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The Spin Tracking Study in RHIC for Polarized Proton

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Spin Note

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In this report, the spin tracking through the RHIC accelerator is present, the numerical calculation of first order is developed from 2- dimensions to 4- dimensions, including the Siberian snake and sextupole magnets. According to calculation, the setup for Siberian snake and the relationship of parameters of Siberian snake and parameters of inject beam are discussed. The calculation results of second order are present and compared with that of first order calculation.

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Abstract

In this report, the spin tracking through the RHIC accelerator is present, the numerical calculation of first order is developed from 2- dimensions to 4- dimensions, including the Siberian snake and sextupole magnets. According to calculation, the setup for Siberian snake and the the relationship of parameters of Siberian snake and parameters of inject beam are discussed. The calculation results of second order are present and compared with that of first order calculation.

1. Introduction

1) *The project of polarization proton:* PHENIX/Spin collaboration is planing, under the RIKEN-BNL agreement, to install Siberian snake magnets and spin rotator in the RHIC accelerator to accelerate polarized protons from 25GeV up to 250GeV, and to study interrelated detectors and physics. Our group worked on researching and making the Siberian snake magnets, studying spin tracking in the snakes and in the RHIC ring.

2) *The structure of RHIC:* [1] The RHIC collider will be constructed in an existing 3.834 km long tunnel, after and taking AGS as injection. The collider consists of two ring of superconducting magnets. The main components of the magnet system are 288 arc-size dipoles and 108 insertion dipoles, and 276 arc and 206 insertion quadrupoles. In addition to dipoles and quadrupoles, There are an inventory of smaller magnets consisting of 72 trim quadrupoles, 288 sextupoles and 492 corrector magnet at each quadrupole. The two ring are symmetric setup with reverse transport beam direction, in a common horizontal plane, oriented to intersect with one another at six crossing points. Each ring consists of three inner arc and three outer arc and six insertions joining the inner and outer arcs. Each arc is composed of 11 FODO cells. The insertion has nine quadrupoles and six dipoles on each side of the crossing point. This kinds of periodic structure will produce the phase of beam resonance, including depolarization of intrinsic resonance for polarization beam. The accelerator design used the periodic structure to maintain the beam, on the other way to avoid the resonance as the possible [2].

3) *Background:* The theory of depolarization resonance were studied by lot of people [3–6], the spin tracking of polarization proton in RHIC was studied by A. Luccio with 2- dimensions (y, y') first order numerical calculation [7], and the high order of spin trace in various magnets was studied in previous report [8]. This report will show the improvements of first order numerical calculation from 2-D to 4-D, including sextupoles magnets and Siberian snake into numerical calculation, and show results of second order calculation.

2. Orbit from Betatron Functions

The property of betatron functions can be used to calculate the parameters of individual particle trajectory anywhere along a beam line [9] if the betatron functions are known. The particle trajectory can be written as

$$u(s) = a\sqrt{\beta}cos\psi + b\sqrt{\beta}sin\psi,$$

and the amplitude factors are given by the initial conditions

$$a = \frac{u_o}{\sqrt{\beta_o}},$$
$$b = \sqrt{\beta_o}u'_o + \frac{\alpha}{\sqrt{\beta_o}}u_o.$$

Then the trajectory and its derivative are

$$\begin{aligned} u(s) = & \sqrt{\frac{\beta}{\beta_o}}(cos\psi + \alpha_o sin\psi)u_o + \sqrt{\beta\beta_o}sin\psi\ u'_o \\ u'(s) = & \frac{1}{\sqrt{\beta\beta_o}}[(\alpha_o - \alpha)cos\psi - (1 + \alpha_o\ \alpha)sin\psi]u_o \\ & + \sqrt{\frac{\beta_o}{\beta}}(cos\psi - \alpha_o sin\psi)u'_o. \end{aligned}$$

The equations can be written as matrix formulation, if we assume that x and y are uncoupled motion, there are:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \begin{vmatrix} \sqrt{\frac{\beta_{2x}}{\beta_{1x}}}(cos\Delta\psi_x + \alpha_{1x}sin\Delta\psi_x) & \sqrt{\beta_{1x}\beta_{2x}}sin\Delta\psi_x \\ \frac{\alpha_{1x}-\alpha_{2x}}{\sqrt{\beta_{1x}\beta_{2x}}}cos\Delta\psi_x - \frac{1+\alpha_{1x}\alpha_{2x}}{\sqrt{\beta_{1x}\beta_{2x}}}sin\Delta\psi_x & \sqrt{\frac{\beta_{1x}}{\beta_{2x}}}(cos\Delta\psi_x - \alpha_{2x}sin\Delta\psi_x) \end{vmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1$$

$$\Delta\psi_x = \psi_{2x} - \psi_{1x}$$

$$\begin{pmatrix} y \\ y' \end{pmatrix}_2 = \begin{vmatrix} \sqrt{\frac{\beta_{2y}}{\beta_{1y}}}(cos\Delta\psi_y + \alpha_{1y}sin\Delta\psi_y) & \sqrt{\beta_{1y}\beta_{2y}}sin\Delta\psi_y \\ \frac{\alpha_{1y}-\alpha_{2y}}{\sqrt{\beta_{1y}\beta_{2y}}}cos\Delta\psi_y - \frac{1+\alpha_{1y}\alpha_{2y}}{\sqrt{\beta_{1y}\beta_{2y}}}sin\Delta\psi_y & \sqrt{\frac{\beta_{1y}}{\beta_{2y}}}(cos\Delta\psi_y - \alpha_{2y}sin\Delta\psi_y) \end{vmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}_1$$

$$\Delta\psi_y = \psi_{2y} - \psi_{1y}$$

where, the β_x, β_y are amplitude function β projected to x and y direction, ψ_x, ψ_y are phase functions, and α_x, α_y are correlation functions ($\alpha_x = (1/2)(\partial\beta_x/\partial s)$, respectively.

The code *spink* uses the *twiss* file as input parameters that is calculated from MAD program. From the matrix, the position and velocity of x and y at entrance (denoted by 1) and exit (denoted by 2) of each element and their average value inside the element are given easily if the initial value are well known.

The momentum for s direction in the RF cavity is increased for increasing a constant energy at $\Delta E = 4 \times 10^{-5}$ GeV/circle. For longitudinal acceleration, the transverse momentums are conservation. There are:

$$x'_o = \frac{M_i}{M_o}\frac{v_{si}}{v_{so}}x'_i; \quad y'_o = \frac{M_i}{M_o}\frac{v_{si}}{v_{so}}y'_i$$

where M, v_s are mass and longitudinal velocity of particle, a prime represents differentiation by s, i and o denote the values of input or output points. The mass part is dominant for relativistic particle, the velocity part is dominant for non-relativistic particle. For RHIC, both of all are very close 1.0 ($\frac{M_i}{M_o} = 1 - 10^{-6}$ to $1 - 10^{-7}$, $\frac{v_{si}}{v_{so}} = 1 - 10^{-9}$ to $1 - 10^{-13}$). For high precision in our calculation, there are:

$$x'_o = (1 - \frac{\Delta\gamma}{\gamma})(\frac{(\gamma+\Delta\gamma)\sqrt{\gamma^2-1}}{\gamma\sqrt{(\gamma+\Delta\gamma)^2-1}})x'_i$$

