BNL-103651-2014-TECH

AGS/RHIC/SN 025;BNL-103651-2014-IR

Numerical and Analytic Studies on Siberian Snake

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April 1996

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U.S. Department of Energy

USDOE Office of Science (SC)

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Spin Note

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Abstract

In this report, the spin transport matrix is deduced for perfect dipole helical magnet. The spin tracking in the Siberian snake are present both analytic expression for first order and numerical method for one and second order. The perfect working point are given. And the symmetry of two Siberian snake is discussed by spin tracking in the ring.

1 Introduction

The design of Siberian snakes and spin rotators based on the helical dipole magnets has been adopted by the maker for LBN-RIKEN cooperation project. The prefect Siberian snake and spin rotators worked as the orbit vertical components are zero at entrance to zero at exit, and the spin change an constant angle. Although the depolarization by one Siberian snake can be recovered by another Siberian snake setup at other side of ring, the inappropriate parameters affect seriously the acceleration process of polarized proton. The parameters of helical magnets and the configuration of Sibarian snake have some errs, it means the Siberian snake doesn't work at prefect point. The Siberian snake with $1.25T, -4.0T, 4.0T, -1.25T$ of four modules and length of each modules at $2.4M$ make it impossible to accelerate the polarized proton with emittance large than $10\mu rad - m$, [1] for example. There are a lot of parameter sets for the setup of four modules that Siberian Snake can be used for this object. We need to find the prefect parameter for spin tracking or operation of RHIC.

This paper analyses the spin tracking in the Siberian snake. The analytic matrix for dipole helical magnet and the prefect parameters of Siberian snake based on four helical modules are considered. The results are compared with numerical calculation. The symmetry of setup of two Siberian snake and that of whole ring are discussed.

2 The Analytic on Helical Magnet

The magnetic field near the axis of the helical magnet modules with the period λ and the field amplitude B can be written as

$$B_x = -B\sin kz$$
$$B_y = B\cos kz$$
$$B_z = 0$$

where wave vector $k = 2\pi/\lambda$, and z is coordinate along axis and the x, y coordinate along horizontal and vertical direction respectively, with the unit vector [$\vec{i}, \vec{j}, \vec{k}$] as normal case.

The spin motion is given by BMT equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_{BMT} \times \vec{S}$$

with

$$\vec{\Omega}_{BMT} = -\frac{e}{m_o\gamma c}[(1+\gamma G)\vec{B}_\perp + (1+G)\vec{B}_{||}]$$

the term of electric field has be ignored. In the helical snake

$$\vec{B}_{||} \approx B_z\vec{k} = 0$$
$$\vec{B}_\perp \approx B_x\vec{i} + B_y\vec{j} \;\; (when\ z=0, \;\; B_y = B)$$

To consider the coordinate system [$\vec{e_x}, \vec{e_y}, \vec{e_z}$], it rotate along the helical snake axis, and with a same rotation period as that of helical magnet. In the system, the snake magnet worked as ordinary dipole magnet. The system transformation have relationship from [$\vec{i}, \vec{j}, \vec{k}$] to [$\vec{e_x}, \vec{e_y}, \vec{e_z}$]:

$$\begin{aligned} \vec{e_x} &= \vec{i}coskz + \vec{j}sinkz \\ \vec{e_y} &= -\vec{i}sinkz + \vec{j}coskz \\ \vec{e_z} &= \vec{k} \end{aligned}$$

after the transformation, the magnet field can be written as

$$\vec{B} = 0\vec{e_x} + B\vec{e_y} + 0\vec{e_z} \ .$$

From

$$\frac{d\vec{S}}{dz} = -\frac{e(1+\gamma G)}{m_o\gamma cv}\vec{B}\times\vec{S}$$
$$\vec{S} = S_1\vec{e_x} + S_2\vec{e_y} + S_3\vec{e_z}$$

then

$$\frac{d(S_1\vec{e_x} + S_2\vec{e_y} + S_3\vec{e_z})}{dz} = -\frac{e(1+\gamma G)}{m_o\gamma cv}B\ \vec{e_y}\times(S_1\vec{e_x} + S_3\vec{e_z})$$

one can get the scalar equation:

$$\frac{dS_1}{dz} = -\frac{e(1+\gamma G)}{m_o\gamma cv}BS_3$$
$$\frac{dS_3}{dz} = \frac{e(1+\gamma G)}{m_o\gamma cv}BS_1$$
$$\frac{dS_3}{dz} = 0 \ .$$

It is .

$$\frac{d^2S_1}{dz^2} + (\frac{eB(1+\gamma G)}{m_o\gamma cv})^2 S_1 = 0$$
$$\frac{d^2S_3}{dz^2} + (\frac{eB(1+\gamma G)}{m_o\gamma cv})^2 S_3 = 0 \ ,$$

the solution is

$$S_1 = Asin(\omega\ z + \theta_o)$$
$$S_2 = \sqrt{1-A^2}$$
$$S_3 = Acos(\omega\ z + \theta_o)$$
$$\omega = \frac{eB(1+\gamma G)}{m_o\gamma cv} \ .$$

The spin vector rotate in the ($\vec{e_x}, \vec{e_z}$) plane along the $\vec{e_y}$ with a angular velocity ω, but coupled with the rotation coordinate system. It result in the quantity A should be change in the coordinate system. The other hands, spin rotate along $\vec{e_z}$ with a angular velocity $-k$, reverse the rotation coordination system. The angular velocity of total rotation is $\sqrt{\omega^2 + k^2}$, the rotation axis is at the ($\vec{e_y}, \vec{e_z}$) plane, a angle at $\theta = tg^{-1}\|\frac{k}{\omega}\|$ between the axis and $\vec{e_y}$. We transfer the coordinate system again with

$$\begin{aligned} \vec{e_1} &= \vec{e_x} \\ \vec{e_2} &= \vec{e_y} cos\theta - \vec{e_z} sin\theta \\ \vec{e_3} &= \vec{e_y} sin\theta + \vec{e_z} cos\theta \ . \end{aligned}$$

In the new system [$\vec{e_1}, \vec{e_2}, \vec{e_3}$], spin rotate in ($\vec{e_1}, \vec{e_3}$) plane along the axis $\vec{e_2}$, angular velocity of spin vector precesses is $\omega_k = \sqrt{\omega^2 + k^2}$. Assume the initial condition of spin at $z = 0$,

$$\begin{aligned} \vec{S_o} &= S_x\vec{i} + S_y\vec{j} + S_z\vec{k} \\ &= S_x\vec{e_x} + S_y\vec{e_y} + S_z\vec{e_z} \end{aligned}$$

the conservation component of spin S_o is

$$[S_y cos\theta + S_z sin\theta]\vec{e_2} \ ,$$

and the rotation component is

$$[S_z cos\theta - S_y sin\theta]\vec{e_3} + S_x\vec{e_1} \ ,$$

the spin rotation can be written as

$$\begin{aligned} S_1 &= \sqrt{S_x^2 + (S_z cos\theta - S_y sin\theta)^2} sin(\omega_k z + cos^{-1}\frac{S_x}{\sqrt{S_x^2+(S_z cos\theta - S_y sin\theta)^2}}) \\ S_2 &= S_y cos\theta + S_z sin\theta \\ S_3 &= \sqrt{S_x^2 + (S_z cos\theta - S_y sin\theta)^2} cos(\omega_k z + cos^{-1}\frac{S_x}{\sqrt{S_x^2+(S_z cos\theta - S_y sin\theta)^2}}) \end{aligned}$$

