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The Spin Trace of Polarized Proton in Various Magnets for RHIC

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Spin Note

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Work report in RIKEN

The Spin Trace of Polarized Proton in Various Magnets for RHIC

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In this work report, we present the spin equation and motion equation. They are expressed in terms of the nature coordinates of motion and the X, Y, Z Cartesian coordinates. The spin equation and motion equation are solved in the 9- dimension spaces for the different magnetic fields: dipole magnet, quadruple magnet, sextupole magnet, octupole magnet, decapole magnet, dodecapole magnet and Siberian snakes of helical magnet. After the trace calculating, the transport matrixes are constructed in linear term and nonlinear relationship, then, the depolarization by the magnet is discussed for polarized proton of RIKEN-BNL collaboration.

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Abstract

In this work report, we present the spin equation and motion equation. They are expressed in terms of the nature coordinates of motion and the X, Y, Z Cartesian coordinates. The spin equation and motion equation are solved in the 9- dimension spaces for the different magnetic fields: dipole magnet, quadruple magnet, sextupole magnet, octupole magnet, decapole magnet, dodecapole magnet and Siberian snakes of helical magnet. After the trace calculating, the transport matrixes are constructed in linear term and nonlinear relationship, then, the depolarization by the magnet is discussed for polarized proton of RIKEN-BNL collaboration.

1. Introduction

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven, now under construction will have the possibility of polarized proton-proton collisions up to a beam energy of 250 GeV, a luminosity of 2 $\times 10^{32} cm^{-2} s^{-1}$ [1)] for BNL-RIKEN collaborated high energy polarized proton project. This report describes the transport trace and matrix of accelerating and transporting the polarized proton beam in the various magnet in RHIC, including a so-call " Siberian snake" magnetic field, a special kind of spin rotator that change the particle spin direction by π around the beam axis lying in the horizontal plane, the Siberian snakes help to keep the polarized beam when the beam are accelerated to cross the region of spin depolarizing resonance.

The first part, the orbit and spin motions are discussed in the magnetic field; then the magnetic fields are described of various magnet; the trace and transport matrix including spin components are given at 9-dimension spaces; and the depolarized action is discussed for various magnets.

2. Orbit and Spin Motion

The motion of the spin vector $\vec{S}$ for proton, in a static field, is a pure precession. It leads to a spin-orbit coupling that is the reason of depolarization phenomena. During acceleration process, the polarization may be lost when the spin precession frequency passes through a depolarizing resonance. These resonance occur when the number of spin precession rotations per revolution $G\gamma$ (G= 1.793 is the anomalous magnetic moment of the proton, γ is the relativistic factor of accelerated particle) is equal to an integer (imperfection resonance) or equal to $kP \pm \nu_y$ (intrinsic resonance). This report presents the basic knowledge of spin trace in the static field that is far from the regions of spin depolarizing resonance.

At relativistic energy, the spin motion in a static electromagnetic field is given by the Bargman-Michel-Telegdi equation [2)]:

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_{BMT} \times \vec{S}$$

with

$$\vec{\Omega}_{BMT} = -\frac{e}{m_o\gamma}[(1+\gamma a)\vec{B}_\perp + (1+a)\vec{B}_\| - (a+\frac{1}{1+\gamma})\gamma\beta \times \frac{\vec{E}}{c}]$$

where $\vec{B}_\perp$ and $\vec{B}_{||}$ are the transverse and longitudinal components of the induction magnetic field $\vec{B}$ relative to the instantaneous velocity of the particle; β is the velocity factor in the units of light velocity; the field $\vec{E}$, $\vec{B}$ and times t are expressed in the laboratory frame; the a is the polarized particle gyromagnet ratio anomalous value.

At same time, the motion of particle obeys the Lorentz force equation,

$$\frac{d\vec{v}}{dt} = \frac{e}{\gamma m}\vec{v} \times \vec{B} + e\vec{E}$$

The Lorentz force has two components originating from the electric field and the magnetic field as same as the spin rotating strength. For the relativistic particle, ($v \sim c$), the rotating strength and the Lorentz force of an electric field $\vec{E}$ are of same order of magnitude as a magnetic field with $\vec{B} = \vec{E}/c$. One-Tesla magnetic field is comparable to an electric field of 3 $\times 10^8 V/m$. Since it is technically straight forward to generate magnetic field of order of Tesla, but rather difficult to establish the equivalent electric field. All apparent that most beam guidance and focusing elements for relativistic particle are based on magnetic field. The term of electric field can be negligible in the beam transport calculations.