$$y'_o = (1 - \frac{\Delta\gamma}{\gamma})(\frac{(\gamma+\Delta\gamma)\sqrt{\gamma^2-1}}{\gamma\sqrt{(\gamma+\Delta\gamma)^2-1}})y'_i$$

$$\gamma = E_{total}/m_oC^2; \quad \Delta\gamma = \Delta E/m_oC^2$$

3. Spin Tracking in Each Elements

The spin motion is given by the BMT equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_{BMT} \times \vec{S}$$

with

$$\vec{\Omega}_{BMT} = -\frac{e}{m_o\gamma c}[(1+\gamma G)\vec{B}_\perp + (1+G)\vec{B}_\parallel - (G + \frac{1}{1+\gamma})\gamma\beta \times \frac{\vec{E}}{c}],$$

the term of electric field can be negligible. The equation is rewritten in the nature coordinates (x,y,s) [8] as follows

$$\frac{d\vec{S}}{ds} = \vec{S} \times \vec{P}$$
$$\vec{P} = \frac{h}{(B\rho)}[(1+\gamma G)\vec{B} - G(\gamma-1)\frac{1}{v^2}(\vec{v} \bullet \vec{B})\vec{v})$$
$$h = 1/\sqrt{1+x'^2+y'^2},$$

the set of scalar equations are

$$\frac{dS_x}{ds} = S_s P_y - S_y P_s$$
$$\frac{dS_s}{ds} = S_y P_x - S_x P_y$$
$$\frac{dS_y}{ds} = S_x P_s - S_s P_x$$

and here

$$P_x = \frac{h}{(B\rho)}[(1+G\gamma)(-x'B_s + B_x) + (1+G)x'(x'B_x + B_s + y'B_y)]$$
$$P_s = \frac{h}{(B\rho)}[(1+G\gamma)(-x'B_x + y'B_y) + (1+G)(x'B_x + B_s + y'B_y)]$$
$$P_y = \frac{h}{(B\rho)}[(1+G\gamma)(-y'B_s + B_y) + (1+G)y'(x'B_x + B_s + y'B_y)].$$

This set of equations yields three order linear equations for the three components of the spin [7]

$$S''' + \omega^2 S' = 0$$
$$\omega^2 = P_x^2 + P_s^2 + P_y^2.$$

If the strength of the field keep a constant, or the transport path length s close 0, the general integral is

$$S = C_1 + C_2 cos\omega\delta s + C_3 sin\omega\delta s,$$

the constants of integration can be found as a function of initial value of spin components using S, S' and S'' and the field of the original system. The variation of spin components also can be written as matrix formulation [7]:

$$\begin{vmatrix} 1-(B^2+C^2)c & ABc+Cs & ACc-Bs \\ ABc-Cs & 1-(A^2+C^2)c & BCc+As \\ ACc+Bs & BCc-As & 1-(A^2+B^2)c \end{vmatrix}$$

with

$$\{ \begin{array}{c} c = 1 - cos\omega\delta s \\ s = sin\omega\delta s \end{array}, A = \frac{P_x}{\omega}, B = \frac{P_y}{\omega}, C = \frac{P_s}{\omega}.$$

The spin rotation strength function P and the magnetic field B are dependent upon position x, y, we can only keep the position terms and ignore the velocitis terms, because the $x' = dx/ds, y' = sy/ds$ are very small at relativistic energy region, and $h \doteq 1$. If the magnetic field is taken as hard boundary field, $B_s = 0$. We reduce to give in first order calculation:

$$P_x = \frac{1}{(B\rho)}(1 + G\gamma)B_x$$

$$P_s = 0$$

$$P_y = \frac{1}{(B\rho)}(1 + G\gamma)B_y.$$

1). *Dipole* (horizontal bend) *magnet*

$$B_x = 0, B_s = 0, B_y = B\rho/\rho$$

$$P_x = 0, P_s = 0, P_y = (1 + G\gamma)/\rho$$

$$A = 0, B = 1, C = 0$$

$$\omega\delta s = (1 + G\gamma)\delta s/\rho = (1 + G\gamma)\delta\theta$$

here $\delta\theta$ is the bend angle. The matrix is

$$\left| \begin{array}{ccc} cos\omega\delta s & 0 & -sin\omega\delta s \\ 0 & 1 & 0 \\ sin\omega\delta s & 0 & cos\omega\delta s \end{array} \right|.$$

2). *Quadrupole magnet*

$$k_1 = -\frac{\partial\ B/\partial\ r}{(B\rho)}, 1/\rho = 0$$

$$B_x = k_1(B\rho)y, B_y = k_1(B\rho)x, B_s = 0$$

$$P_x = k_1(1 + G\gamma)y, P_y = k_1(1 + G\gamma)x, P_s = 0$$

$$A = y/r, B = x/r, C = 0, r = \sqrt{x^2 + y^2}$$

$$\omega\delta s = k_1(1 + G\gamma)r\delta s$$

the matrix is

$$\frac{1}{r^2}\left| \begin{array}{ccc} y^2 + x^2 cos\omega\delta s & xy(1 - cos\omega\delta s) & -xr sin\omega\delta s \\ xy(1 - cos\omega\delta s) & x^2 + y^2 cos\omega\delta s & yr sin\omega\delta s \\ xr sin\omega\delta s & -yr sin\omega\delta s & r^2 cos\omega\delta s \end{array} \right|$$

if $x = 0$, the matrix become very simple as the code *spink*.

3). *Sextupole magnet*

$$k_2 = -\frac{\partial^2\ B/\partial\ r^2}{(B\rho)}, 1/\rho = 0$$

$$B_x = k_2(B\rho)xy, B_y = \tfrac{1}{2}k_2(B\rho)(x^2 - y^2), B_s = 0$$

$$P_x = k_2(1 + G\gamma)xy, P_y = \tfrac{1}{2}k_2(1 + G\gamma)(x^2 - y^2), P_s = 0$$

$$A = 2xy/r^2, B = (x^2 - y^2)/r^2, C = 0, r = \sqrt{x^2 + y^2}$$

$$\omega\delta s = \tfrac{1}{2}k_2(1 + G\gamma)r^2\delta s$$

the matrix is

$$\frac{1}{r^4}\begin{vmatrix} 4x^2y^2+(x^2-y^2)^2cos\omega\delta s & 2xy(x^2-y^2)(1-cos\omega\delta s) & (y^2-x^2)r^2sin\omega\delta s \\ 2xy(x^2-y^2)(1-cos\omega\delta s) & (x^2-y^2)^2+4x^2y^2cos\omega\delta s & 2xyr^2sin\omega\delta s \\ (x^2-y^2)r^2sin\omega\delta s & -2xyr^2sin\omega\delta s & r^4cos\omega\delta s \end{vmatrix}$$

if $x=0$, the matrix also become very simple. Old *spink* code neglected the action of all sextupole magnets.