where S_1, S_2, S_3 are three components for $\vec{e_1}, \vec{e_2}, \vec{e_3}$ directions. If we simplify with

$$A_1 = \sqrt{S_x^2 + (S_z cos\theta - S_y sin\theta)^2}, \quad \theta_c = cos^{-1}\frac{S_x}{A_1}$$

and the coordinate system return [$\vec{e_x}, \vec{e_y}, \vec{e_z}$] system, the spin $\vec{S}$ can be written as

$$\begin{aligned} \vec{S} = & \ A_1 sin(\omega_k z + \theta_c)\vec{e_x} + [-A_1 cos(\omega_k z + \theta_c) sin\theta + (S_y cos\theta + S_z sin\theta) cos\theta]\vec{e_y} \\ + & \ [A_1 cos(\omega_k z + \theta_c) cos\theta + (S_y cos\theta + S_z sin\theta) sin\theta]\vec{e_z} \end{aligned}$$

and return lab system [$\vec{i}, \vec{j}, \vec{k}$], the spin $\vec{S}$ is

$$\begin{aligned} \vec{S} = & \ [A_1 sin(\omega_k z + \theta_c) coskz + (-A_1 cos(\omega_k z + \theta_c) sin\theta + (S_y cos\theta + S_z sin\theta) cos\theta) sinkz]\vec{i} \\ + & \ [-A_1 sin(\omega_k z + \theta_c) sinkz + (-A_1 cos(\omega_k z + \theta_c) sin\theta + (S_y cos\theta + S_z sin\theta) cos\theta) coskz]\vec{j} \\ + & \ [A_1 cos(\omega_k z + \theta_c) cos\theta + (S_y cos\theta + S_z sin\theta) sin\theta]\vec{k} \end{aligned}$$

One can deduce the spin transport matrix for any part as well as a whole according to the initial conditon.

$$\begin{vmatrix} \cos kz\ \cos\omega_k z & -\cos kz\ \sin\theta\ \sin\omega_k z + (\cos^2\theta & \cos kz\ \cos\theta\ \sin\omega_k z \\ +\sin\omega_k z\ \sin\theta\ \sin kz & +\sin^2\theta\ \cos\omega_k z)\sin kz & -\sin\theta\ \cos\theta\ (\cos\omega_k z - 1)\sin kz \\ & & \\ -\sin kz\ \cos\omega_k z & \sin kz\ \sin\theta\ \sin\omega_k z + (\cos^2\theta & -\sin kz\ \cos\theta\ \sin\omega_k z \\ +\sin\omega_k z\ \sin\theta\ \cos kz & +\sin^2\theta\ \cos\omega_k z)\cos kz & -\sin\theta\ \cos\theta\ (\cos\omega_k z - 1)\cos kz \\ & & \\ -\sin\omega_k z\ \cos\theta & -\sin\theta\ \cos\theta\ (\cos\omega_k z - 1) & \cos^2\theta\ \cos\omega_k z + \sin^2\theta \end{vmatrix}$$

this matrix is right test by the numerical calculation. The Matrix from the entrance to exit can be simplify as following if initial spin direction with same direction of magnetic field at entrance point:

$$\begin{vmatrix} \cos\omega_k z & -\sin\theta\ \sin\omega_k z & \cos\theta\ \sin\omega_k z \\ \sin\omega_k z\ \sin\theta & \cos^2\theta + \sin^2\theta\ \cos\omega_k z & -\sin\theta\ \cos\theta\ (\cos\omega_k z - 1) \\ -\sin\omega_k z\ \cos\theta & -\sin\theta\ \cos\theta\ (\cos\omega_k z - 1) & \cos^2\theta\ \cos\omega_k z + \sin^2\theta \end{vmatrix}$$

for the helical magnet have integer period ($kl = N\ 2\pi$, l is the length of magnet). The θ are replaced by $-\theta$ when the magnet field is negative B_y at entrance and by $\pi - \theta$ when the helicity is negative sign. If the magnet setup is rotated about the $\vec{k}$ axis by an angle α, the transport matrix should be

$$\begin{vmatrix} \cos\omega_k z & -\sin\theta\ \sin\omega_k z & \cos\theta\ \sin\omega_k z \\ \sin\omega_k z\ \sin\theta & \cos^2\theta + \sin^2\theta\ \cos\omega_k z & -\sin\theta\ \cos\theta\ (\cos\omega_k z - 1) \\ -\sin\omega_k z\ \cos\theta & -\sin\theta\ \cos\theta\ (\cos\omega_k z - 1) & \cos^2\theta\ \cos\omega_k z + \sin^2\theta \end{vmatrix} \times \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}.$$

It means the angle of setup snake have same ability to adjust the the spin matrix.

The spin matrix of whole Siberian snake is obtained by times the four matrixes of helix magnet together.

3 Nemerical Calculation of Spin Track on Siberian Snake

BMT equation of spin motion can be written in the as matrix formulation [1)] :

$$\begin{vmatrix} 1-(B^2+C^2)c & ABc+Cs & ACc-Bs \\ ABc-Cs & 1-(A^2+C^2)c & BCc+As \\ ACc+Bs & BCc-As & 1-(A^2+B^2)c \end{vmatrix}$$

with

$$\left\{ \begin{array}{c} c = 1 - \cos\omega\delta s \\ s = \sin\omega\delta s \end{array} \right. , A = \frac{P_x}{\omega}, B = \frac{P_y}{\omega}, C = \frac{P_s}{\omega}.$$

in the terms of spin rotation strength P:

$$P_x = \tfrac{h}{(B\rho)}[(1+G\gamma)(-x'B_s + B_x) + (1+G)x'(x'B_x + B_s + y'B_y)]$$
$$P_s = \tfrac{h}{(B\rho)}[(1+G\gamma)(-x'B_x + y'B_y) + (1+G)(x'B_x + B_s + y'B_y)]$$
$$P_y = \tfrac{h}{(B\rho)}[(1+G\gamma)(-y'B_s + B_y) + (1+G)y'(x'B_x + B_s + y'B_y)],$$

if the magnetic field is well known.

This matrix is suit for local part of magnet or the thin magnet element. The spin matrix of transport through whole element should integrate all local matrix along the path of transport line, that be got from multiplication all the local matrix. The number of parts divided the element is more large, the total matrix is more high precision.