We assume that the particle moves in the planar orbit and its position is characterized by the nature coordinates (x,s,z), and the field in that position is written as

$$\vec{r} = r_o(s)\vec{s} + x\vec{x} + z\vec{z}$$

$$\vec{B} = B_s\vec{s} + B_x\vec{x} + B_z\vec{z},$$

we have the following relations:

$$\vec{v} = \frac{d\vec{r}(s)}{dt} = \frac{ds}{dt}[x'\vec{x} + (1 + \frac{x}{\rho})\vec{s} + z'\vec{z}]$$

and

$$\frac{d\vec{v}}{dt} = \frac{d^2 s}{dt^2}[x'\vec{x} + (1 + \frac{x}{\rho})\vec{s} + z'\vec{z}] + (\frac{ds}{dt})^2[(x'' - \frac{1}{\rho} - \frac{x}{\rho^2})\vec{x} + (1 + \frac{x'}{\rho})\vec{s} + z''\vec{z}]$$

we can deduce that ($\rho \to \infty$):

$$\vec{v} \times \vec{B} = \tfrac{ds}{dt}[((1 + \tfrac{x}{\rho})B_z - z'B_s)\vec{x} + (z'B_x - x'B_z)\vec{s} + (x'B_s - (1 + \tfrac{x}{\rho})B_x)\vec{z}]$$

$$\vec{B}_\perp = \tfrac{1}{v^2}(\vec{v} \times \vec{B}) \times \vec{v}$$

$$\approx [(-x'B_s + B_x)\vec{x} + (-x'B_x - z'B_z)\vec{s} + (-z'B_s + B_z)\vec{z}]\tfrac{1}{1+x'^2+z'^2}$$

$$\vec{B}_{||} = \tfrac{1}{v^2}(\vec{v} \bullet \vec{B})\vec{v}$$

$$\approx \tfrac{x'B_x + B_s + z'B_z}{1+x'^2+z'^2}(x'\vec{x} + \vec{s} + z'\vec{z})$$

where a prime denotes differentiation by s. Finally We get the motion in x, z, x', z',

S_x, S_z, S_s 7- dimensions trace equations:

$$\frac{dx}{ds} = x'$$
$$\frac{dz}{ds} = z'$$
$$\frac{dx'}{ds} = c_1(-z'B_s - x'z'B_x + (1+x'^2)B_z)$$
$$\frac{dz'}{ds} = c_1(x'B_s - x'z'B_z + (1+z'^2)B_x)$$
$$\frac{dS_x}{ds} = S_sP_z - S_zP_s$$
$$\frac{dS_s}{ds} = S_zP_x - S_xP_z$$
$$\frac{dS_z}{ds} = S_xP_s - S_sP_x$$

and here

$$P_x = \frac{c_1}{1+x'^2+z'^2}[(1+G\gamma)(-x'B_s + B_x) + (1+G)x'(x'B_x + B_s + z'B_z)$$
$$P_s = \frac{c_1}{1+x'^2+z'^2}[(1+G\gamma)(-x'B_x + z'B_z) + (1+G)(x'B_x + B_s + z'B_z)$$
$$P_z = \frac{c_1}{1+x'^2+z'^2}[(1+G\gamma)(-z'B_s + B_z) + (1+G)z'(x'B_x + B_s + z'B_z)$$
$$c_1 = \frac{e}{m_o\gamma v}\sqrt{1+x'^2+z'^2}$$

The calculation is simpler in nature coordinates of 7-dimension spaces than in right angle coordinates of 9-dimension spaces. But the motion equations of high precision is very complicated in nature coordinates, the results can be given is in one approximation that the radii of curve is infinite. If we need calculate the trace in a ring for a long time, or for high precision results, the Cartisan coordinates need be used.

Positions are characteristic by (x,y,z) in Cartisan coordinates, they are functions of time t, the axis (x,y,z) are relative to (x,z,s) at originate point of Cartisan coordinates. The equation sets are similarly written as:

$$\frac{dx}{dt} = x'$$
$$\frac{dy}{dt} = y'$$
$$\frac{dz}{dt} = z'$$
$$\frac{dx'}{dt} = c_1v(y'B_z - z'B_y)$$
$$\frac{dy'}{dt} = c_1v(z'B_x - x'B_z)$$
$$\frac{dz'}{dt} = c_1v(x'B_y - z'B_x)$$
$$\frac{dS_x}{dt} = -c_1v[S_y((1+G\gamma)B_{cz} + (1+G)B_{pz}) - S_z((1+G\gamma)B_{cy} + (1+G)B_{py})]$$
$$\frac{dS_y}{dt} = -c_1v[S_z((1+G\gamma)B_{cx} + (1+G)B_{px}) - S_x((1+G\gamma)B_{cz} + (1+G)B_{pz})]$$
$$\frac{dS_z}{dt} = -c_1v[S_x((1+G\gamma)B_{cy} + (1+G)B_{py}) - S_y((1+G\gamma)B_{cx} + (1+G)B_{px})]$$

hare

$$B_{pi} = (x'B_x + y'B_y + z'B_z)\frac{i'}{v^2} \quad (i = x, y, z)$$
$$B_{ci} = B_i - B_{pi} \quad (i = x, y, z)$$

3. Initial Conditions and Transport Matrix

The orbit and spin traces can be easy to calculate in the previous equation sets. But the seemingly arbitrary parameters of initial conditions according to distribution of real beam package make it impossible to formulate a general solution of the differential equations of motion. To describe particle trajectories results analytically through a beam transport line composed of shift space, bending magnets, quadrupoles and Siberian snake of helical magnets, or each one of magnet, independent with initial condition, we necessarily to construct matrix of beam transport.