4) *The Siberian Snake*

In Siberian snake, because the magnetic field is rotated $4\times 2\pi$ degree in all four modules, we only can think a thin part of field, the field keep a constant. If the length of each module is L, the field rotation is 2π in each model, and the strength of magnetic field is B_{SN}, we analyse the spin matrix at s to $s+\delta s$:

$$B_x=-B_{SN}sin(\phi), B_y=B_{SN}cos(\phi), B_s=0, \phi=\frac{2\pi}{L}s$$

It is

$$f_x=-\frac{(1+G\gamma)B_{SN}}{(B\rho)}sin\phi, f_y=\frac{(1+G\gamma)B_{SN}}{(B\rho)}cos\phi, f_s=0$$
$$\omega=\frac{(1+G\gamma)B_{SN}}{(B\rho)}$$
$$A=-sin\phi, B=cos\phi, C=0$$

producing the matrix

$$\begin{vmatrix} 1-cos^2\phi\,(1-cos\delta\psi) & -sin\phi\,cos\phi\,(1-cos\delta\psi) & -cos\phi\,sin\delta\psi \\ -sin\phi\,cos\phi\,(1-cos\delta\psi) & 1-sin^2\phi\,(1-cos\delta\psi) & -sin\phi\,sin\delta\psi \\ cos\phi\,sin\delta\psi & sin\phi\,sin\delta\psi & cos\delta\psi \end{vmatrix}$$

with the spin rotation angle

$$\delta\,\psi=\omega\delta\,s=\frac{(1+G\gamma)B_{SN}}{(B\rho)}\delta\,s$$

The previous code made a matrix used input angular parameter. We give the matrix by following way: divided the length L into N parts, the field value of each part is thought as a constant that dependent the position. Matrix is constructed for each part and multiplied together, the total matrix of Siberian snake is given by multiplying four matrix respectively for each module. Comparing with the precise calculation [8)], if we divided each part in the length of $0.1cm$, the precise matrix of Siberian snake is easy to be got.

4. Compare Results Between Improved Code and *SPINK*

The previous code *spink* was developed by: < 1 > The orbit motion changes from y, y' two dimensions into x, x', y, y' four dimensions. < 2 > The spin depolarization is calculated that is affected by x and y in various magnets. < 3 > The sextupole magnets are take into account in depolarization calculation. < 4 > The spin matrix of Siberian snake, that is dependent on beam energy, from described parameters, are instead of from numerical calculation for a real Siberian snake. < 5 > The focusing action of RF is renew.

Figure 1 shows the depolarization spectrum in the procedure of acceleration with pervious code (a), with improvement *RF* but two dimensions (b), and improvement code four dimensions including sextupole (c). Initial conditions are: y emittance is 10 $\mu rad-m$, y and y' are maximum value with a angle $\theta_y = 60^o$ between the direction and position axis in initial phase ellipse for (a) and (b), x and y emittance are 10 $\mu rad-m$ $x, x'y, y'$ are maximum value with $\theta_y = 60^o$ and $\theta_x = 60^o$ for (c). The spin flippers didn't use in all calculation.

The variation of y direction spin mainly dependent on the magnetic field of x and s direction. In general case, the field of s direction can be neglected. The depolarization can be neglected in dipole magnets, because it have only y direction field if spin direction only along y. The most quadrupole magnets, except skewed quadrupole, the field of x direction is only dependent on the position of y. The 2- dimensions calculation predicted the depolarization peaks of position correctly in some words, but the amplitude is quite different. The difference between (b) and (c) mainly produced by the periodic y spin lost and recover, but the recover process is dependent on the magnetic field of y direction, thus is relative to x position in qudrupole, but that is neglected in 2- dimensions calculation. The total rotation strength of spin direction is proportion to $r = \sqrt{x^2 + x^2}$ in quadrupole magnets, and betatron amplitude of x is large the that of y for beam orbit. The little bit difference produced by the sextupoles, because the strength of magnetic field is weak, and the length along transport line is short.

The depolarization peaks in *spink* calculation are lost in higher energy part for a incorrect focusing action in *RF* (a). The 2- dimensions calculation (*spink*) also give us the tree strongest peaks with correct RF (b). The 4- dimensions calculation give more precision peak positions and strengths (c), compared with the results from the *Depol* code (Fig. 5).

5. Work Status of Siberian Snake

If one polarized particle is transported along the center line of ring, the orbit just change the direction in the bend magnets, the polarization will maintain a constant. If the initial position and velocity aren't zero relative to the center line of transport, the values of x, x', y, y' will be changed by dipole, quadrupole, sextupole magnets, the values at any position of transport line are keep in a line of phase ellipse, the spin little bit lost and recover in the process of acceleration. At resonance area, the polarization will lost. The Siberian snake help beam keep beam polarization through depolarization resonance area. But the imprefect working of Siberian snake also produce depolarization.

The all magnets are taken as thin slice in first order calculation, the magnetic field was determined by using that of one position of a particle orbit. But Siberian snake is very different, its field change direction, the strength of rotation spin change although the transverse position keep same approximately. In our work, matrixes of Siberian snake given both numerical and analytic calculation.

We compare the energy dependent matrix in Siberian snake for first order calculation. The parameters are: length of each of four modules is $240cm$, hard edge field approach is set, magnetic field are $1.25T, -4.00T, 4.00T, -1.25T$ for each module, respectively.

The matrix at $25GeV$ is

$$\begin{vmatrix} 0.048495 & -0.014904 & 0.998712 \\ 0.014896 & -0.999767 & 0.015643 \\ 0.998712 & 0.015635 & -0.048262 \end{vmatrix}$$

and that at $250GeV$ is

$$\begin{vmatrix} 0.040900 & 0.010418 & 0.999109 \\ -0.010425 & -0.999896 & 0.009991 \\ 0.999109 & -0.010007 & 0.041005 \end{vmatrix}.$$

there are little change. Taking a set of constant magnetic field in our first order calculation haven't meet trouble.

We have calculated that one magnet is depolarization, and the polarization for periodic setup should be lost and recover periodically [8]. In the Fig. 2(a), the depolarization spectra is given for one circle for RHIC in first order calculation, at non-resonance area ($G\gamma = 50$), no Siberian snake and spin flipper involved, with initial condition: x emittance is 10, y emittance is 10 unit with $\mu rad - m$, the position and velocity are at point of phase ellipse with a angle $\theta_y = 60^o$, $\theta_y = 60^o$ between position axis's. Although the strength of spin rotation is one way and another way reverse in the DOFO periodic cell, liking focusing and dispersing in DOFO periodic cell, there are a lot of frequency periodic structure adding together, the periodic behavior isn't remarkable. At depolarization resonance area, the polarization is decreasing or increasing mainly, it is difficult to see the fine structure of lost and recover.

In the Fig. 2(b), depolarization in process of acceleration shows in a small emittance x is 5, and y is 5, no Siberian snake and flipper involved. There are some periodic structure, the polarization approximately keep a constant in one period, vibration appear at the resonance region of depolarization spectrum with Siberian snake. The same spectrum from 2- dimensions calculation can give the constant keeping very well. The value of variant polarization is unknown between two periods. If the beam is injected RHIC with polarization beam, and it con't be keep in the ring during acceleration process, the polarization will be lost and recover, at some special energy region the polarization recover to close to 1. But it is difficult to find where and how much polarization.