First order calculation of spin tracking used the magnetic field of helix magnet of Siberian snake or spin rotator as following:

$$B_x = -B_{SN} sin(\phi), B_y = B_{SN} cos(\phi), B_s = 0, \phi = \frac{2\pi}{L} s$$

and the all velocity terms in expression of spin rotation strength are ignore, the local matrix can be reduced simple formulae:

$$\begin{vmatrix} 1 - cos^2\phi\,(1 - cos\delta\psi) & -sin\phi\ cos\phi\,(1 - cos\delta\psi) & -cos\phi\ sin\delta\psi \\ -sin\phi\ cos\phi\,(1 - cos\delta\psi) & 1 - sin^2\phi\,(1 - cos\delta\psi) & -sin\phi\ sin\delta\psi \\ cos\phi\ sin\delta\psi & sin\phi\ sin\delta\psi & cos\delta\psi \end{vmatrix}$$

with the spin rotation angle

$$\delta\ \psi = \omega\delta\ s = \frac{(1 + G\gamma)B_{SN}}{(B\rho)}\delta\ s$$

Second order calculation of spin tracking used the magnetic field of helix as:

$$\begin{aligned} B_x &= -B_0[(1 + \tfrac{1}{8}k^2(3x^2 + y^2))sin(kz) - \tfrac{1}{4}k^2xycos(kz)] \\ B_y &= B_0[(1 + \tfrac{1}{8}k^2(x^2 + 3y^2))cos(kz) - \tfrac{1}{4}k^2xysin(kz)] \\ B_z &= -B_0k[(1 + \tfrac{1}{8}k^2(x^2 + y^2))(xcos(kz) + ysin(kz))]. \end{aligned}$$

Spin tracking need to know the transverse position and velocity inside helix for each part of magnet. The precision method is multiple integration from Lorentz force equation [2], but it is too much CPU time to track spin. The simple analytic expression of the position and velocity are written approximately as:

$$\begin{aligned} &x = x_c + x'_c s + \tfrac{eB_0}{m_oC^2k^2}(cosks - 1); \quad y = y_c + y'_c s + \tfrac{eB_0}{m_oC^2k^2}(sinks - ks) \\ &x' = x'_c - \tfrac{eB_0}{m_oC^2k}sinks; \quad y' = y'_c - \tfrac{eB_0}{m_oC^2k}(1 - cosks). \end{aligned}$$

The spin matrix of Siberian snake is obtained to integrate along four helix magnets. Fig. 1 and Fig. 2 shows the spin trajectory inside the Siberian snake for first order and second order approximation. Although the behave of two figures are same, the difference of final results can be found obviously, it means the status of working Siberian snake have some little shifts.

4 Search Working Point of Siberian Snake

Imperfect working status of Siberian snake make the depolarization spectrum unstable when the particles with large emittance are accelerated. One need search perfect working point of a set of parameters for spin tracking or real operation the RHIC of polarized beam. The length of each of four helix modules is $2.4m$, field direction of magnet field rotate 2π for each module with same helicity, $B_4 = -B_1$ and $B_2 = -B_3$, the initial spin direction is same direction of magnetic field at entrance of first snake and the gap between two helixes is $0.4m$, are set for the Siberian snake. The matrixes are calculated various B_1 and B_3 by step $1Gauss$ in the range of $0 - 10Tesla$ for beam energy at $25GeV$.

Fig. 3 shows the relationship between spin components of initial direction and the magnetic field B_1 and B_3 (unit by *Tesla*). The theory of depolarization request Siberian snake change the $S_y = 1$ into $S_y = -1$, it means the A_{22} term of spin matrix should be -1. Fig. 4 (magnetic field unit by *Gauss*) shows different precision of A_{22} term to close -1.0, the special area being suit for the condition change to a closed curve line, then to the part of line. The spin tracking request the $A_{22} = -1.0$ and $A_{31} = A_{13} = 1.0$, this condition split the working area (in Fig. 4) separate area, and the separate area change to several point for high precision (Fig 5). This point is perfect working point for Siberian snake. Table 1 lists 7 working points of this magnetic field range for three kinds of calculation. The difference between analytic calculation of first order and numerical calculation of first order produced by the step length of divided the helix. The working points are shift from one to second order calculations

Table 1 The perfect working parameter of Siberian snake, with B_2=-B_3, B_4=-B_1 at $25GeV$ polarized proton, unit by Tesla, when the spin direction is same direction of magnetic field at entrance point. Numerical calculation used step at $0.1cm$.

		1	2	3	4	5	6	7
First order analytic	B_1	1.2319	3.2282	5.0892	5.3788	6.3878	8.0004	8.5256
	B_3	3.9580	5.5300	2.3861	8.8272	6.6088	4.2691	5.1514
First order numerical	B_1	1.2336	3.2265	5.0865	5.3769	6.3847	7.9988	8.5171
	B_3	3.9570	5.5273	2.3836	8.8225	6.6115	4.2765	5.1463
Second order numerical	B_1	1.0723	3.3872	5.1324	5.5050	6.4517	8.0139	8.4701
	B_3	3.9328	5.6587	2.2711	8.9256	6.9018	4.4816	5.1915

5 Precision of Making Siberian Snake Require by RIHC

Two Siberian snake install the RHIC ring, to help acceleration polarized proton from $25GeV$ to $250GeV$, it used for avoid depolarization in resonance area. But the ideal same of Siberian snakes is impossible to get, that maybe produced by technical deficiency of making snake or imperfect working of power supply... . The effect by asymmetrical Siberian snakes can't be given by spin tracking only Siberian snakes, but by spin tracking all the process of acceleration.

Fig. 6-9 shows the variation of depolarization from the perfect working point to asymmetry setup. The depolarization peaks become high, and background noise become much when the setup of Siberian snake is more asymmetry. The stability of polarized beam is decreasing, and polarization is lost at larger asymmetry.

The rings are symmetry along the transport line (close orbit), the polarization can be keep in the ring although the Siberian snake haven't work at perfect working point or two Siberian snake at asymmetry, when the rings are working as the storage. But the symmetry are broken by rf working for acceleration. Part transport line have same energy the other have different energy if the ring is view from Siberian snakes. The asymmetry of ring itself made different depolarization spectrum between increasing some parameter of the first Siberian snake and increasing that of second Siberian snake. In Fig. 6-9, the spectrum become worse slowly one way of change parameters, but rapidly another way, it is results in the asymmetry of ring at acceleration. To accelerate the proton with emittance 6 $\mu rad - m$, the err of magnetic field of making Siberian snake should be no more than 1%, The high precision of making Siberian snake request for acceleration large emittance of polarized particle.

Acknowledgments

We would like to thank M. Ishihara, H. Sakurai, Y. Watanabe, T. Kishida, T. Ichihara for their hospitality and useful discuss.

Reference

1) H. Wu *The Spin Tracking Study in RHIC for Polarized Proton*, Work report 4 in RIKEN, Mar. 1996.

2) H. Wu *The Spin Trace of Polarized Proton in Various Magnets for RHIC*, Work report 3 in RIKEN, Oct. 1995.

