BNL-103676-2014-TECH

AGS/RHIC/SN 050;BNL-103676-2014-IR

Optimization of a Nominal 4-Helix Snake at Various Beam Energies

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December 1996

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U.S. Department of Energy

USDOE Office of Science (SC)

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Spin Note

AGS/RHIC/SN No. 050

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December 30, 1996

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OPTIMIZATION OF A NOMINAL 4-HELIX SNAKE AT VARIOUS BEAM ENERGIES *

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1. Introduction

Siberian snakes designed for RHIC are built with four superconducting helical dipole magnets. They should flip the spin vector by 180^0 around a given precession axis, that for RHIC lies in the horizontal plane at $\pm 45^0$ with respect to the longitudinal axis. Another requirement is that the orbits should be compensated, i.e. a particle entering on axis with zero angle should emerge on axis and with zero angle.

In this note we will analyze the optimum value of the magnetic fields of the helices from the injection to the top energy.

2. Spin rotation matrix

For a complete analysis , we have revisited the E.Courant [1] and M.Syphers [2] formalism to find a slightly more general form of the spin matrix. A first order Spin rotation matrix for a perfect helix with hard edges and without fringe field effects has been derived on the assumption of a nominal 360^0 field rotation as we proceed along the axis of the magnet.

The matrix for a snake depends on $G\gamma$. We shall calculate the values of the magnetic fields of the outer (B_1) and inner (B_2) helices that optimize the snake -i.e. that produce the required spin rotation and precession axis-. Note also that for this idealized snake the orbits are automatically compensated.

3. Matrix analysis

Based on the formalism by E.Courant, let us write a general expression for the spin matrix of a helix , when its precession axis and rotation angle are not nominal. To lowest order, considering the coordinate frame shown in Fig. 1, the magnetic field components [3] are:

$$\begin{cases} B_x = -B_0 \cos\left[k(s - s_1) + \alpha_1\right] \\ B_y = B_0 \sin\left[k(s - s_1) + \alpha_1\right] \\ B_z \simeq 0 \end{cases} \tag{1}$$

x,y,z are radial axis, vertical axis and longitudinal axis respectevely.
While:
- B_0 is the highest value of the magnetic field;
- s_1 and α_1 are the longitudinal coordinate and the angle of the field with the vertical at the entrance of the helix;

*Work Performed Under the Auspices of the U.S.Department of Energy and of the University of Genova, Italy

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- It is

$$k = \frac{2\pi}{\lambda} \tag{2}$$

with $k > 0$ for a right-handed helix, and $k < 0$ for a left-handed. λ is the lenght of the magnetic field inside the helix.

The spin of a charged particle obeys the BMT Equation [4].

$$\frac{d\vec{S}}{ds} = \vec{S} \times \vec{\Omega}, \tag{3}$$

where the precession frequency vector $\vec{\Omega}$ is

$$\vec{\Omega} = [(1 + G\gamma)\vec{B}_\perp + (1 + G)\vec{B}_{||}]/B\rho, \tag{4}$$

$\vec{B}_\perp$ and $\vec{B}_{||}$ being the parts of the field perpendicular and parallel to the particle velocity.

Following [1], let us write in spinor notation

$$\vec{S} = \Psi^\dagger \vec{\sigma} \Psi, \tag{5}$$

with σ the Pauli matrices.

From Eqs. (3) and (5) we obtain

$$\frac{d\Psi}{ds} = \frac{i}{2}(\vec{\sigma} \cdot \vec{\Omega})\Psi, \tag{6}$$

with

$$\kappa = \frac{1 + G\gamma}{B\rho} B_0 \tag{7}$$

where $G = 1.7928$ is the proton gyromagnetic factor, $\gamma = \frac{E}{m_p} \approx E(GeV)$ the Lorentz factor and $B\rho = \frac{p}{e} \simeq \frac{\gamma mc}{e}$. Inserting the expression for the field of Eq. (1), the diff. Eq. (6) becomes

$$\frac{d\Psi}{ds} = \frac{i}{2}\kappa \left\{\sigma_3 \cos\left[k(s - s_1) + \alpha_1\right] - \sigma_2 \sin\left[k(s - s_1) + \alpha_1\right]\right\} \Psi. \tag{8}$$

Using the relation between Pauli matrices $\sigma_2 = -i\sigma_3\sigma_1$ we can write

$$\sigma_3 \cos\left[k(s - s_1) + \alpha_1\right] - \sigma_2 \sin\left[k(s - s_1) + \alpha_1\right] = \sigma_3 e^{i\sigma_1[k(s-s_1)+\alpha_1]} \tag{9}$$

Eq. (8) becomes

$$\frac{d\Psi}{ds} = \frac{i}{2}\kappa\sigma_3 e^{i\sigma_1[k(s-s_1)+\alpha_1]}\Psi. \tag{10}$$

Following [1], let us introduce the quantity

$$\varphi(s) = e^{\frac{i}{2}\sigma_1[k(s-s_1)+\alpha_1]}\Psi(s) \tag{11}$$

From (8), after some algebra, we obtain a diff. equation for φ:

$$\frac{d\varphi}{ds} = \frac{i}{2}(k\sigma_1 + \kappa\sigma_3)\varphi \tag{12}$$