A spin matrix in prefect Siberian snake have the terms $A13 = A31 = 1$, $A22 = -1$, the other terms equal 0. But the real situation is different, imperfect terms will produce little bit depolarization. If there just use one or two but with same setup, the depolarization by Siberian snake should add together, polarization lost little by little up to all during the acceleration process. If two Siberian snake used special setup, the depolarization produced by one and it recover by another when neglected the other magnets in the ring. The depolarization for Siberian snake will reduce to a minimum for working at different beam energy and technical deficiency of making snake. Fig. 3 shows two kinds of Siberian snake setup and its spectrum of depolarization. The prefect setup of Siberian snake should have reverse magnetic field and with reverse helicity.

The status of imperfect working of Siberian snake make the spectrum of depolarization bad. The Fig. 4 shows depolarization spectrum with imperfect working of Siberian snake with parameters $1.25T, -4.00T, 4.00T, -1.25T$ at different emittance.

The Siberian snake with Imperfect working status make it impossible to acceleration the particle with large emittance. The parameters of working of Siberian snake is more far from perfect working point, the emittance of particle is more small that can be acceleration and keep polarization. In the first order calculation, the Siberian snake consits of four perfect dipole helix with B_x, B_y only simple sine and cosine term, have perfect working point at $1.2319T, -3.9580T, 3.9580T, -1.2319T$ with analytic calculation, and at $1.2336T, -3.9570T, 3.9570T, -1.2336T$ with numerical calculation. But it is at $1.0723T, -3.9328T, 3.9328T, -1.0723T$ in second order calculation for a describe of precise magnetic field. This will discuss in detail in next report.

6. Second Order Calculation of Spin Tracking

In the section 2, the position and velocity of particle can be given at the entrance and exit point of each elements from the lattice functions for closed orbit of RHIC, if initial condition is well known. The spin tracking of first order used the position and velocity of exit point or average value between exit point and entrance point as the that for the element as a whole. The second order should use the position and velocity as well as the magnetic field to calculate spin in detail inside of elements. But the 9-dimensions or 7- dimensions intergral calculations [8] use too much CPU time to do for spin tracking, we try to use the analytic expression for particle orbit in my second order calculation. It reduce the CPU time of computation and give those precision quantities those orbit is independent with integral step.

According to the Lorentz equation, the orbit of particle in nature coordinate system can be written as the analytic expression, for an initial condition of $[x_c, y_c, x'_c, y'_c]$ and final condition of $[x_s, y_s, x'_s, y'_s]$, (here, the prime denotes differentiation by s). The length of element is L, and transport distance from entrance point is s:

1). *Dipole* (horizontal bend) *magnet*

$$x' = x'_c + \frac{x'_s - x'_s}{L} s; \quad y' = y'_c = y'_s$$
$$x = x_c + x'_c s + \frac{1}{2}\frac{x'_s - x'_s}{L} s^2; \quad y = y_c + y'_c s$$

here, the motion of uniform acceleration is assumed.

2). *Quadrupole magnet*

For focusing of horizontal plane x, there is $K_1 < 0$ of the strength parameter:

$$x = x_c\, cosks + \frac{x'_c\, sinks}{k}; \quad y = y_c\, coshks + \frac{y'_c\, sinhks}{k}$$
$$x' = x'_c\, cosks + K_1 \frac{x_c\, sinks}{k}; \quad y' = y'_c\, coshks - K_1 \frac{x_c\, sinks}{k}$$
$$k = \sqrt{|K_1|}.$$

For focusing of vertical plane y, there is $K_1 > 0$ of focus strength:

$$x = x_c\, coshks + \frac{x'_c\, sinhks}{k}; \quad y = y_c\, cosks + \frac{y'_c\, sinks}{k}$$
$$x' = x'_c\, coshks + K_1 \frac{x_c\, sinhks}{k}; \quad y' = y'_c\, cosks - K_1 \frac{x_c\, sinks}{k}$$
$$k = \sqrt{|K_1|}.$$

3). *Sextupole magnet*

If the strength parameter is K_2, the orbit can be written (as [2]):

$$x = x_c + sx'_c - \frac{K_2 s^2}{24}[6({x_c}^2 - {y_c}^2) + 2s(x_c x'_c - y_c y'_c) + s^2({x'_c}^2 - {y'_c}^2)]$$
$$y = y_c + sy'_c + \frac{K_2 s^2}{24}[6x_c y_c + 2s(x_c y'_c + y_c x'_c) + s^2 x'_c y'_c]$$
$$x' = x'_c - \frac{K_2 s}{24}[12({x_c}^2 - {y_c}^2) + 6s(x_c x'_c - y_c y'_c) + 4s^2({x'_c}^2 - {y'_c}^2)]$$
$$y' = y'_c + \frac{K_2 s}{24}[12x_c y_c + 6s(x_c y'_c + y_c x'_c) + 4s^2 x'_c y'_c].$$

The last term is very small in normal case for the thin sextupole, one can only use the linear relationship as the space shift:

$$x = x_c + sx'_c \quad y = y_c + sy'_c$$
$$x' = x'_c \qquad y' = y'_c$$

4) *The Siberian Snake*
The position and velocity are written approximately as

$$x = x_c + x'_c s - \frac{eB_0}{m_o C^2 k^2}(1 - cosks); \quad y = y_c + y'_c s + \frac{eB_0}{m_o C^2 k^2}(sinks - ks)$$
$$x' = x'_c - \frac{eB_0}{m_o C^2 k} sinks; \quad y' = y'_c - \frac{eB_0}{m_o C^2 k}(1 - cosks).$$

This is the motion of particle in the prefect dipole helical magnet. From the approximate orbit position, one can give the magnetic field of more precision value as:

$$B_x = -B_0[(1 + \tfrac{1}{8}k^2(3x^2 + y^2))sin(kz) - \tfrac{1}{4}k^2 xycos(kz)]$$
$$B_y = B_0[(1 + \tfrac{1}{8}k^2(x^2 + 3y^2))cos(kz) - \tfrac{1}{4}k^2 xysin(kz)]$$
$$B_z = -B_0 k[(1 + \tfrac{1}{8}k^2(x^2 + y^2))(xcos(kz) + ysin(kz))]$$

5) *The spin rotation strength*
From section 3, the spin rotation strength can be given more precision using original formulae. In the second order calculation, the terms of first order of x' and y' are keep, and the hard boundary is taken, $B_s = 0$. The spin rotation strength can be reduced as:

$$P_x = \frac{1}{(B\rho)}(1 + G\gamma)B_x$$
$$P_s = \frac{1}{(B\rho)}[(G - G\gamma)x'B_x + (2 + G + G\gamma)y'B_y]$$
$$P_y = \frac{1}{(B\rho)}(1 + G\gamma)B_y.$$

The matrix of spin transport in each piece of one elements is constructed, the spin tracking is given by times those matrixes together one by one to whole the element, to whole the ring, to whole the process of acceleration.

The figure 5 shows the depolarization spectrum with second order calculation (c), there are more noise than first order (b) in the background of depolarization spectrum, and there are larger strength of depolarizing peaks than first order at same emittance value. Fig. 5 (a) shows the intrinsic depolarization for RHIC by the program *Depol* at emittance 10 $\mu rad - m$. The first order and second order calculations of spin tracking conform to *Depol* analysis very well at the peaks position and relative strength of peaks. But in the second order calculation, it is difficult to track the spin with a large emittance more than 10 $\mu rad - m$, for the depolarizing spectrum become unstable adding much noise in the background, although one order calculation can track the spin easily up to emittance 20 $\mu rad - m$.