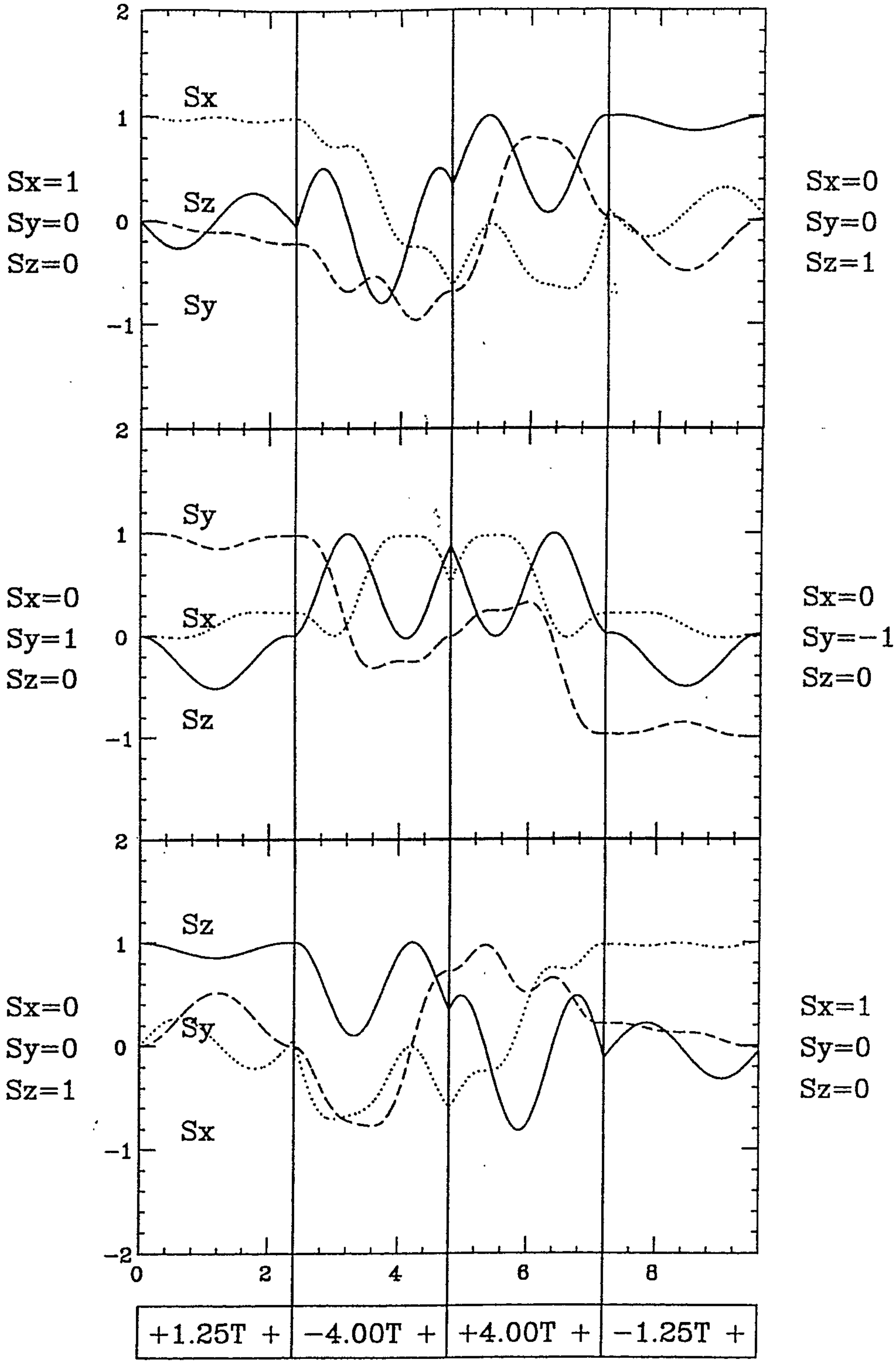

Fig. 1 Spin tracking inside the Siberian snake for $25GeV$ polarized proton, with first order approximation and method of local matrixes times together.

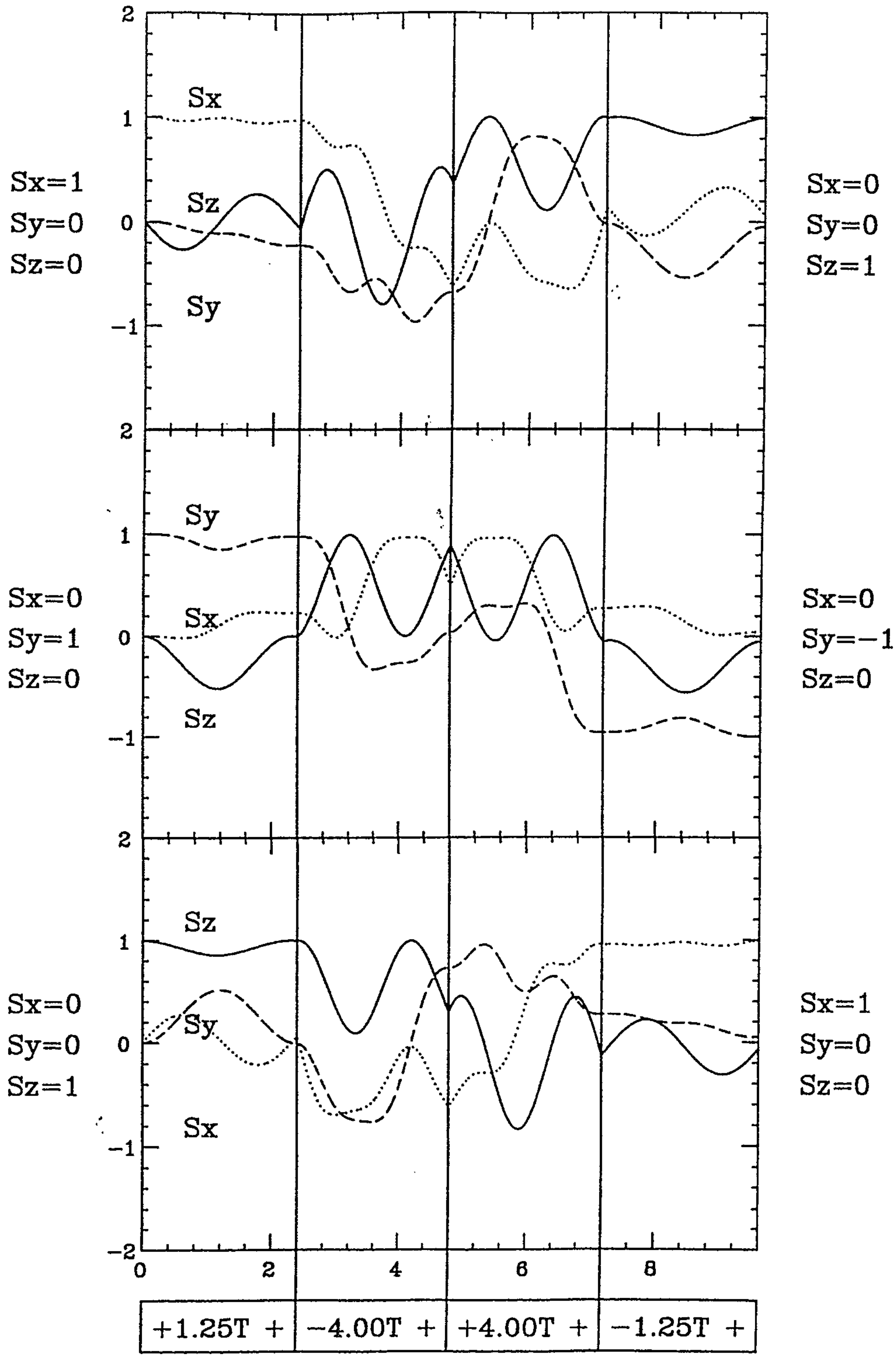

Fig. 2 Spin tracking inside the Siberian snake same as Fig. 1 but with two order approximation.