A general solution is

$$\varphi(s) = e^{\frac{i}{2}(k\sigma_1+\kappa\sigma_3)(s-s_1)}\varphi(s_1). \tag{13}$$

Using again the definition (11) to express $\Psi(s)$ and $\Psi(s_1)$, we arrive at the spinor transformation

$$\Psi(s) = M\Psi(s_1) \tag{14}$$

with M a 2×2 matrix,

$$M = e^{-\frac{i}{2}\sigma_1[k(s-s_1)+\alpha_1]}e^{\frac{i}{2}(k\sigma_1+\kappa\sigma_3)(s-s_1)}e^{\frac{i}{2}\sigma_1\alpha_1}. \tag{15}$$

Here, $\alpha_2 = k(s - s_1) + \alpha_1$ is the orientation angle between the helical field and the vertical at the end of the helix.

Let us explicitly calculate the matrix for the full helix, i.e. for $L = s - s_1$, where L is the length of the helix. A general expression for the spin precession matrix of an angle μ around an axis $\vec{b}$ is [5]

$$M = e^{-\frac{i}{2}\mu(\vec{\sigma}\cdot\vec{b})} \tag{16}$$

or

$$M = cos\frac{\mu b}{2} + i\frac{\vec{\sigma}\cdot\vec{b}}{b}sin\frac{\mu b}{2} \tag{17}$$

with $b = |\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2}$. Using this relation we can obtain the expression for μ and $\vec{b}$ as

$$\cos\left(\frac{\mu}{2}\right) = \frac{1}{2}Tr(M). \tag{18}$$

$$\vec{b} = \left[\frac{i}{2}\sin\left(\frac{\mu}{2}\right)\right] Tr(\vec{\sigma}M). \tag{19}$$

Consider the ideal case of a hard edge helix (no fringe field) with a magnetic field starting vertical (snake-like helix) and a field rotation of 360^0

$$\begin{cases} L = \lambda \\ B_x(s_1) = 0 \\ B_y(s_1) = B_0 \\ \alpha_1 = 0 \end{cases} \tag{20}$$

In this case, the matrix for the full helix becomes

$$M = -e^{\frac{i}{2}(k\sigma_1+\kappa\sigma_3)} \tag{21}$$

In the ideal case, from Eq.(21) we obtain the following expression for the precession angle

$$\cos\left(\frac{\mu}{2}\right) = \cos\left[\frac{kL}{2}\sqrt{1+\left(\frac{\kappa}{k}\right)^2}\right]\cos\frac{kL}{2} + \frac{1}{\sqrt{1+\left(\frac{\kappa}{k}\right)^2}}\sin\left[\frac{kL}{2}\sqrt{1+\left(\frac{\kappa}{k}\right)^2}\right]\sin\frac{kL}{2} \tag{22}$$

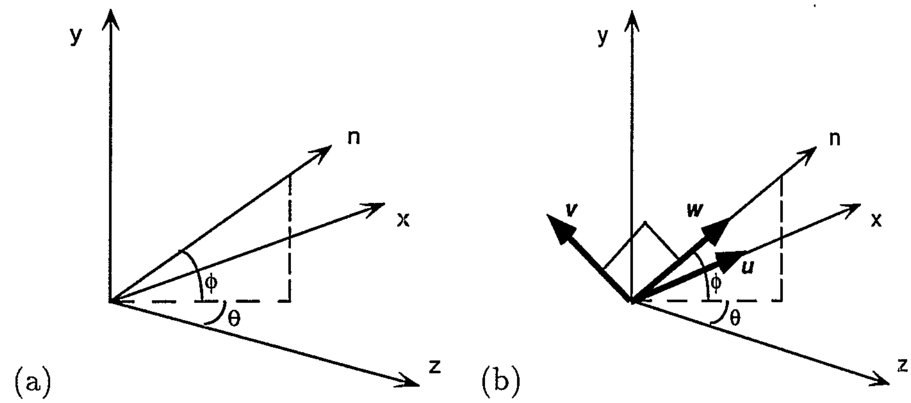

Figure 1: (a) (xyz) is the accelerator coordinate frame and n the normalized precession axis. (b)(uvw) is a coordinate system where the spin precesses around the $\vec{b} \equiv \vec{w}$ axis.

and for the components of the precession axis

$$\begin{aligned}
b_x &= -\frac{\kappa}{k}\frac{1}{\sqrt{1+\left(\frac{\kappa}{k}\right)^2}} \sin\left[\frac{kL}{2}\sqrt{1+\left(\frac{\kappa}{k}\right)^2}\right] \sin\frac{\alpha_2+\alpha_1}{2} \\
b_y &= \frac{\kappa}{k}\frac{1}{\sqrt{1+\left(\frac{\kappa}{k}\right)^2}} \sin\left[\frac{kL}{2}\sqrt{1+\left(\frac{\kappa}{k}\right)^2}\right] \cos\frac{\alpha_2+\alpha_1}{2} \\
b_z &= \frac{1}{\sqrt{1+\left(\frac{\kappa}{k}\right)^2}} \sin\left[\frac{kL}{2}\sqrt{1+\left(\frac{\kappa}{k}\right)^2}\right] \cos\frac{\alpha_2+\alpha_1}{2} - \cos\left[\frac{kL}{2}\sqrt{1+\left(\frac{\kappa}{k}\right)^2}\right] \sin\frac{\alpha_2+\alpha_1}{2}
\end{aligned} \tag{23}$$

