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Spin Behavior with Partial Rotators

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Spin Note

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SPIN BEHAVIOR WITH PARTIAL ROTATORS

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In operating RHIC with polarized protons one needs spin rotators to convert the vertical spin in the arcs to horizontal (usually longitudinal, sometimes transverse) orientation at the interaction points. These rotators will be located in pairs, in the long $Q3-Q4$ gaps upstream and downstream of the interaction points; the downstream rotator just canceling the action of the upstream one.

For commissioning purposes, and probably also in machine operations, it should be possible to turn the rotators on while the polarized beam is already stored in the ring. During the process of turning the rotators on the spin orientation will change. In this note we investigate the spin orientation at the interaction point during the stages of the turn-on process, when the two rotators are partially energized, equally or unequally.

In the RHIC spin design[1] the spin rotators (as well as the snakes) are to be composed of four helical magnets each; each magnet is a complete 360° helix, and the four magnets of the rotator are energized so that the net orbit distortion produced is zero.

We assume that in the turn-on process the four magnets of each rotator are ramped up in proportion, so that each rotator still produces zero net orbit distortion; therefore the global orbit is unaffected by the degree of excitation of the rotators. The orbit excursion in the interior of the rotator is, of course, proportional to its excitation.

In calculating the spin at the IP when the rotators are partially excited we use the idealized spin matrices corresponding to perfect helices without fringing fields[2]. We also assume that in the absence of the rotators the spin matrix[3] for the whole ring, with two snakes, is exactly $i\sigma_3$, corresponding to a perfect ring with two perfect snakes. The bending between the rotator and the IP (due to magnets $D0$ and DX) is taken as 3.6745 milliradians.

The spin at the IP may be determined as follows:

Calculate the spin matrix for one revolution beginning at the IP, continuing with the matrix $\mathbf{D}$ for passage through the DX and D0 magnets which bend through an angle $\varphi = 3.6745mr$, the matrix $\mathbf{R}_1$ for the downstream rotator, the unperturbed one-turn spin matrix $i\sigma_3, \mathbf{R}_2$ for the upstream rotator, and $\mathbf{D}^{-1}$ for passage through the upstream D0 and DX magnets. We consider

[1] RHIC Spin design report
[2] E. D. Courant, AD/RHIC 133, Spin Note #10
[3] We use the spinor (SU_2) representation of the rotation matrices.

the case where the two rotators are excited equally so that $\mathbf{R}_2 = \mathbf{R}_1^{-1}$, and the case where $\mathbf{R}_2$ is absent. The resulting one-turn spin matrix $\mathbf{M}$ may be parametrized in the form

$$\mathbf{M} = \exp \frac{i}{2}\mu(\vec{n} \cdot \vec{\sigma}). \tag{1}$$

where $\vec{n}$ is a unit vector and $(\mu/2\pi)$ is the spin tune ν_s. Then the spin at the IR is just the vector $\vec{n}$. If the two rotators are equal the spin tune is 1/2 just as when there are no rotators; if the two rotators are energized unequally the spin tune is generally different from 1/2.

The matrix $\mathbf{D}$ for the downstream magnets DX and D0 is

$$\mathbf{D} = \exp \frac{i}{2}\gamma G\varphi\sigma_3. \tag{2}$$

If with the full rotators the spin is to be longitudinal at the IR, then the matrix for the rotator $\mathbf{R}_1$ has to be equal to

$$\mathbf{R}_1 = (\pm i\sigma_2 \pm e^{-i\gamma G\varphi\sigma_3})/\sqrt{2} \tag{3}$$

as can easily be verified by multiplying out the matrix

$$\mathbf{M} = \mathbf{D}^{-1}\mathbf{R}_2(i\sigma_3)\mathbf{R}_1\mathbf{D} \tag{4}$$

with $\mathbf{R}_2 = \mathbf{R}_1^{-1}$.

Using the spreadsheet program described in ref. 2 we can compute the strengths of the four magnets in the helix rotator necessary to make the matrix $\mathbf{R}_1$ equal to (3); we find values in agreement with those of the RHIC spin design report. Now if we excite the magnets to a fraction of their nominal strength we can again calculate the matrix (4), either with $\mathbf{R}_2 = \mathbf{R}_1^{-1}$(equal excitations) or with $\mathbf{R}_2$ omitted (only one rotator excited). We can do this at injection energy, $\gamma = 27$, and top energy ($\gamma = 268$), or of course any other energy. The results are in the following tables. Here B_1 and B_2 are the field strengths in helices 1 (and 4) and 2 (and 3) in the rotator; the spin components at the IP are S_1(longitudinal), S_2(transverse horizontal), S_3(vertical); the spin tune is ν_s.

Table 1: $\gamma = 27$

	B_1	B_2	Both rotators S_1	S_2	S_3	ν_s	One rotator S_1	S_2	S_3	ν_s
100%	2.09	2.71	1.00	0.00	0.00	.50	-.70	-.13	.70	.46
90%	1.88	2.44	.93	.07	.36	.50	-.56	-.10	.58	.47
75%	1.56	2.04	.67	.09	.73	.50	-.36	-.06	.93	.49
50%	1.04	1.36	.23	.04	.97	.50	-.12	-.02	.99	.498
25%	.52	.68	.03	.006	.999	.50	.02	.003	1.00	.500

Table 2: $\gamma = 268$

	B_1	B_2	Both rotators S_1	S_2	S_3	ν_s	One rotator S_1	S_2	S_3	ν_s
100%	3.52	3.25	1.00	0.00	0.00	.50	.19	-.96	-.19	.26
90%	3.17	2.92	.82	.52	-.23	.50	.19	-.94	.27	.30
75%	2.64	2.44	.27	.96	.07	.50	.14	-.71	.69	.39
50%	1.76	1.63	-.07	.53	.84	.50	.06	-.27	.96	.48
25%	.88	.81	-.02	.06	.998	.50	.01	-.04	.999	.499