7. Imperfect Depolarization Spin Tracking

The spin tracking calculation in before are taking beam injected RHIC as a single energy beam. For beam acceleration at no energy dispersion, the *Twiss* calculation gives that all *COD* parameters all are zero, the depolarization of spin tracking contributed only by the intrinsic issue. If the momentum of beam have dispersion, the *COD* parameters, that indicate this energy of particle have relative motion to the centered energy particle, can give the imperfect depolarization in spin tracking. Fig. 6 shows spin tracking at different momentum dispersion that means the depolarizing spectrum are the intrinsic depolarization adding different imperfect depolarization.

The betatron orbit is coupled with COD orbit if COD parameter aren't zero. The trajectory have large position value of vertical coordinate if two orbit move at same side, it result in a large depolarization. Depolarization maybe reduced by contrary orbit. Because COD motion orbit with the same period of beam transport in ring, but the betatron motion orbit with period determined by the tune value, the increasing and decreasing depolarization are appeared periodically at the process of acceleration. If the position of increasing depolarization happen to at one of largest depolarization peaks, that dependent initial status at injected point of RHIC and the momentum dispersion, the depolarization spectrum of spin tracking become unstable. The position value of COD is smaller than that of betatron orbit, the spin can be tracked with a large energy dispersion, but the accelerating voltage for this particle is diffcult to keep same as energy centered particle.

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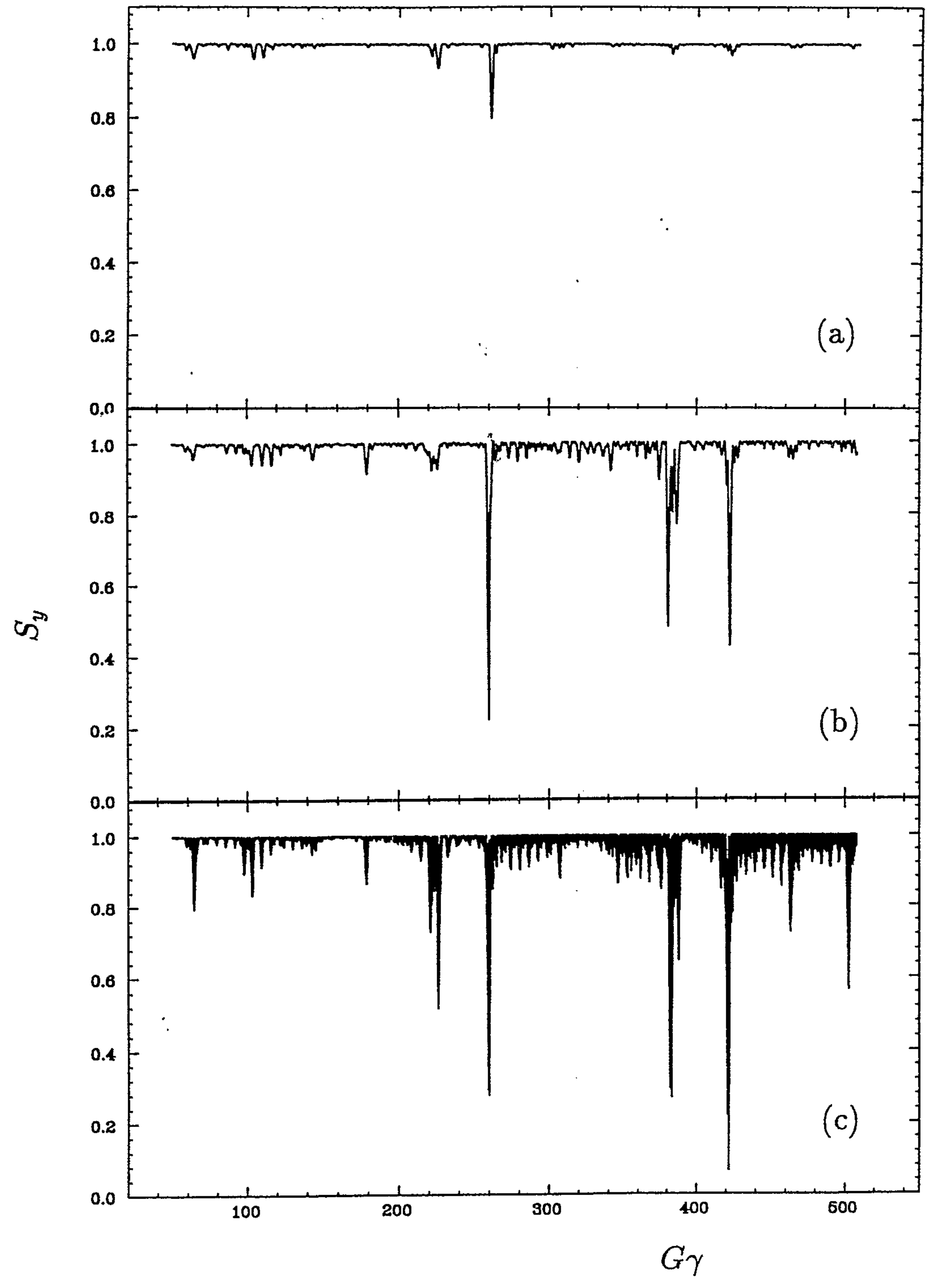
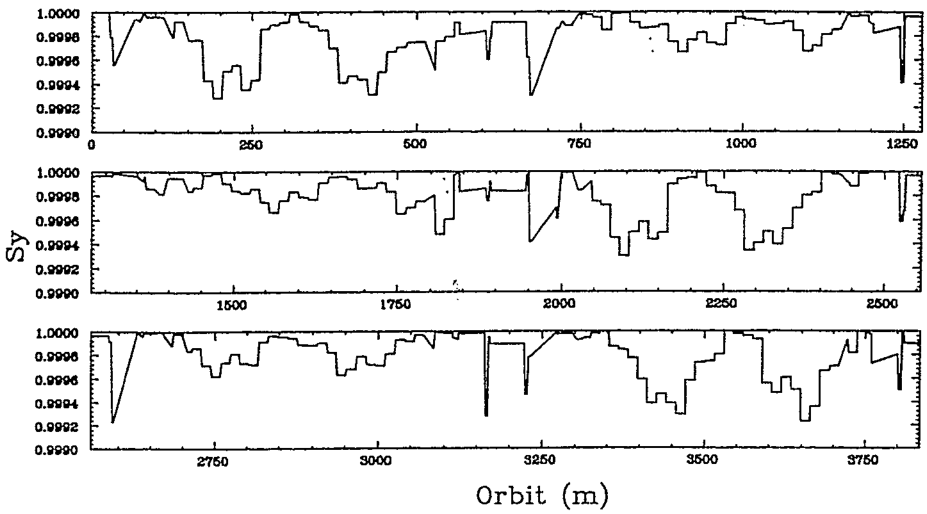
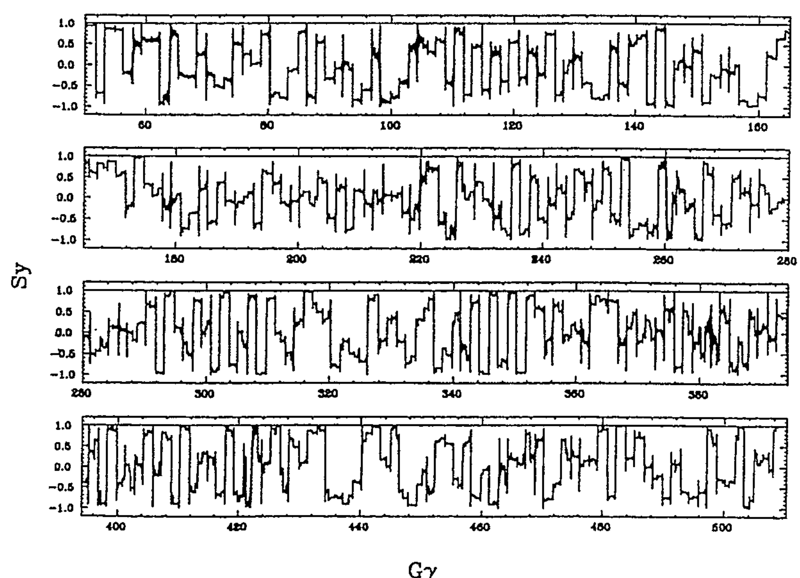