For spin tracking with *Spink* [6] we need a 3×3 matrix in the coordinate space. This matrix can be expressed by transforming a rotation around $\vec{b}$ by a rotation of the coordinate system

$$M = R^{-1}PR \tag{24}$$

$$M = \begin{pmatrix} \lambda_1 & \eta_1 & \nu_1 \\ \lambda_2 & \eta_2 & \nu_2 \\ \lambda_3 & \eta_3 & \nu_3 \end{pmatrix} \times \begin{pmatrix} \cos\mu & -\sin\mu & 0 \\ \sin\mu & \cos\mu & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \eta_1 & \eta_2 & \eta_3 \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix} \tag{25}$$

With reference to Fig. 1 let us consider two coordinate systems. (xyz) is the accelerator istantaneous frame, and (uvw) a frame where $\vec{w} \equiv \vec{b}$ ($\vec{u}$ and $\vec{v}$ are arbitrary). λ, μ, ν are the direction cosines of (uvw) with respect to (xyz) given by

$$\lambda = \begin{pmatrix} \cos\theta \\ -\sin\phi\sin\theta \\ \cos\phi\sin\theta \end{pmatrix}, \quad \eta = \begin{pmatrix} 0 \\ \cos\phi \\ \sin\phi \end{pmatrix}, \quad \nu = \begin{pmatrix} -\sin\theta \\ -\sin\phi\cos\theta \\ \cos\phi\cos\theta \end{pmatrix} \tag{26}$$

An explicit expression for the 3×3 rotation matrix is

$$M^{(3)} = \begin{pmatrix} \nu_1^2 + A_{11}c & \nu_1\nu_2 + A_{12}c + B_{12}s & \nu_1\nu_3 + A_{13}c + B_{13}s \\ \nu_1\nu_2 + A_{12}c - B_{12}s & \nu_2^2 + A_{22}c & \nu_2\nu_3 + A_{23}c + B_{23}s \\ \nu_1\nu_3 + A_{13}c - B_{13}s & \nu_2\nu_3 + A_{23}c - B_{23}s & \nu_3^2 + A_{33}c \end{pmatrix} \quad (27)$$

with

$$\left[\begin{array}{l} A_{i,j} = \lambda_i\lambda_j + \eta_i\eta_j \\ B_{i,j} = \lambda_i\eta_j - \eta_i\lambda_j \\ c = \cos\mu \\ s = \sin\mu \end{array} \right. \quad (28)$$

The spin precession is finally characterized by three angles

$$\begin{array}{l} \mu \;\; (\textit{ precession angle around } \vec{b}) \\ \theta = \arctan \frac{b_x}{b_z} \\ \phi = \arctan \frac{b_y}{\sqrt{b_x^2+b_z^2}} \end{array} \quad (29)$$

The matrix form for the whole snake is obtained by multiplying the matrices of the four modules

$$M_{snake} = M_4 \cdot M_3 \cdot M_2 \cdot M_1 \quad (30)$$

For an ideal snake the three angles will be

$$\begin{array}{l} \mu = 180^0 \\ \theta = 45^0 \\ \phi = 0^0 \end{array} \quad (31)$$

4. Optimization

To verify the behaviour of the magnetic field for different energies a fortran program *Mini* has been built. This program is based on the Powell method (without derivatives) [7] for the minimization of a multidimentional function.

We want minimize for instance the analytical expression

$$f(B_1, B_2) \;=\; (\mu_0 - \mu)^2 + (\phi_0 - \phi)^2 + (\theta_0 - \theta)^2$$

where the index 0 refers to the ideal angles given in (31), while μ, ϕ and θ are the angles for a generic rotation. They can be obtained from the snake matrix (30) as:

$$\mu \;=\; \arccos\left[\frac{Tr(M) - 1}{2}\right] \quad (32)$$

$$\phi \;=\; -\arctan\left[\frac{M(1,2) + M(2,1)}{M(1,3) + M(3,1)}\right] \quad (33)$$

$$\theta = \arccos\left[\sqrt{\frac{2\,(1-M(1,1))}{3-Tr(M)}}\right] \tag{34}$$

Powell uses some subroutines for the unidimensional minimization along a set of linearly indipendent directions.

5. Magnetic fields

Because of the presence of κ in the snake matrix, the fields will change with proton energy. This means that to obtain (31) for the spin rotation angles, B_1 and B_2 shall vary. The results are shown in Fig. 2 and Fig. 3.