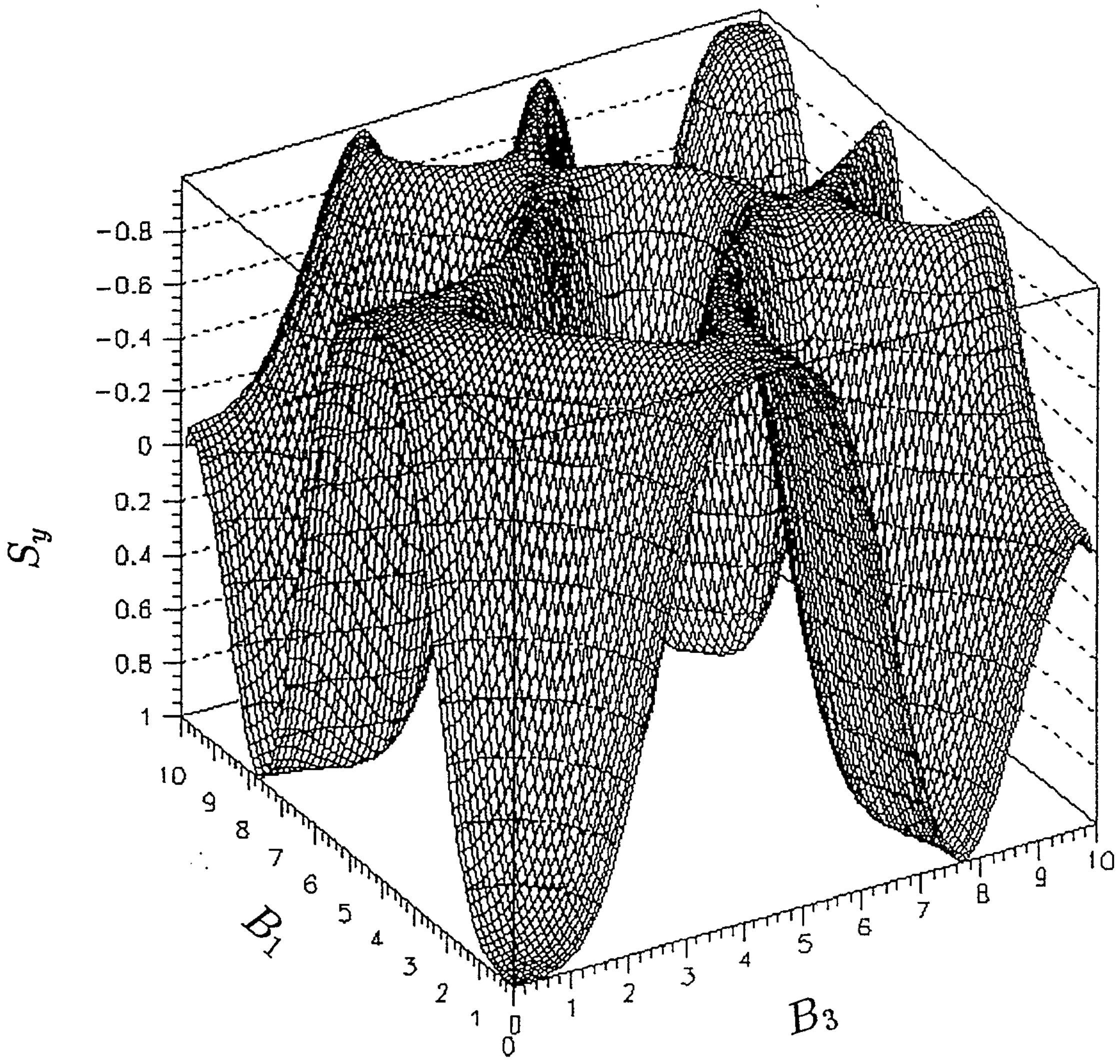

Fig. 3 Relationship of spin component of initial direction S_y and the magnetic field B_1 and B_3.

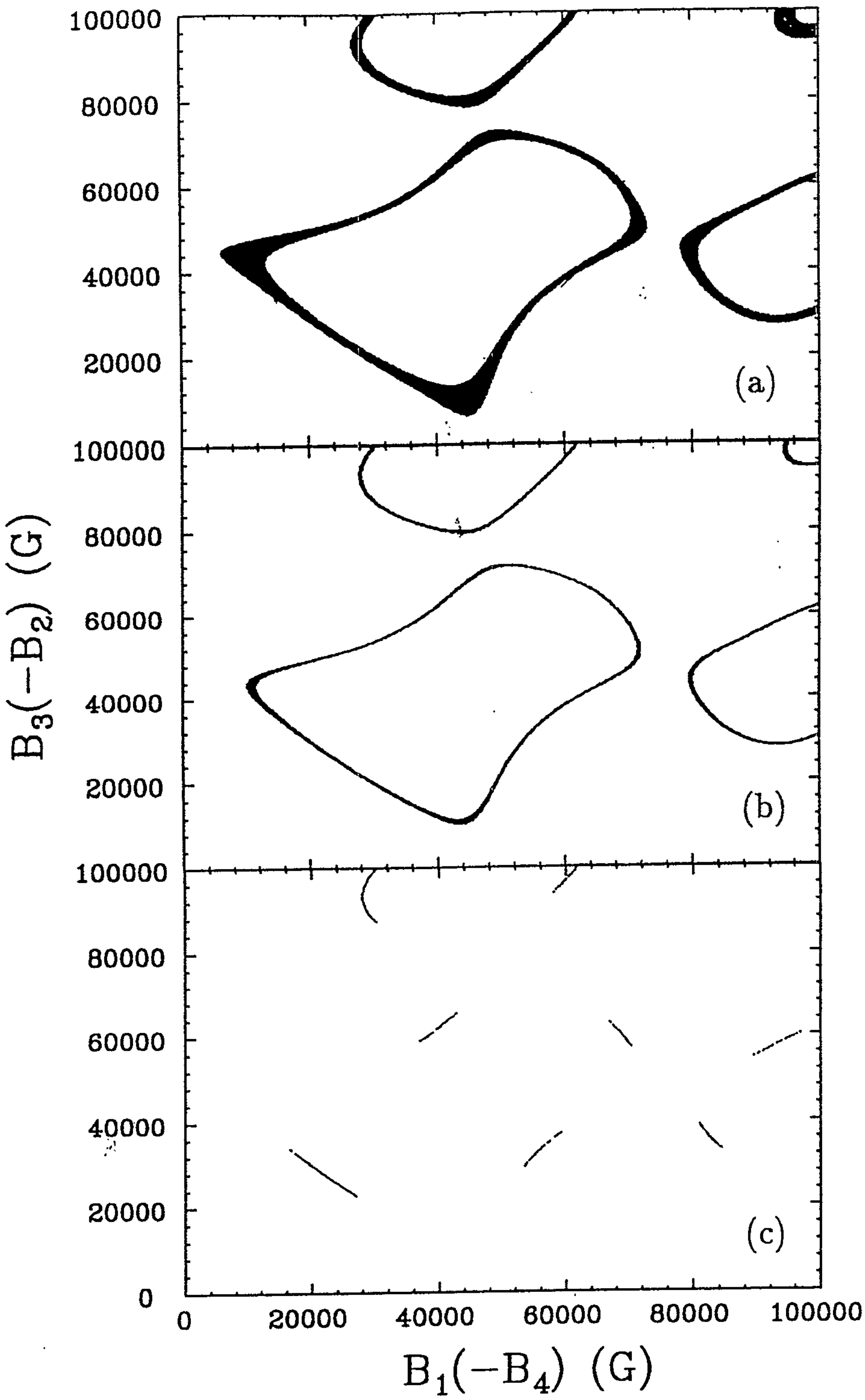

Fig. 4 Siberian snake working point with different precision, with different the condition of matrix term $A_{22} < -\alpha$. (a) $\alpha = 0.995$, (b) $\alpha = 0.9995$, (c)$\alpha = 0.99995$.