We create lot of initial condition rays of individual particle. The initial conditions include initial positions, velocities, spin directions in the three dimension spaces. The random positions around start point within radii less 3 cm, the random velocities around beam velocity within small velocity radii less than 0.0003 v, and random spin components with total spin equal 1, are generated by Mont-Calo method.

One particle with initial coordinates $u_i(x, y, z, v_x, v_y, v_z, S_x, S_y, S_z)$ comes to end point of apparent with coordinates $u_o(x, y, z, v_x, v_y, v_z, S_x, S_y, S_z)$ after solving differential equation sets, According to the concept of transport matrix, there is

$$u_o = T \times u_i$$

here T is 9 $\times$ 9 matrix, is so call transport matrix. u_o, u_i is coordinate vector of 1 $\times$ 9 matrix. A sets of initial coordinate vector U form n $\times$ 9 matrix, we can get output U_o n $\times$ 9 matrix. From a complete matrix (n=9), we easily deduce transport matrix as

$$T = U_i \times U_o^{-1}$$

For some case, we need the nonlinear of the transport matrix, here we partly give the transport matrix of second order, it defines as

$$u_o = T_1 \times u_i + T_2 \times (u_i \times u_i^+)$$

The equation need two complete matrixes U_{i1}, U_{i2} at least and those solution matrixes U_{o1}, U_{o2}, we deduce as

$$\begin{aligned} T_1 = & (U_{o1} \times [U_{i1} \times U_{i1}^+]^{-1} - U_{o2} \times [U_{i2} \times U_{i2}^+]^{-1}) \times \\ & [U_{i1} \times [U_{i1} \times U_{i1}^+]^{-1} - U_{i2} \times [U_{i2} \times U_{i2}^+]^{-1}]^{-1} \\ T_2 = & [U_{o1} \times U_{i1}^{-1} - U_{o2} \times U_{i2}^{-1}] \times [U_{i1} \times U_{i1}^{-1} - U_{i2} \times U_{i2}^{-1}]^{-1} \end{aligned}$$

In this work, we only care about the particle shift from the prefect beam path. The initial condition differences $(u_{i1} - u_{i2})$ and final output difference $(u_{o1} - u_{o2})$ are taken as input and output coordinate vectors, the velocity united by v_{beam} for Δv_z, and by $0.002 v_{beam}$ for v_x, v_y.

4. Polarized Proton in Helical Field

The general form of the field of the helical can be described by using the scalar potential $\psi(r, \phi, z)$ in the cylindrical coordinate system. For helical symmetry, the solution of Laplases equation can is written as an infinite sum of harmonics:

$$\psi(r, \theta) = \sum_{m=1}^{\infty} I_m(mkr)(a_m cos(m\theta) + b_m sin(m\theta))$$

where I_m are modified Bessel function, and the coefficients a_m, b_m depend on the magnet coil configuration, $\theta = \phi - kz$, k is helical phase vector ($= 2\pi/\lambda, \lambda$ is period of helical magnet in length). From field $\vec{B} = -\nabla\psi$, one can obtain:

$$B_r = -k \sum_{m=1}^{\infty} mI'_m(mkr)(a_m cos(m\theta) + b_m sin(m\theta))$$
$$B_z = k \sum_{m=1}^{\infty} mI_m(mkr)(b_m cos(m\theta) - a_m sin(m\theta))$$
$$B_\phi = -\frac{1}{kr} B_z$$

I' is the derivative value of the Bessel function. In particular if we consider a helical dipole, from dipole symmetry, all A_m harmonics with even m should be zero and all a_m are equal zero. Farther to eliminate higher harmonics with $m = 3, 5...$, the field can be described:

$$B_r = 2B_0[I_0(kr) - \frac{1}{kr} I_1(kr)]cos(kz)$$
$$B_\phi = -\frac{2B_0}{kr} I_1(kr) sin(kz)$$
$$B_z = 2B_0 I_1(kr) sin(kz)$$

B_0 is transverse magnetic field on-axis. When the field is closed to the axis, it is expressed in Cartesian coordinates (x,y,z) as

$$B_x = -B_0[(1 + \frac{1}{8}k^2(3x^2 + y^2))sin(kz) - \frac{1}{4}k^2 xycos(kz)]$$
$$B_y = B_0[(1 + \frac{1}{8}k^2(x^2 + 3y^2))cos(kz) - \frac{1}{4}k^2 xysin(kz)]$$
$$B_z = -B_0 k[(1 + \frac{1}{8}k^2(x^2 + y^2))(xcos(kz) + ysin(kz))]$$

within $kr < 0.8$, the maximum error is estimated at 1.0%.

We calculate trace of the 25GeV polarized proton, the Siberian snake consists of four helical magnets with hard boundary, the length of each one is 2.5 meters and all the helicity is right-handed (positive k), the maximum magnetic field are 1.25T, -4.00T, 4.00T, -1.25T respectively, here magnetic field at enter point is only z direction.