The two fields grow quickly at the injection and for highest energies they become constant. Considering the expression of κ, we have
- Low energies, $G\gamma \approx 1 \Rightarrow \kappa \propto \kappa(\gamma)$.
- High energies, $G\gamma \gg 1 \Rightarrow \kappa \simeq \frac{GeB_0}{mc}$ $(constant)$.

If magnetic fields don't change, the angles would not maintain the ideal values. In Fig. 4 and Fig. 5 we consider a constant value of magnetic fields for every energy. We have a perfect spin rotation only for a given energy while for different energies the angles are no more the right ones.

6. Spin tracking

As a first exercise we can analyze the spin tracking for two different range of energy applying the optimized B_1 and B_2 obtained from the minimization program. If we reverse the fields, we can compared the results in Fig. 6 and Fig. 7. There is a clear difference between the two cases. This means that optimum field values for a high energy range can be applied also to the lowest range, but the opposite is not true. To explain this, recall that the resonance amplitudes are in general stronger at higher proton energy as shown in Fig. 8 [8]. If we decide to vary the magnetic fields with energy, this variation is larger at the injection than at the highest energies, where, as previously obtained, B_1 and B_2 are almost constant. Therefore, it may be preferable to use fixed field values.

7. Acknowledgments

Discussions with A. Luccio, M. Syphers, E. Courant are acknowledged.

8. References

1. E.D.Courant, Rept. BNL-61920,AD/RHIC-133, Upton, NY, June 1995.
2. M.J.Syphers, Spin Note AGS/RHIC/SN No 020, Upton, NY, February 1996. ed.1962, p.558.
3. J.P. Blewett and R. Chasman, Journal Appl. Phys. 48 (1977) 2692.
4. V. Bargman, L. Michel and V.L. Teledgi, Phys. Rev. Letters, 2 (1959) 435.
5. B.W.Montague, *Polarized Beams in High Energy Storage Rings*, Phys. Reports 113, No.1 (1984) p.1-96.
6. A.U.Luccio, Rept. BNL-52481, Upton, NY, September 1995.
7. W.H. Press et al., *Numerical Recipes*, Cambridge (1992).
8. E.D. Courant e R.D. Ruth, Rept. BNL-51270, Upton, NY September 1980.

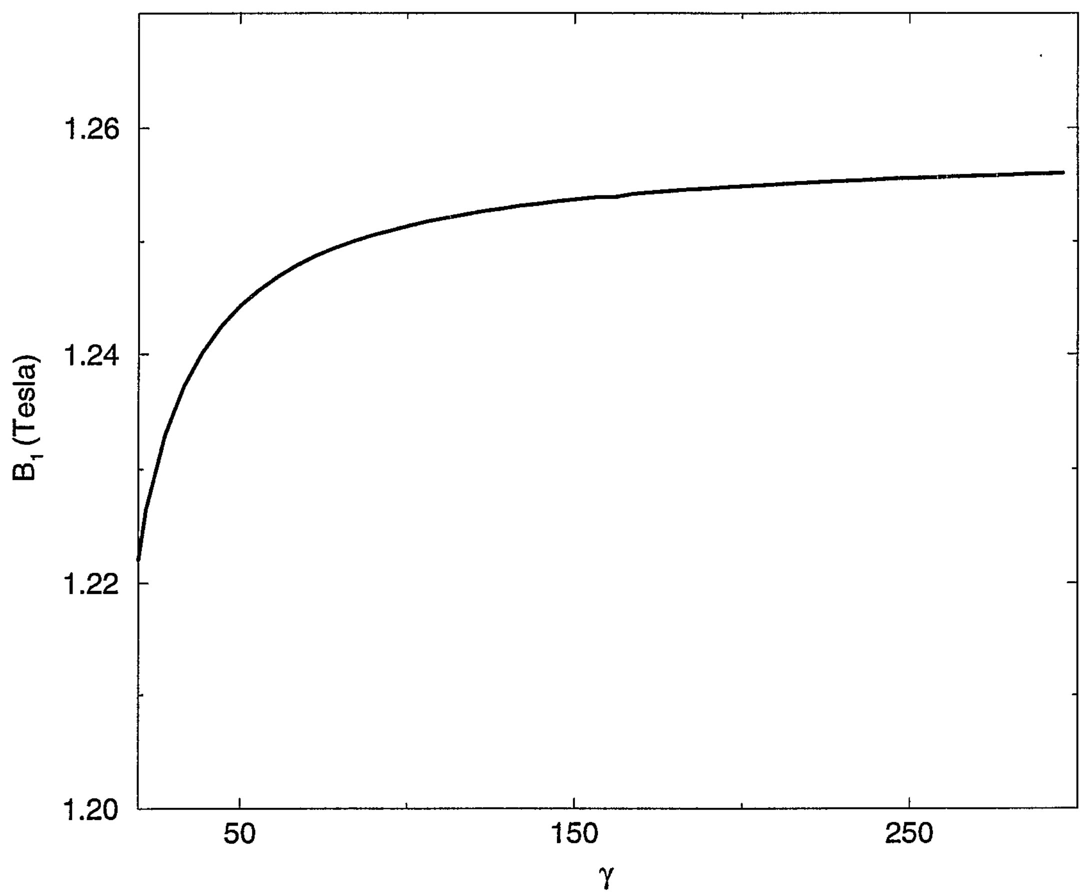

Figure 2: Optimized magnetic field in the outer helices as a function of the energy