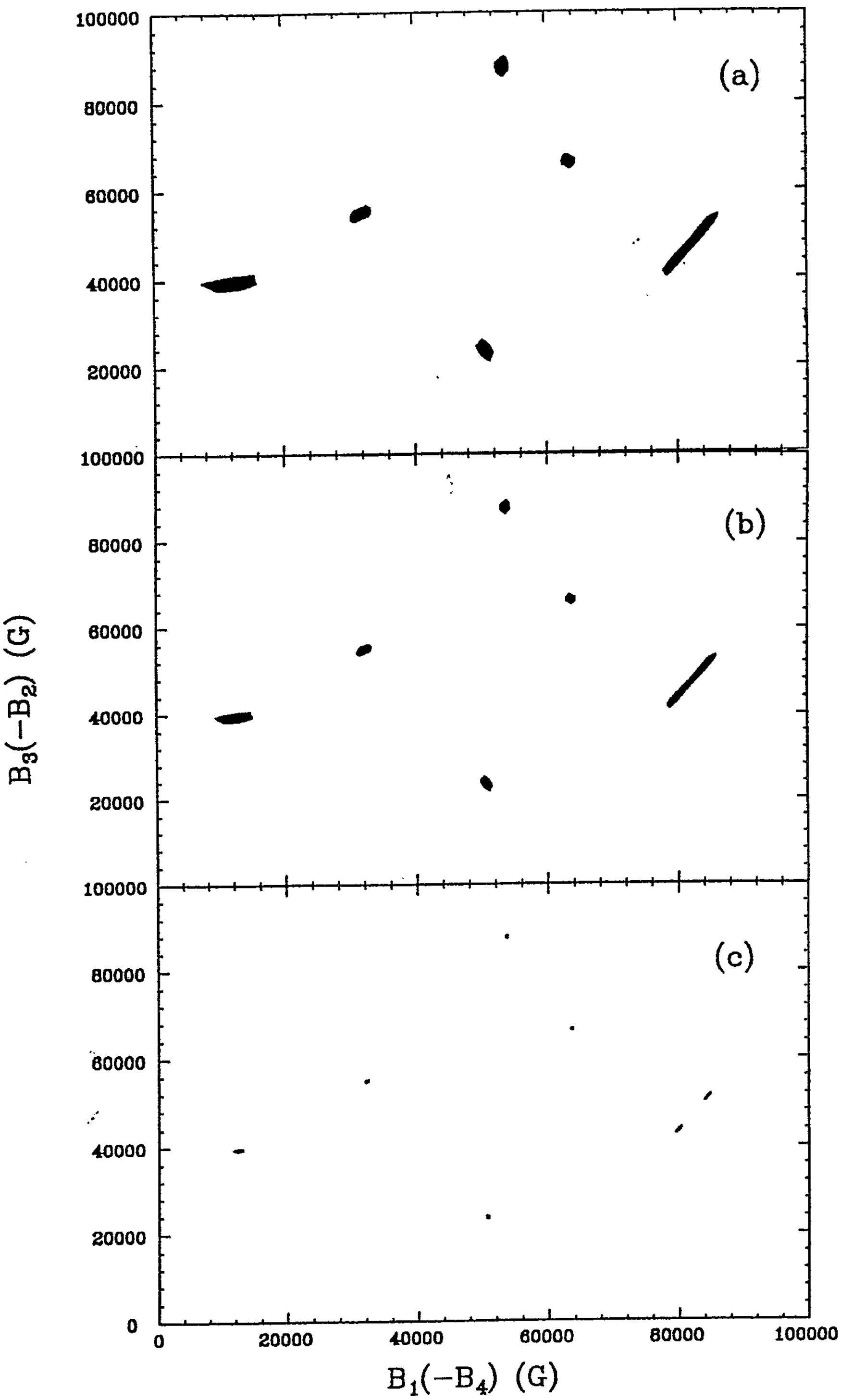

Fig. 5 Siberian snake working point with different precision, with different the condition of matrix terms: $A_{22} < -\alpha$, $A_{13} > \alpha$, $A_{31} > \alpha$. (a) $\alpha = 0.98$, (b) $\alpha = 0.99$, (c)$\alpha = 0.999$.