Fig. 1 Spin tracking of emittance 10 μ *rad m* through RHIC accelerator compare the *spink* code (a) with two dimensions for orbit y, y', that but renew the focusing action of RF (b), and our y, y', x, x' dimensions result with the parameters of Siberian snake at $1.2336T, -3.9570T, 3.9570T, -1.2336T$.



(a)



(b)

Fig. 2 Depolarization in one circle of RHIC at non-resonance area (a), and the pocess of acceleration (b), no Siberian snake in the ring. the emittance is 5 μ *rad m* for initial condition.

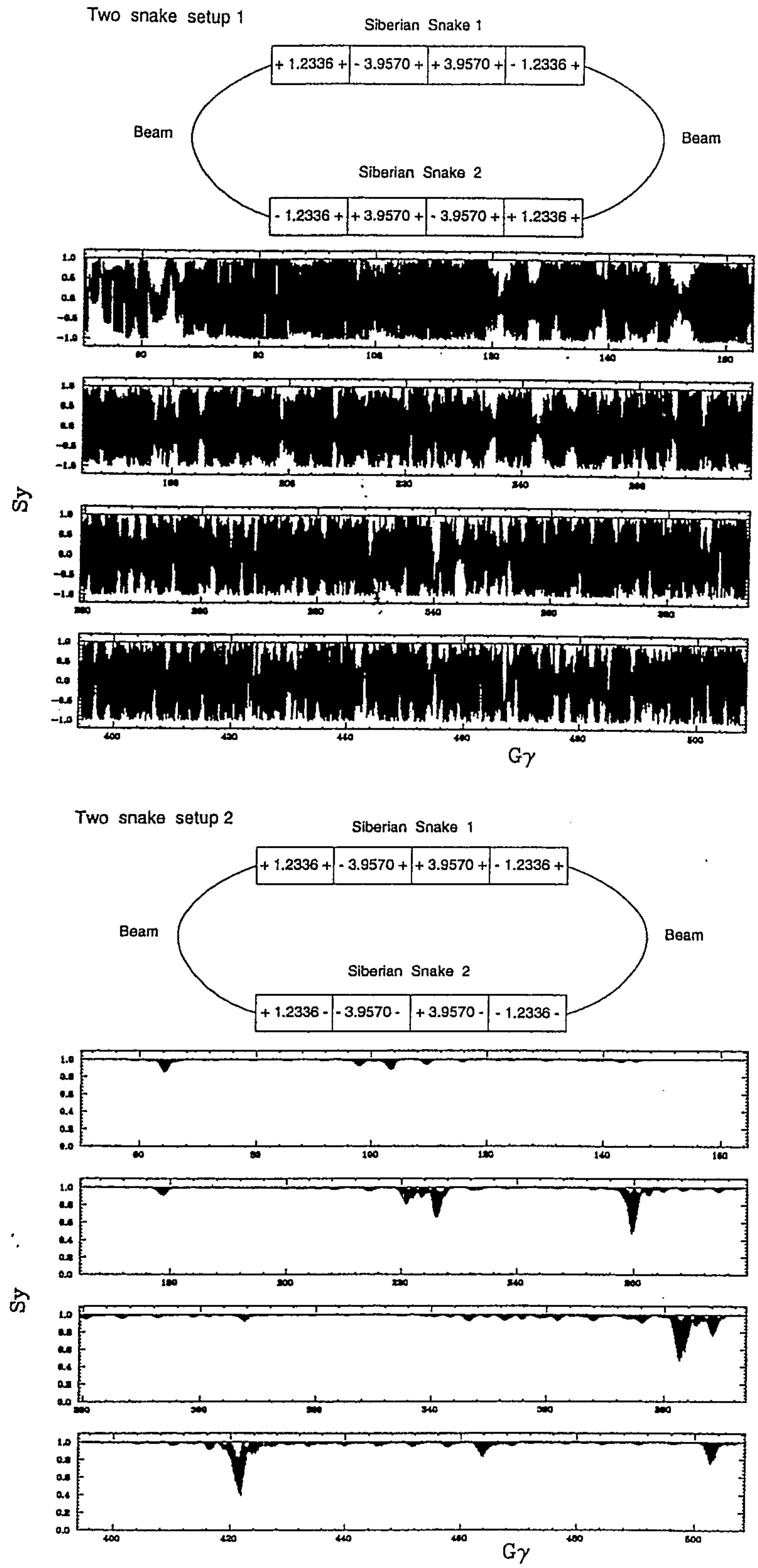


Fig. 3 Spin tracking for two kinds setup for Siberian snake, the emittance is 5 μ *rad m* for initial condition.