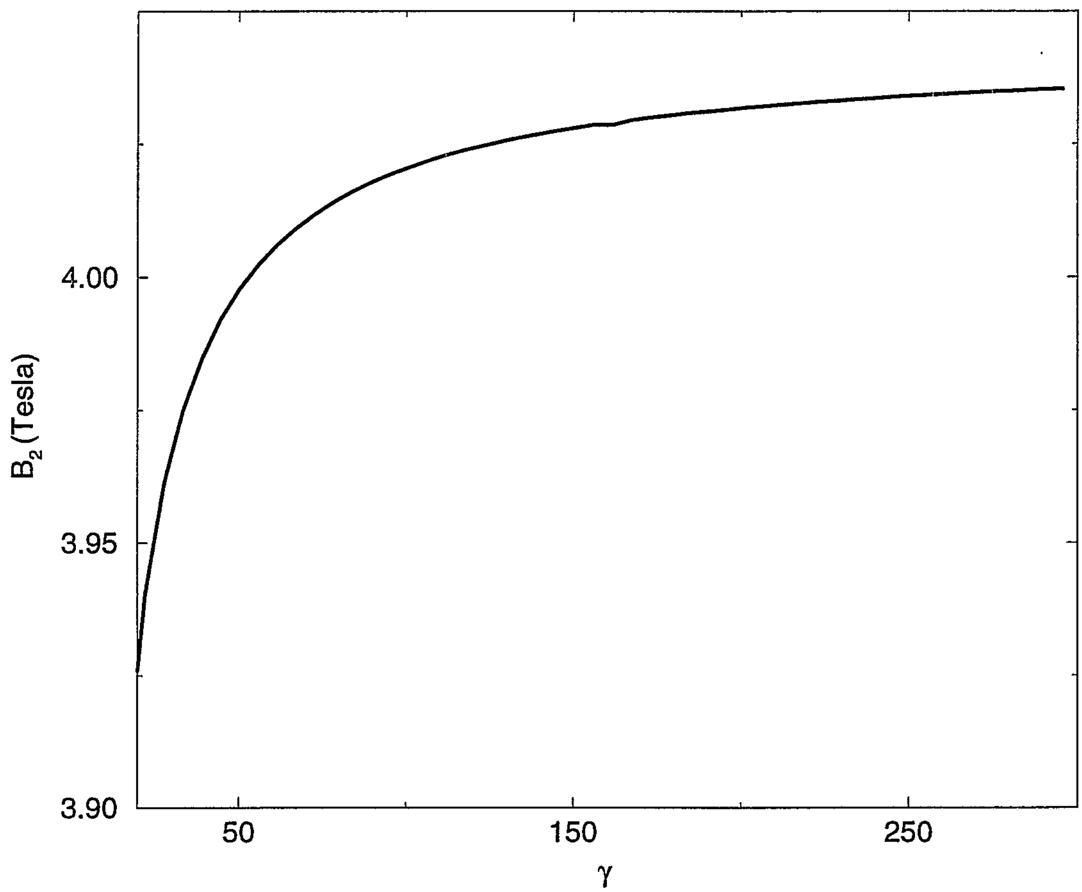

Figure 3: Optimized magnetic field in the inner helices as a function of the energy