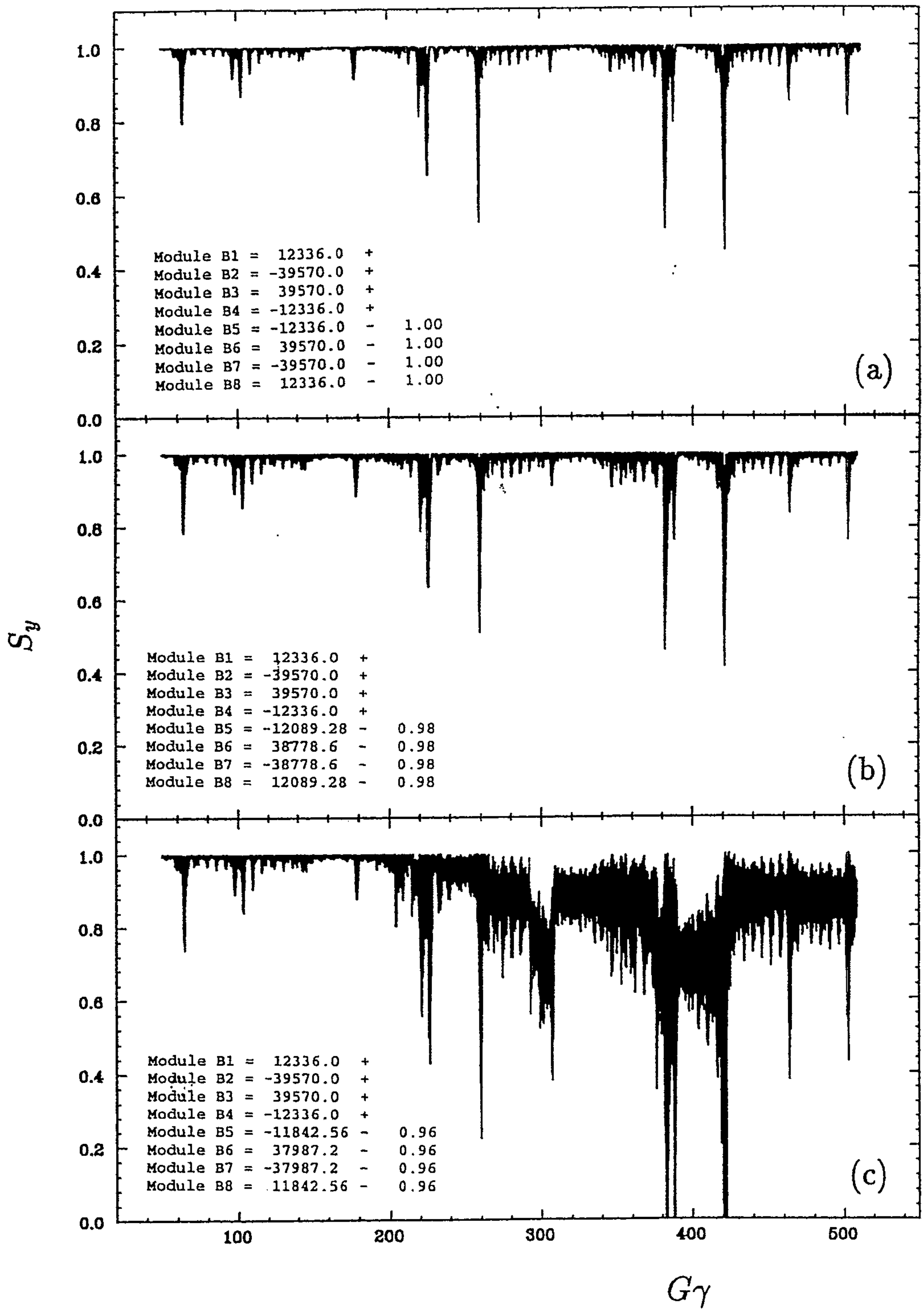

Fig. 6 Spin tracking for decreasing the magnetic field of all helixes of second Siberian snake from perfect working point (a), to 2% (b) then 4% (c), with initial emittance 6 $\mu radm$.

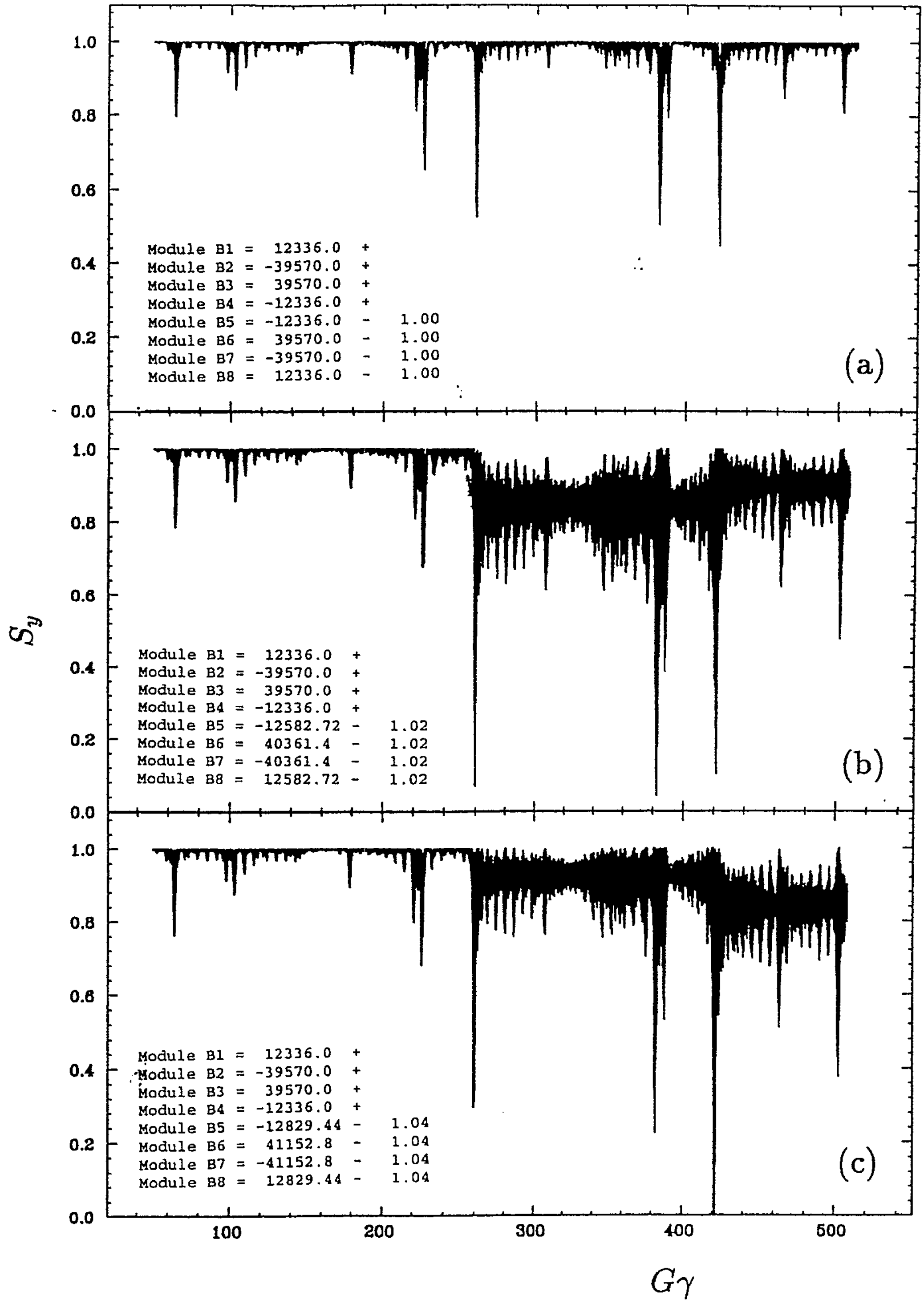

Fig. 7 Spin tracking for increasing the magnetic field of all helixes of second Siberian snake from perfect working point (a), to 2% (b) then 4% (c), with initial emittance 6 $\mu radm$.