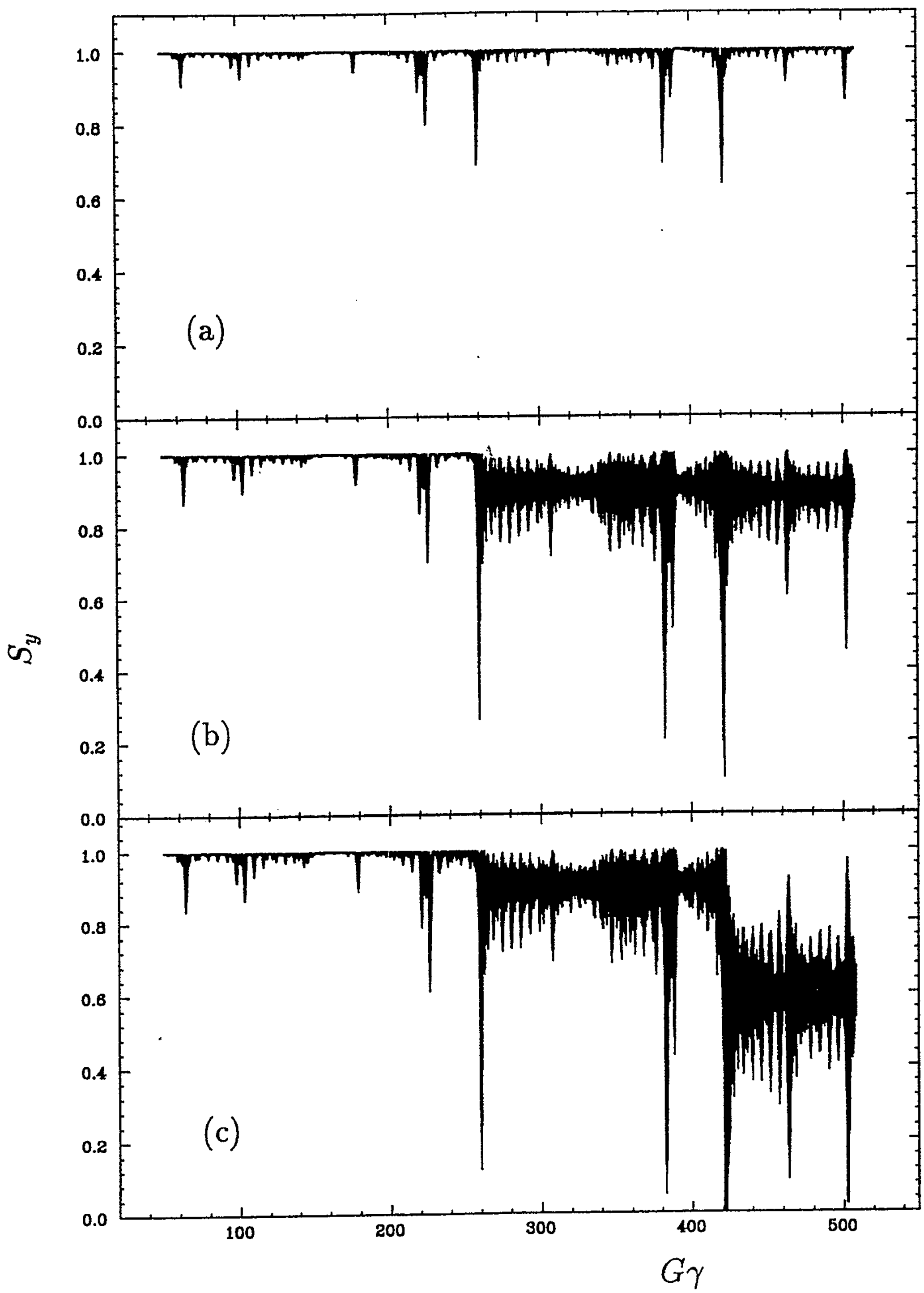


Fig. 4 Spin tracking at imperfect working status of Siberian snake at a set parameters $1.25T, -4.00T, 4.00T, -1.25T$ for different initial condition (a) for emittance 4, (b) for emittance 6, (c) for emittance 8 unit by $\mu\ rad\ m$.

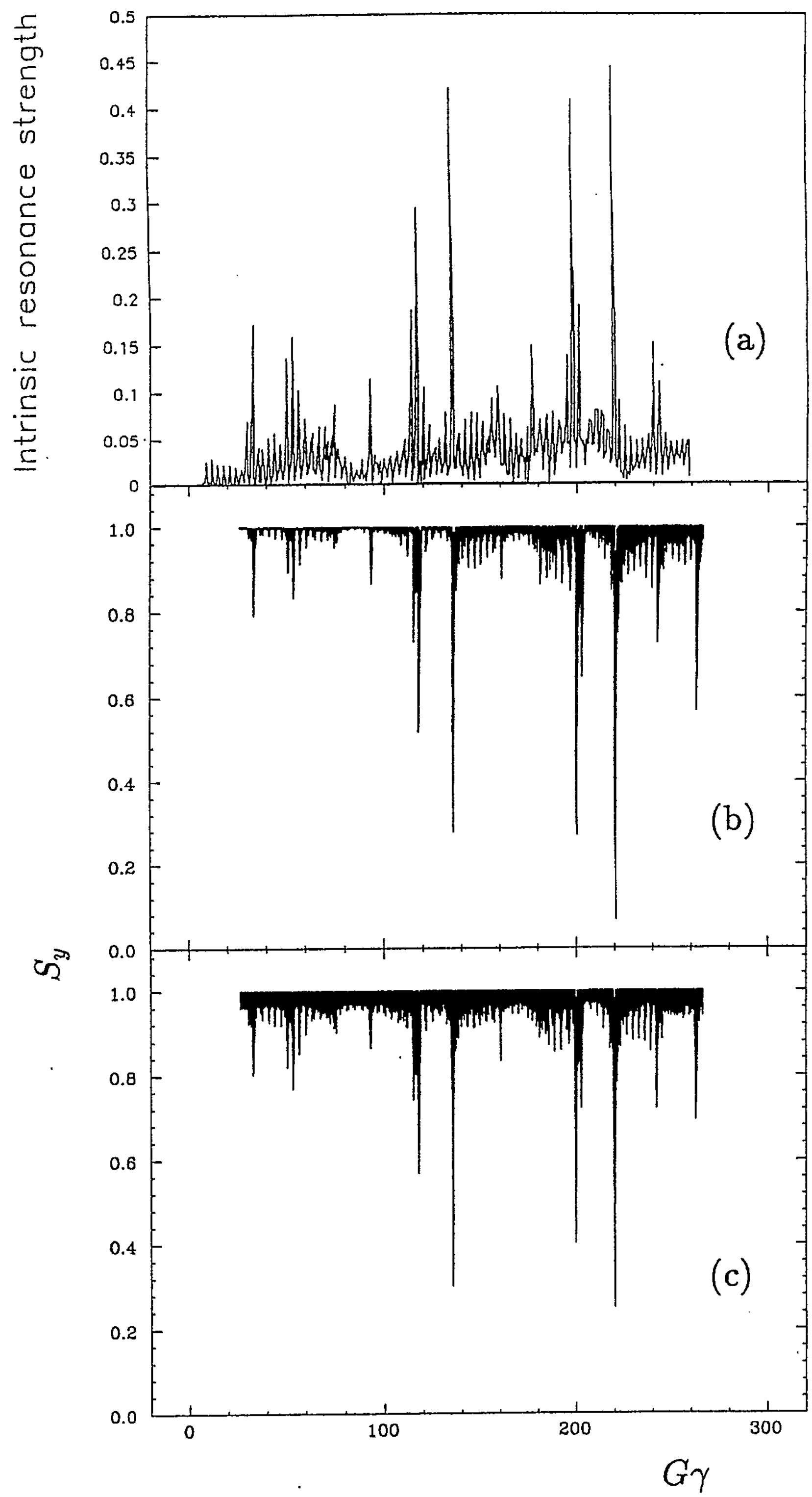


Fig. 5 The strengths of the intrinsic depolarizing resonance by *depol* with emittance 10 $\mu rad - m$ (a), compared with spin tracking for first order with 10 $\mu rad - m$ (b) and for second order with 8 $\mu rad - m$ (c).