Fig. 1 shows the field on particle projectile, Fig. 2 and Fig. 3 are the trace in position and velocity space respectively, for all x, z, x', z' of initial value equal zero. For RHIC accelerator, the minimum energy is 25GeV, the maximum shift in the space close to 3cm, so the design size of snake should greater or equal 3cm. The velocity shift is symmetric with 180^o at center point, the position shift is equal zero at final. If initial velocity is not equal zero the shape of velocity is same but shift a constant, and the position is shift to the fix direction, the helical snakes haven't focusing function.

Fig. 4-6 are the spin trajectory in the helical snake, Sz shift into -Sz, Ss shift into Sx and Sx shift into Ss if with initial condition Sz, Ss, Sx equal to 1 individually. The important is Sz shift to -Sz, this shift makes the process of acceleration avoids the polarized resource and keeps the beam polarized.

The transport matrix of 9×9 is obtained

$$\begin{bmatrix}
0.9856 & 0.0037 & 0.0000 & -0.1496E-01 & -0.4452E-03 & 0.1559E-09 & 0.0001 & 0.0053 & -0.0067 \\
-0.0053 & 0.9572 & -0.0001 & -0.1933E-03 & -0.4457E-01 & -0.2479E-08 & -0.0011 & 0.0044 & -0.0069 \\
0.0000 & 0.0000 & 1.0000 & -0.1150E-05 & -0.3431E-05 & -0.3496E-10 & 0.0002 & -0.0012 & 0.0012 \\
0.0000 & 0.0000 & 0.0000 & 0.1645E-07 & 0.3885E-10 & -0.7913E-17 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.3207E-10 & 0.1597E-07 & 0.2013E-14 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.3412E-13 & -0.2465E-11 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.2248E-05 & 0.1334E-06 & 0.1037E-10 & 0.1372 & 0.0303 & 0.9897 \\
0.0000 & 0.0000 & 0.0000 & 0.2910E-05 & 0.1326E-04 & 0.1747E-09 & -0.0543 & -0.9933 & 0.0326 \\
0.0000 & 0.0000 & 0.0000 & 0.1526E-06 & 0.7619E-05 & 0.6389E-10 & 0.9888 & -0.0576 & -0.1388
\end{bmatrix}$$

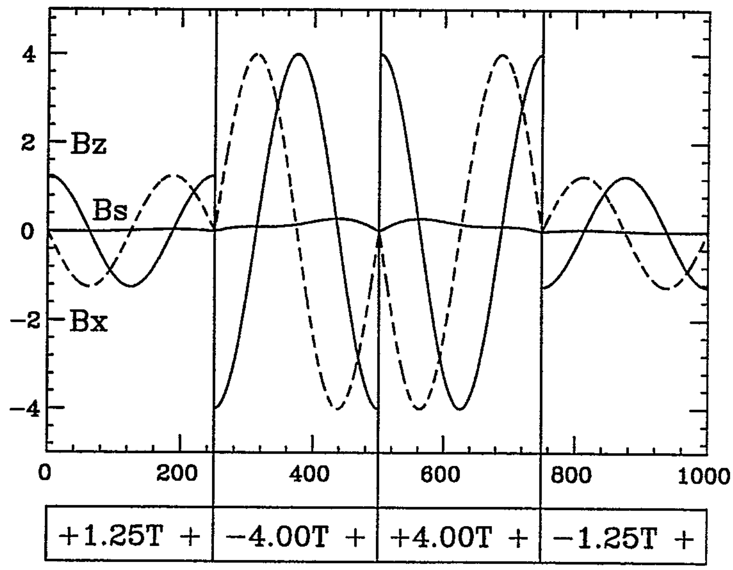

Fig. 1 The magnet field along particle orbit in the snake with Sz change into Sx.

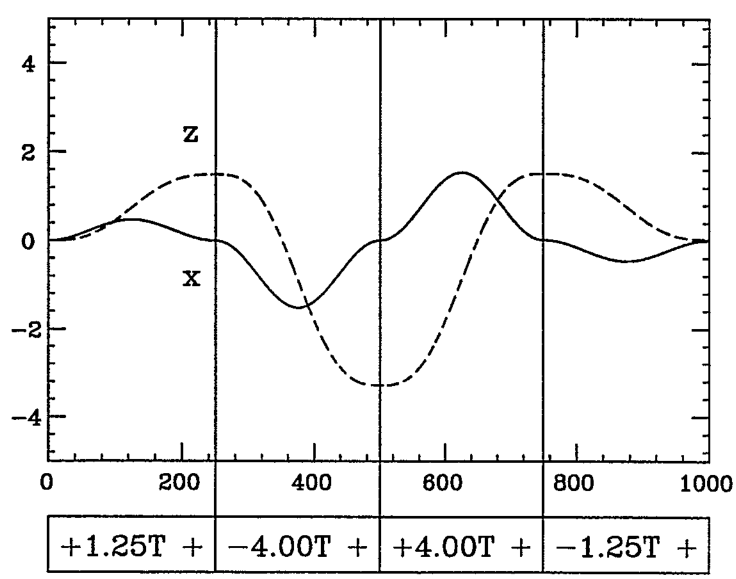

Fig. 2 Particle orbit inside the snake with Sz change into −Sz for 25GeV proton.