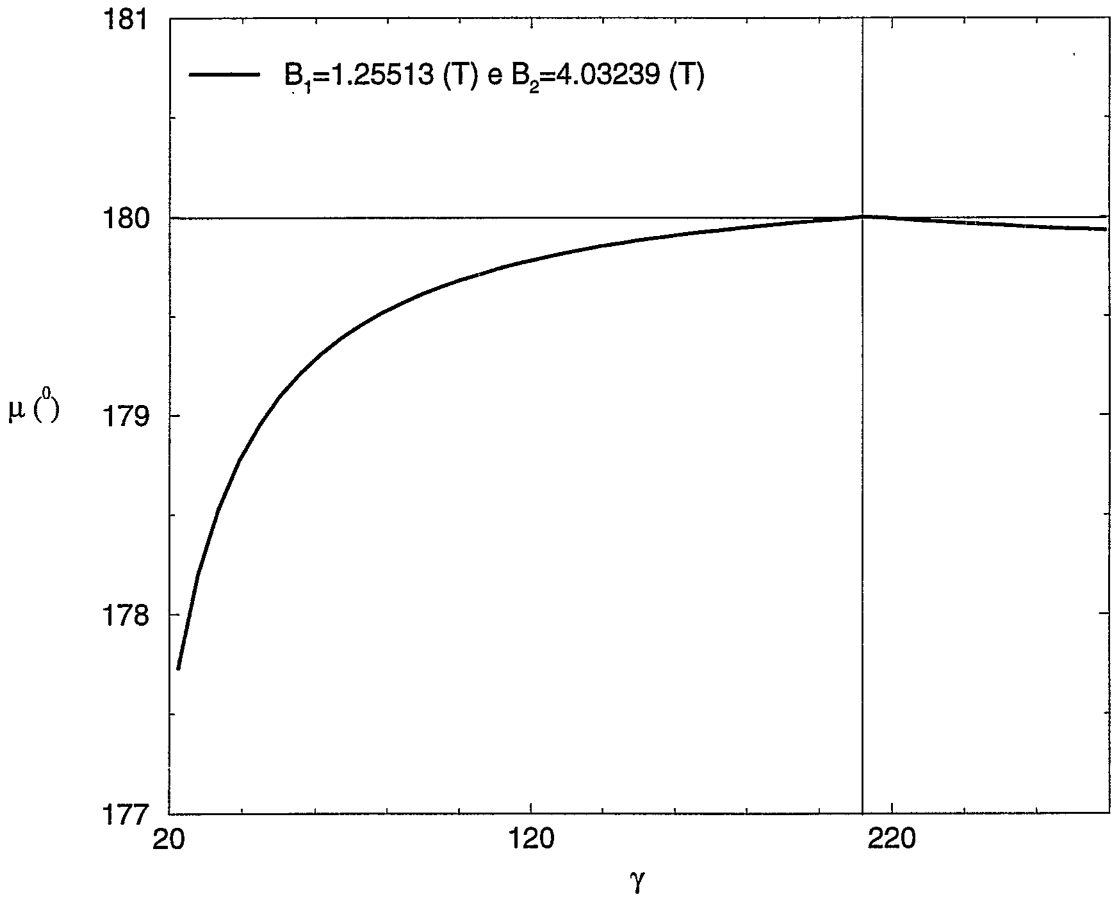

Figure 4: Rotational angle as a function of energy for a fixed value of B_1 and B_2

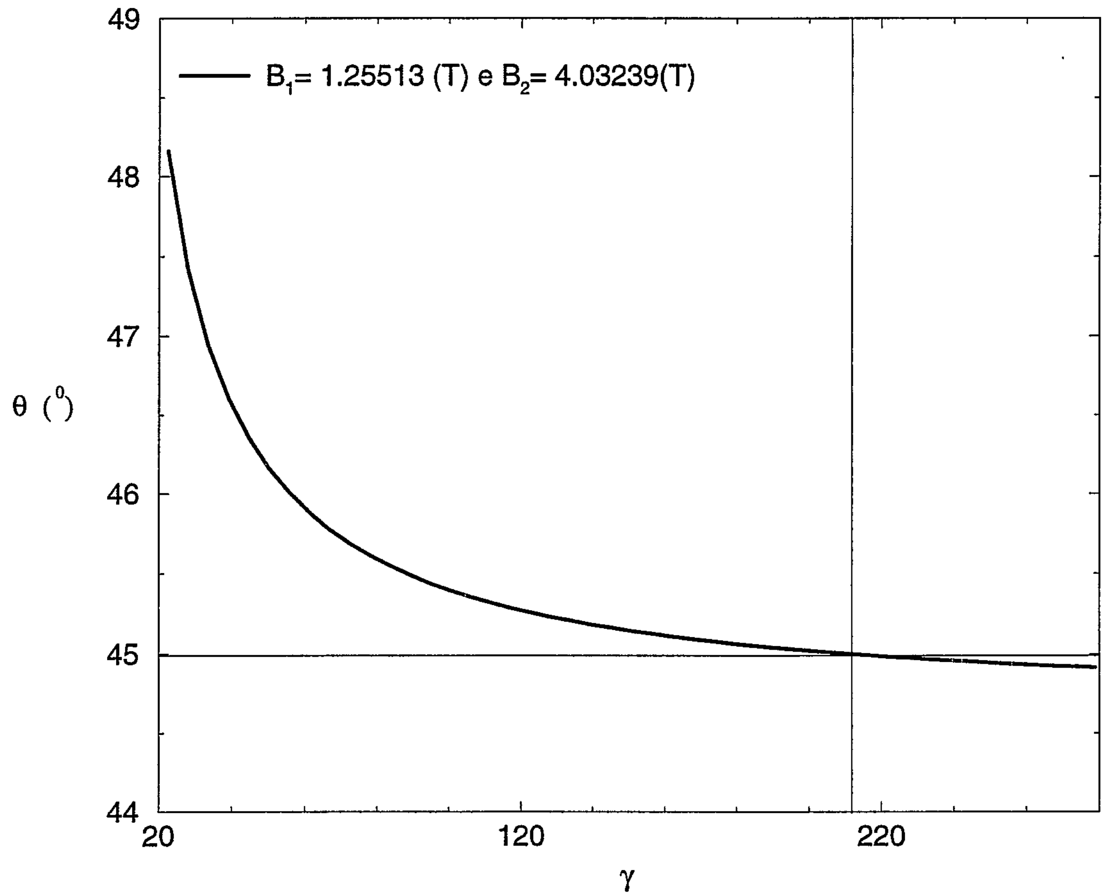

Figure 5: Precession axis angle as a function of energy for a fixed value of B_1 and B_2