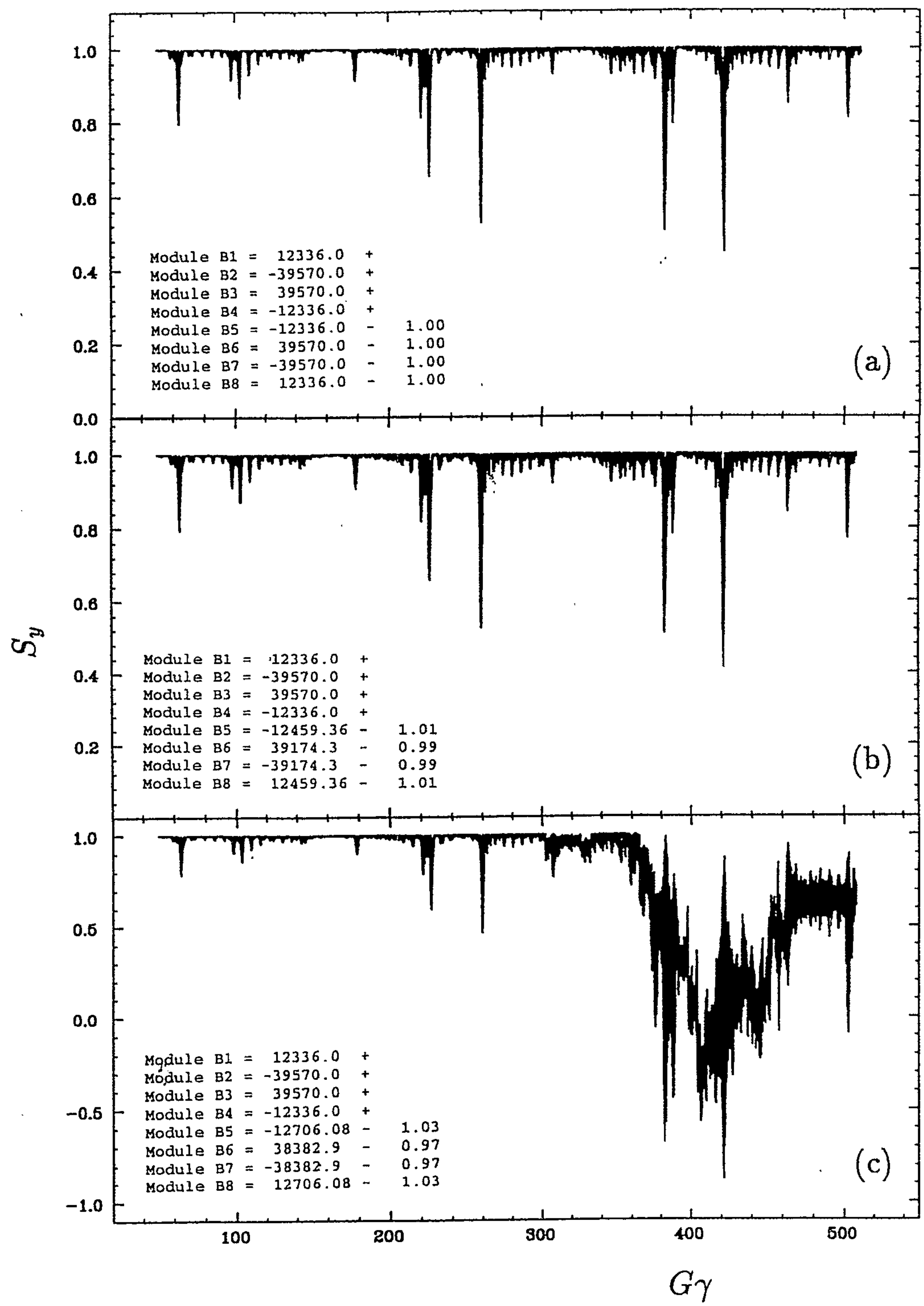

Fig. 8 Spin tracking for increasing the magnetic field of outside two helixes but decreasing that of inner two helixes of second Siberian snake from perfect working point (a), to 1% (b) then 3% (c), with initial emittance 6 $\mu radm$.

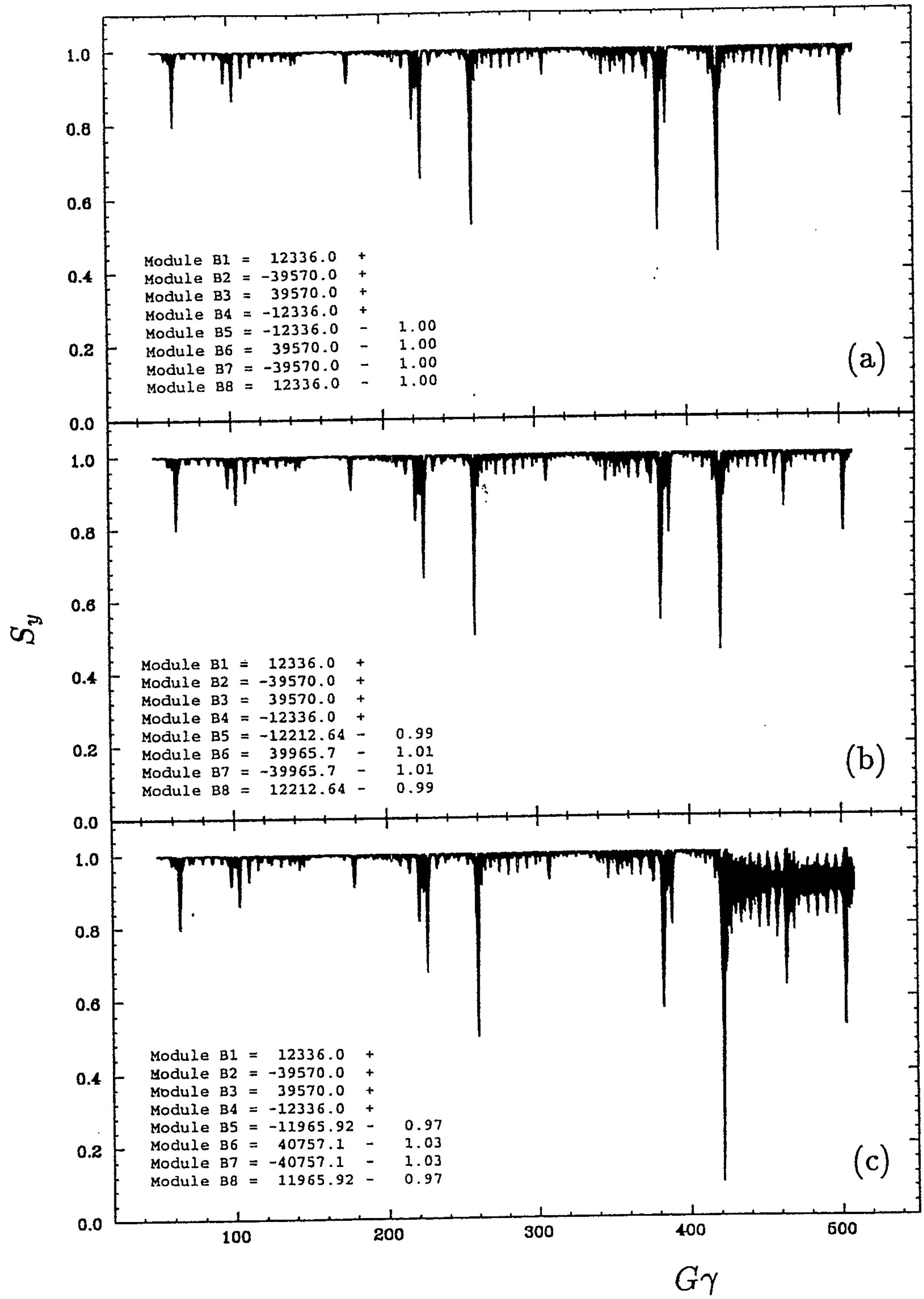

Fig. 9 Spin tracking for increasing the magnetic field of inside two helixes but decreasing that of outside two helixes of second Siberian snake from perfect working point (a), to 1% (b) then 3% (c), with initial emittance 6 $\mu radm$.