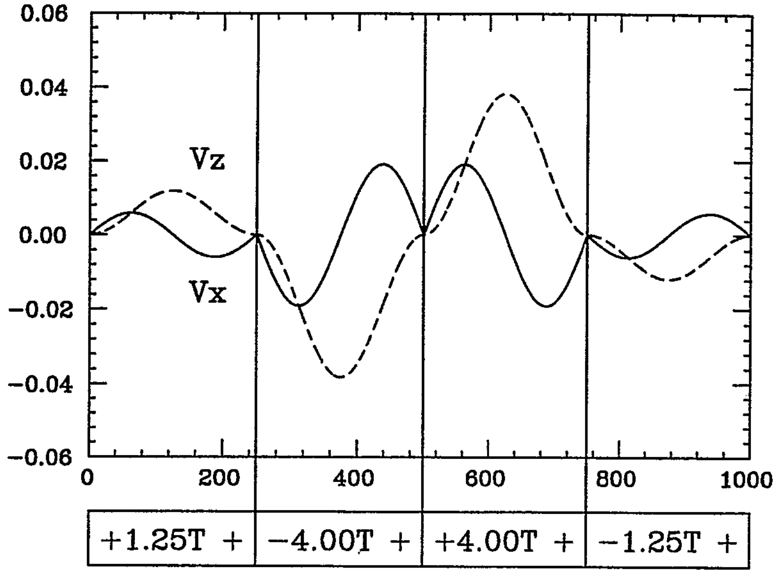

Fig. 3 Vertical velocity change inside the snake with Sz change into −Sz for 25GeV proton.

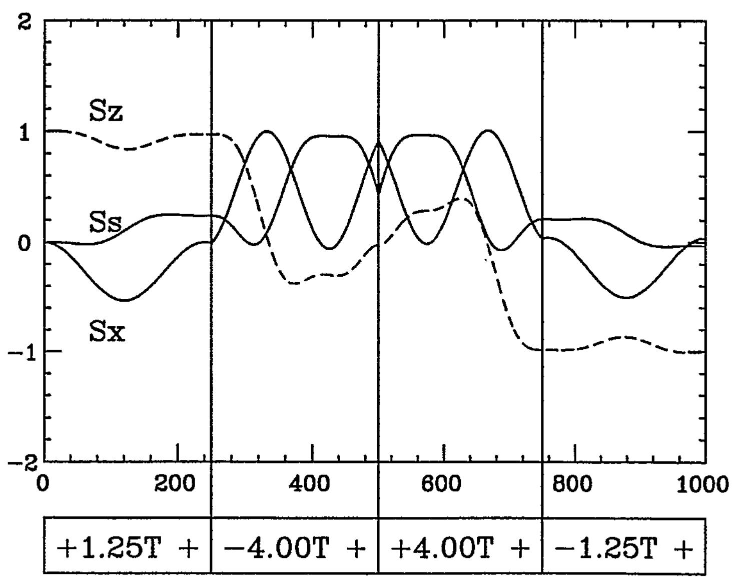

Fig. 4 Spin trajectory inside the snake with Sz change into −Sz for 25GeV proton.

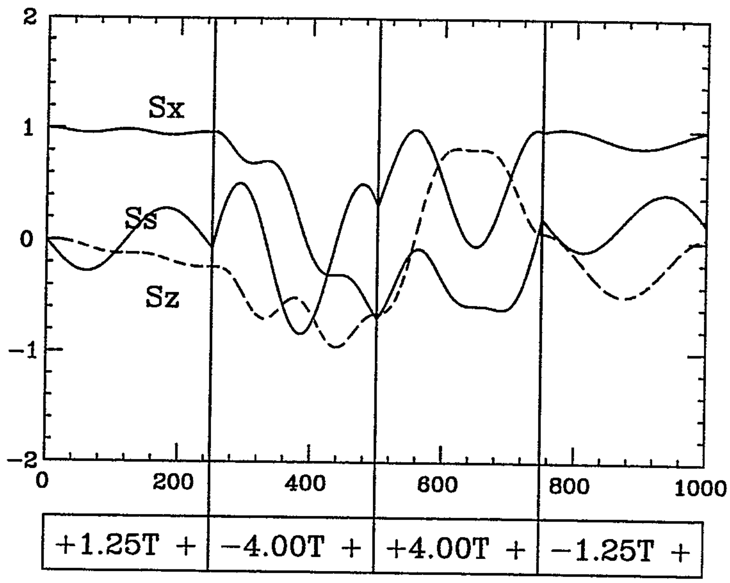

Fig. 5 Spin trajectory inside the snake with Sx change into Ss for 25GeV proton.