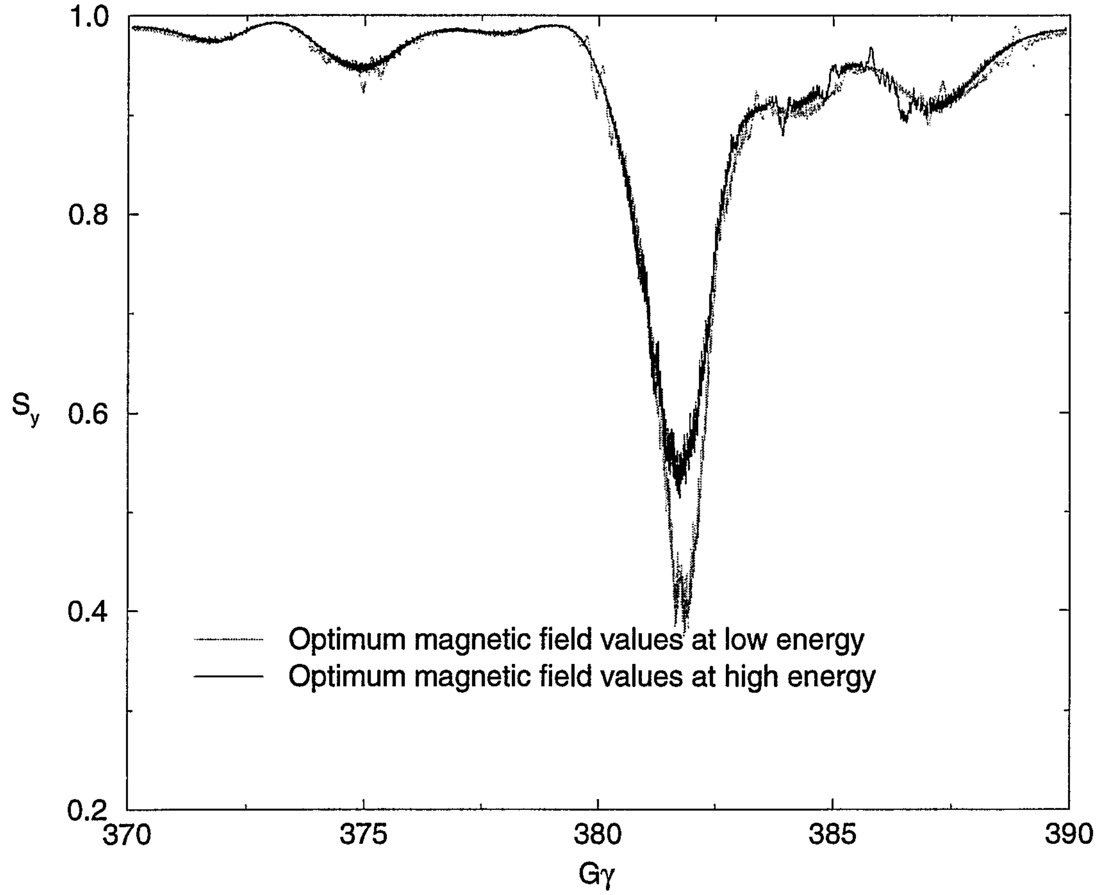

Figure 6: Comparison between different magnetic fields values to overcome the $G\gamma = 381.82$ resonance.

