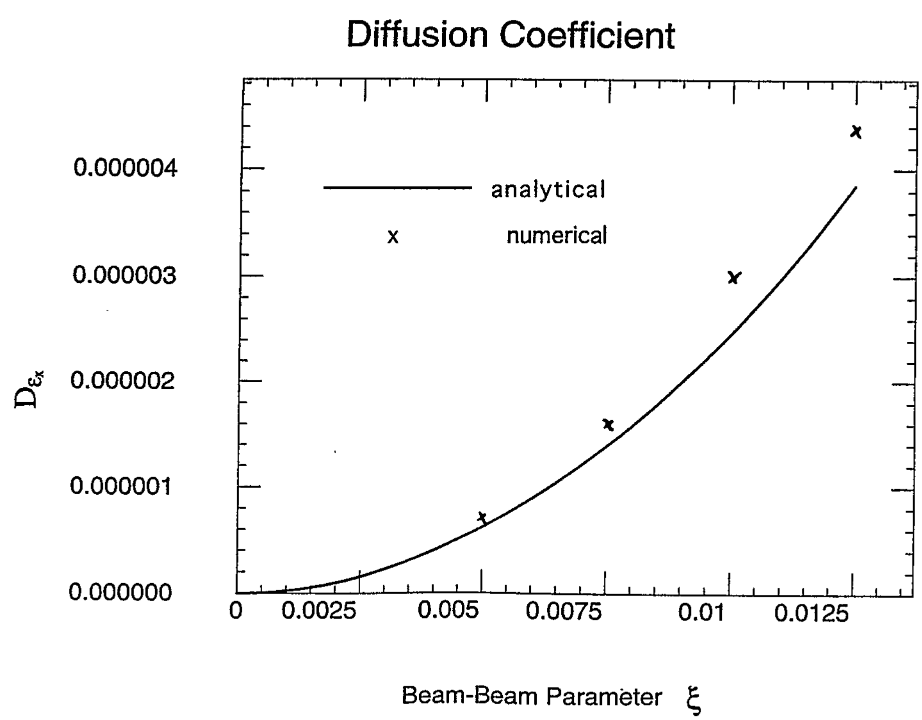


Fig.6. Diffusion coefficient D_{ε_x} as a function of beam-beam parameter ξ for the value of noise amplitude in parameter σ 5%.