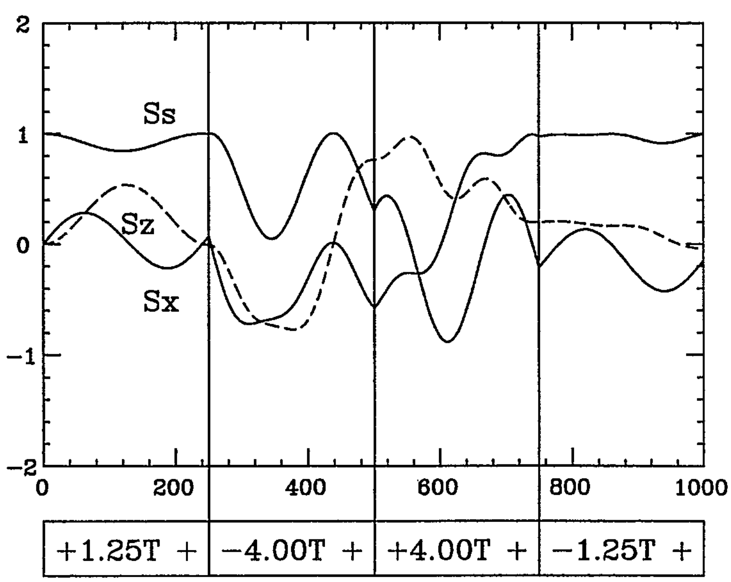

Fig. 6 Spin trajectory inside the snake with Ss change into Sx for 25GeV proton.

and nonlinear of matrix, T_1 is

$$\begin{bmatrix}
0.9856 & 0.0037 & 0.0000 & -0.1500E-01 & -0.4447E-03 & -0.2257E-09 & -0.0012 & 0.0007 & -0.0005 \\
-0.0053 & 0.9571 & -0.0001 & -0.1699E-03 & -0.4463E-01 & -0.2299E-08 & 0.0029 & 0.0004 & 0.0003 \\
0.0000 & 0.0000 & 1.0000 & -0.1656E-04 & 0.1502E-04 & -0.2274E-09 & -0.0010 & 0.0003 & 0.0001 \\
0.0000 & 0.0000 & 0.0000 & 0.1645E-07 & 0.3861E-10 & -0.3232E-17 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.3204E-10 & 0.1597E-07 & 0.2012E-14 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.1020E-12 & -0.2213E-11 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.2684E-05 & -0.1666E-04 & 0.4346E-10 & 0.1387 & 0.0307 & 0.9908 \\
0.0000 & 0.0000 & 0.0000 & -0.2608E-04 & 0.3595E-04 & -0.2867E-09 & -0.0553 & -0.9971 & 0.0379 \\
0.0000 & 0.0000 & 0.0000 & -0.1601E-05 & 0.1838E-04 & -0.5501E-10 & 0.9885 & -0.0594 & -0.1365
\end{bmatrix}$$

and T_2

$$\begin{bmatrix}
0.0000 & 0.0003 & 0.0001 & -0.4487E-04 & 0.5375E-05 & -0.2714E-06 & -0.0024 & 0.0008 & -0.0011 \\
0.0000 & 0.0002 & -0.0003 & -0.4036E-04 & -0.3998E-05 & -0.4940E-06 & -0.0070 & 0.0046 & -0.0015 \\
0.0000 & -0.0007 & 0.0009 & 0.1350E-03 & 0.1657E-04 & 0.1479E-05 & 0.0213 & -0.0118 & 0.0046 \\
0.0000 & 0.0000 & 0.0000 & -0.6669E-19 & -0.2490E-19 & -0.6017E-21 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.1203E-19 & -0.9312E-20 & -0.1719E-21 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.1583E-19 & -0.1204E-19 & 0.1616E-21 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0003 & -0.0002 & -0.3249E-04 & 0.5958E-04 & -0.4566E-06 & -0.0060 & -0.0011 & 0.0013 \\
0.0000 & -0.0010 & 0.0015 & 0.2610E-03 & 0.6416E-04 & 0.2751E-05 & 0.0319 & -0.0208 & 0.0071 \\
0.0000 & 0.0000 & -0.0002 & -0.3419E-04 & -0.5082E-04 & -0.2383E-06 & -0.0027 & 0.0028 & -0.0008
\end{bmatrix}$$

5. Polarized Proton in Multipole Magnets

Central part of field:

Same as previous section, the potential can be describe in the term of a Taylor expansion at $r \to \infty$:

$$\psi(r,\theta,z) = -\sum_{m>1} \frac{1}{m!} A_m(z) r^m e^{im\theta}$$

the field coefficients A_m are derived from the charge free Laplace equation. The field of center party of multipole magnets being independent z is assumed for beam transport issue. The magnetic field components for n-th order of multipoles, derived from the imaginary solution, are given in the Cartesian coordinates:

$$B_{nx} = \textstyle\sum_{m=1}^{n/2} A_{n-2m+1,2m-1} \frac{x^{n-2m}}{(n-2m)!} \frac{y^{2m-1}}{(2m-1)!}$$
$$B_{ny} = \textstyle\sum_{m=1}^{(n+1)/2} A_{n-2m+1,2m-1} \frac{x^{n-2m+1}}{(n-2m+1)!} \frac{y^{2m-2}}{(2m-2)!}$$