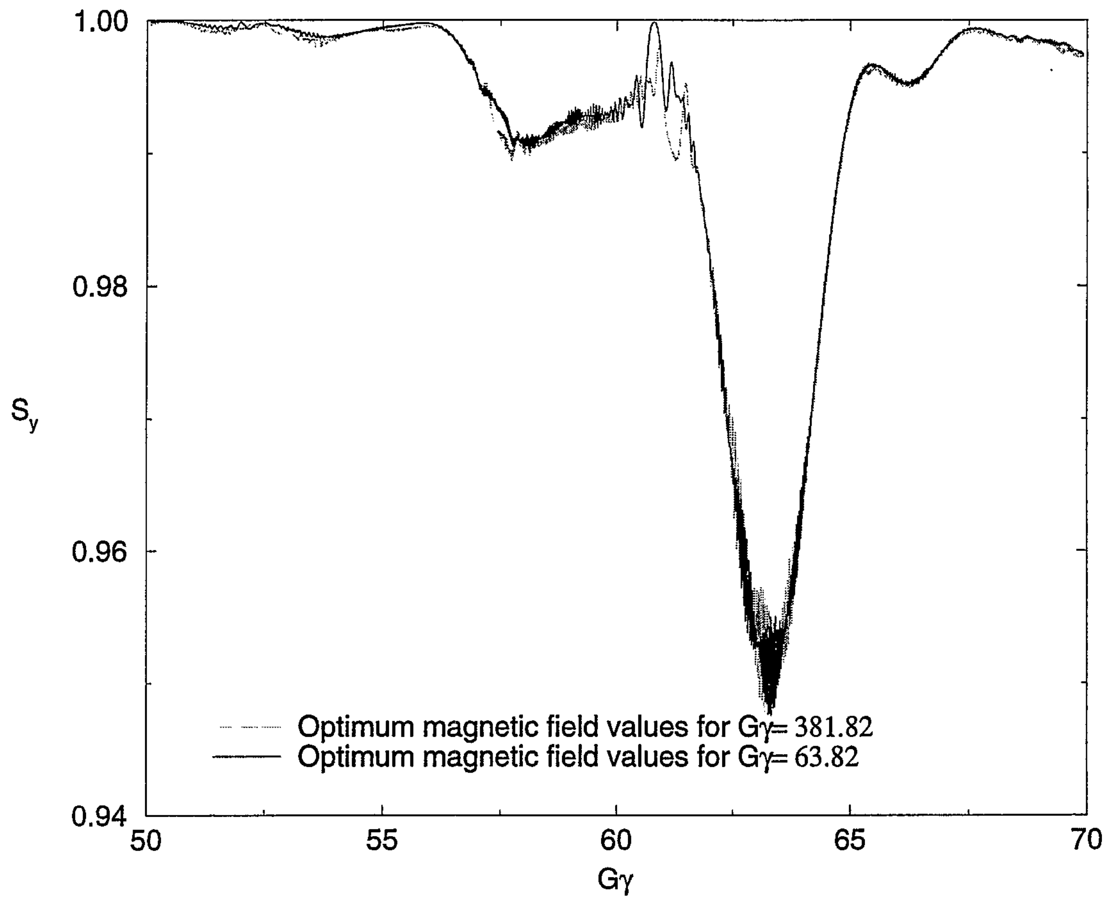

Figure 7: Comparison between different magnetic fields values to overcome the $G\gamma = 63.82$ resonance.

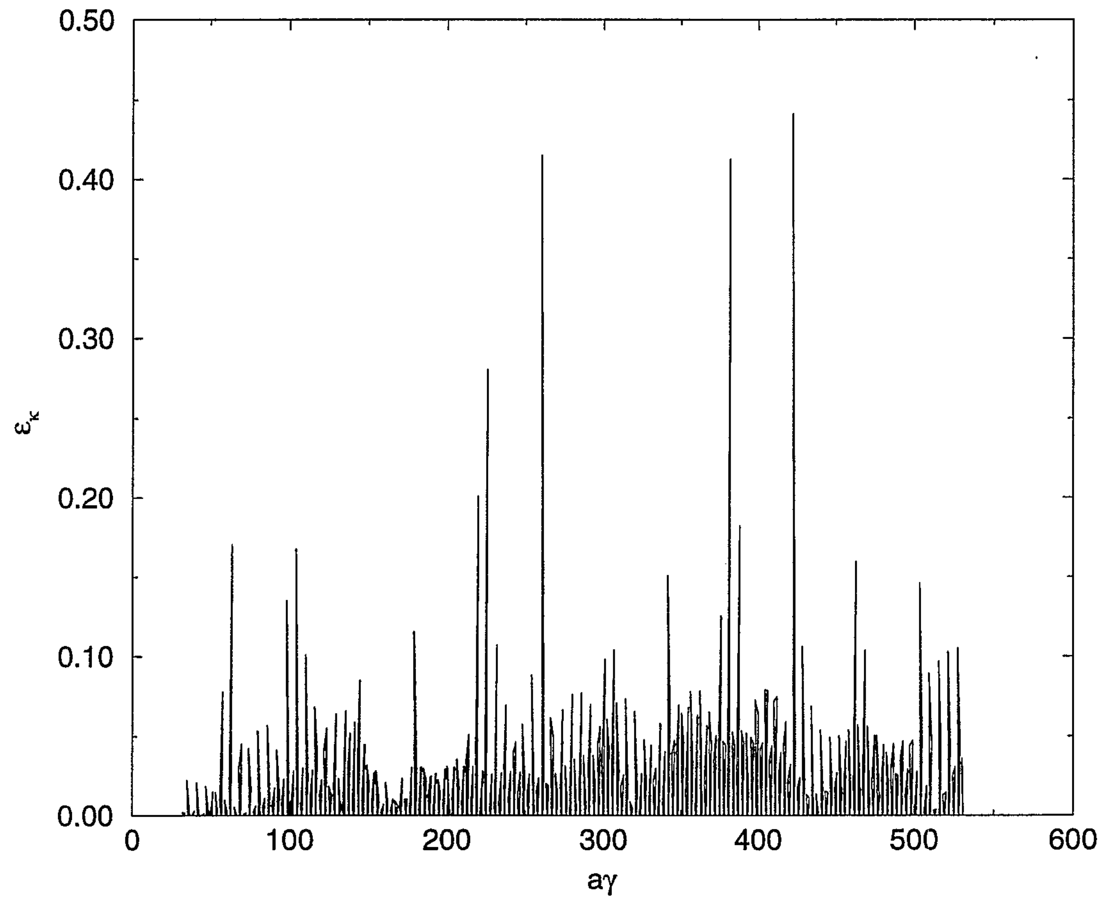

Figure 8: Intrinsic resonance strenght