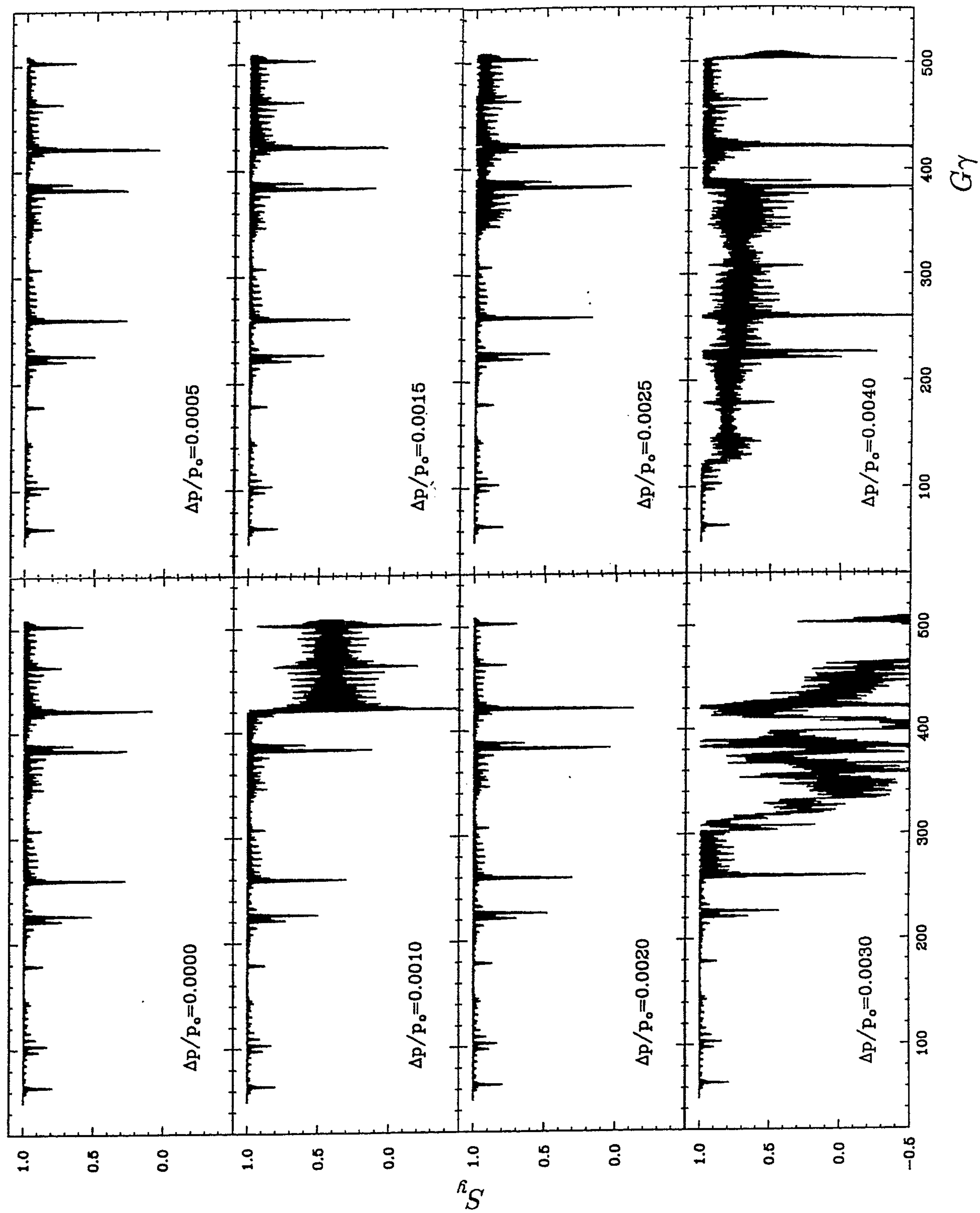


Fig. 6 Spin tracking with emittance 10 $\mu rad - m$, with different energy (or momentum) dispersion ($\Delta E/p_0 c = \Delta p/p_0$) that are indicated in frame.

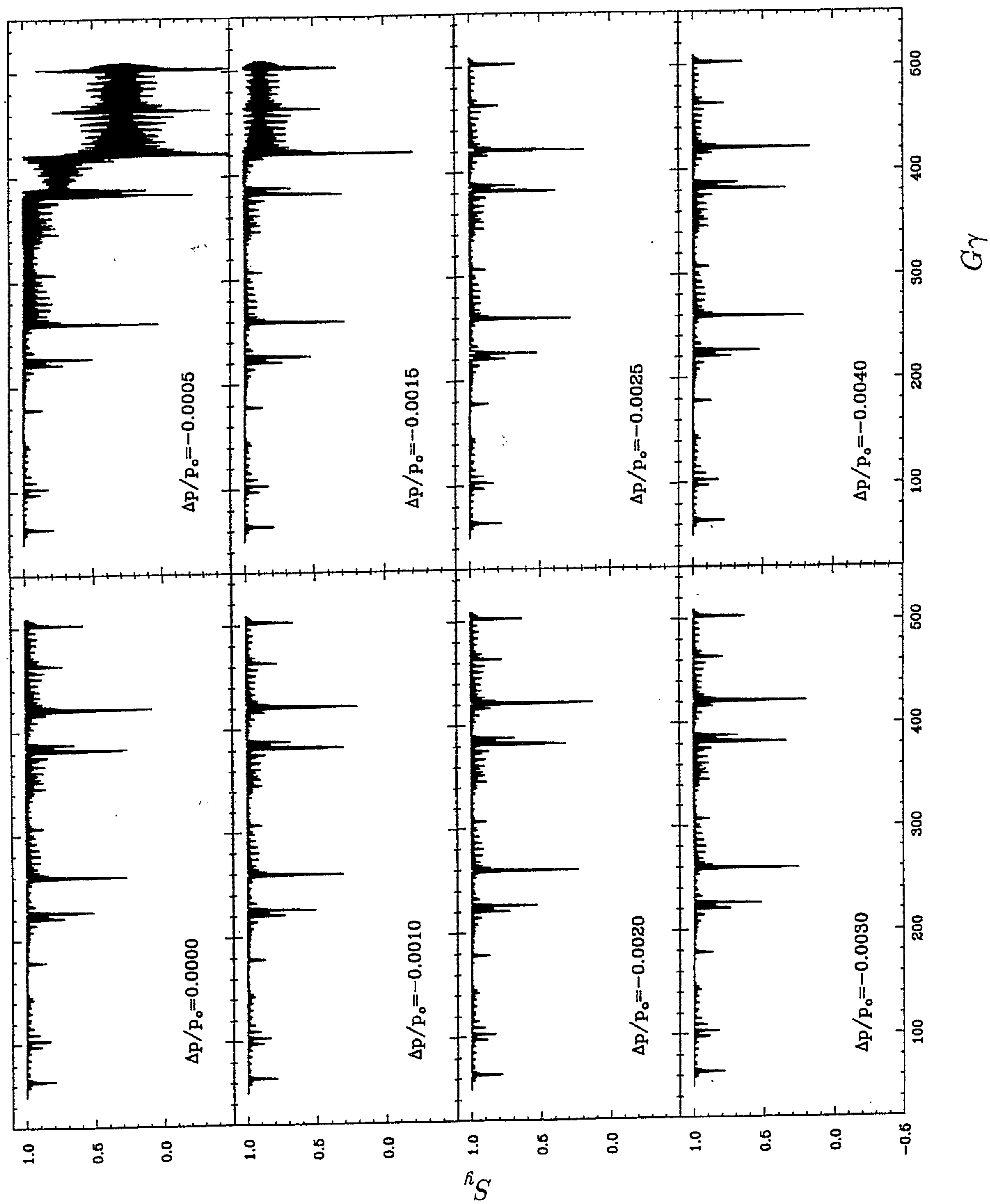


Fig. 6 (continue)