One can easily get the field solution (upright) for lower multipoles:

Dipole (2):

$$B_x = 0$$
$$B_y = \kappa$$

Quadrupole (4):

$$B_x = ky$$
$$B_y = kx$$

Sextupole (6):

$$B_x = mxy$$
$$B_y = \tfrac{1}{2} m(x^2 - y^2)$$

Octupole (8):

$$B_x = \tfrac{1}{6}r(3x^2y - y^3)$$
$$B_y = \tfrac{1}{6}r(x^3 - 3xy^2)$$

Decapole (10):

$$B_x = \tfrac{1}{6}d(x^3y - xy^3)$$
$$B_y = \tfrac{1}{24}d(x^4 - 6x^2y^2 + y^4)$$

dodecapole (12):

$$B_x = \tfrac{1}{24}ey(5x^2(x^2 - y^2) - y^2(5x^2 - y^2))$$
$$B_y = \tfrac{1}{24}ex(5y^2(x^2 - y^2) + x^2(5y^2 - x^2))$$

The parameters of multipole strength can be related to the derivatives of magnetic field:

$$s_n(m^{-n}) = (\frac{\partial^{n-1} B_y}{\partial x^{n-1}})_{x=0,y=0}$$

and s_n is the strength parameter of n-th order of multipole, i,e., $s_2 = k, s_3 = m, s_4 = r, s_5 = d....$ In our calculation, the strength parameters are taken as input parameters.

Fringe field:

The field decay of edge of multipole magnets is dependent on the equipment radius D and the distance z of effective magnetic boundary with a factor of z/D, and the field decay is in term of exponential function, all are well known. One can used a set of parameters c_0, c_1, c_2, c_3, c_4 to create distance function:

$$f = c_1 + c_2X + c_3X^2 + c_3X^3 + c_4X^4; \quad X = \frac{z}{D}$$

and its derivative function $f^{(n)}$ in the order $n(n < 4)$ for z. The exponential form function are use as:

$$P_1 = \frac{1}{1 + exp(f)} \qquad P_2 = \frac{exp(f)}{1 + exp(f)}.$$

After mathematical deducing, the field can be written in the term of following coefficients that are n-th derivative value of shape function P_1:

$$\begin{aligned}
G_1 = & -f^{(1)}P_1P_2 \\
G_2 = & -(f^{(2)} + (f^{(1)})^2)P_1P_2 + 2(f^{(1)})^2P_1P_2^2 \\
G_3 = & -(f^{(3)} + 3f^{(2)}f^{(1)} + (f^{(1)})^3)P_1P_2 + 6(f^{(1)}f^{(2)} + (f^{(1)})^3)P_1P_2^2 - 6(f^{(1)})^3P_1P_2^3 \\
G_4 = & -(f^{(4)} + 4f^{(1)}f^{(3)} + 3(f^{(2)})^2 + 6(f^{(1)})^2f^{(2)} + (f^{(1)})^4)P_1P_2 + (8f^{(1)}(f^{(3)}+ \\
& 36(f^{(1)})^2f^{(2)} + 6f^{(2)})^2 + 14(f^{(1)})^4)P_1P_2^2 - 36((f^{(1)})^2f^{(2)} + (f^{(1)})^4)P_1P_2^3+ \\
& 24(f^{(1)})^4)P_1P_2^4
\end{aligned}$$

Quadrupole (4):

$$B_x = ky(P_1 - \tfrac{1}{12}G_2(3x^2 + y^2) + \tfrac{1}{384}G_4(x^2 + y^2)(5x^2 + y^2))$$
$$B_y = kx(P_1 - \tfrac{1}{12}G_2(x^2 + 3y^2) + \tfrac{1}{384}G_4(x^2 + y^2)(x^2 + 5y^2))$$
$$B_z = kxy(G_1 - \tfrac{1}{12}G_3(x^2 + y^2))$$

Sextupole (6):

$$B_x = mxy(P_1 - \tfrac{1}{14}G_2(3x^2 + y^2))$$
$$B_y = \tfrac{1}{2}m(P_1(x^2 - y^2) - \tfrac{1}{48}G_2(x^2(3x^2 + y^2) + 5y^2(x^2 - y^2)))$$
$$B_z = \tfrac{1}{2}my(3x^2 - y^2)(\tfrac{1}{3}G_1 - \tfrac{1}{48}G_3(x^2 + y^2))$$

Octupole (8):

$$B_x = \tfrac{1}{6}ry(P_1(3x^2 - y^2) - \tfrac{1}{20}G_4(x^2(5x^2 + y^2) - y^2(x^2 + y^2)))$$
$$B_y = -\tfrac{1}{6}rx(P_1(3y^2 - x^2) - \tfrac{1}{20}G_4(y^2(5y^2 + x^2) - x^2(x^2 + y^2)))$$
$$B_z = \tfrac{1}{6}rG_1xy(x^2 - y^2)$$

Decapole (10):

$$B_x = \tfrac{1}{6}dP_1(x^3y - xy^3)$$
$$B_y = \tfrac{1}{24}dP_1(x^4 - 6x^2y^2 + y^4)$$
$$B_z = \tfrac{1}{24}dG_1y(x^2(x^2 + y^2) - \tfrac{1}{2}y^2(5x^2 - y^2))$$

dodecapole (12):

$$B_x = \tfrac{1}{24}eP_1y(5x^2(x^2 - y^2) - y^2(5x^2 - y^2))$$
$$B_y = \tfrac{1}{24}exP_1(5y^2(x^2 - y^2) + x^2(5y^2 - x^2))$$
$$B_z = 0$$

In the Cartesian coordinates, the field of multipole magnets makes the spin direction changed, if the field of transverse components of spin direction isn't zero. The individual magnet is depolarized for polarized beam, we calculate spin trace in every magnet and turn the results into matrix. We used the parameters as $c_1 = 0.476; c_1 = 5.980; c_2 = -5.610; c_3 = 2.20; c_4 = 0.00; c_5 = 0.00$, and all magnets have equipment radius of 3.5cm and effective length of 40cm for the central part for example. The proton energy is set at 25GeV. The strength parameters of multipole magnetic field are $k = 0.2 Tesla/cm$ for quadrupole, $\frac{1}{2}m = 0.2 Tesla/cm^2$ for sextupole, $\frac{1}{6}r = 0.2 Tesla/cm^3$ for octupole, $\frac{1}{24}d = 0.2 Tesla/cm^4$ for decapole and $\frac{1}{24}e = 0.2 Tesla/cm^5$ for dodecapole magnets. Integral step are taken as 0.05cm in solving the differential equations.

From the results of matrixes, the area of $n = 7 \sim 9, m = 7 \sim 9$ shows that the spin direction can be maintained mainly in each magnets in the error %5. Special quadrupole, a lot of that are used in RHIC for beam focusing, the spin direction can keep in %99.9. The area of $n = 7 \sim 9, m = 1 \sim 6$ shown the spin affect the orbit trace, it for positions are clearly greater that for velocity, that for velocity can be ignored in the order of 1E-4 ; the affect are decreasing significant increasing the beam energy. The area of $n = 1 \sim 6, m = 7 \sim 8$ shown the trace affect spin, mainly by the velocity of x, y. The column $n = 7$ show the affect for status of particle by velocity of z is clearly small than that of x, y.

for **Quadrupole** , the transport matrix:

$$\begin{bmatrix}
0.9304 & 0.0001 & 0.0000 & -0.4966E+00 & 0.9013E-03 & 0.2852E-09 & -0.0003 & -0.0003 & -0.0004 \\
-0.0001 & 1.0704 & 0.0000 & -0.6297E-03 & 0.5037E+00 & 0.2868E-09 & -0.0002 & 0.0001 & -0.0003 \\
0.0000 & 0.0000 & 1.0000 & 0.1765E-03 & -0.1517E-03 & -0.5937E-10 & -0.0002 & -0.0001 & 0.0002 \\
0.0000 & 0.0000 & 0.0000 & 0.1603E-07 & -0.8491E-12 & 0.5268E-18 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.5940E-11 & 0.1737E-07 & 0.3227E-17 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.3848E-11 & -0.6310E-11 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.1027E-03 & 0.1255E-03 & 0.8588E-10 & 0.9997 & 0.0000 & 0.0000 \\
-0.0001 & 0.0001 & 0.0000 & -0.5507E-03 & 0.6617E-03 & 0.2738E-09 & -0.0002 & 0.9998 & -0.0002 \\
0.0000 & 0.0000 & 0.0000 & -0.2319E-03 & 0.3172E-03 & 0.1604E-09 & 0.0000 & 0.0001 & 0.9991
\end{bmatrix}$$

and two order of matrix, T_1 is

$$\begin{bmatrix}
0.9305 & 0.0000 & 0.0000 & -0.4956E+00 & -0.1328E-03 & 0.1058E-09 & 0.0002 & 0.0000 & 0.0003 \\
0.0002 & 1.0702 & 0.0000 & 0.1730E-02 & 0.5024E+00 & 0.1018E-08 & -0.0001 & -0.0013 & -0.0013 \\
-0.0002 & 0.0000 & 1.0000 & -0.1368E-02 & 0.2324E-04 & -0.3237E-09 & -0.0004 & 0.0005 & 0.0008 \\
0.0000 & 0.0000 & 0.0000 & 0.1605E-07 & 0.1363E-11 & 0.7787E-17 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.4220E-11 & 0.1737E-07 & -0.5907E-17 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.1973E-10 & 0.1689E-10 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0001 & 0.0000 & 0.0000 & 0.4950E-03 & -0.1955E-03 & 0.3093E-09 & 0.9996 & 0.0005 & 0.0006 \\
-0.0002 & 0.0001 & 0.0000 & -0.1483E-02 & 0.5028E-03 & -0.6413E-09 & -0.0006 & 0.9996 & 0.0005 \\
0.0000 & 0.0000 & 0.0000 & -0.1537E-03 & 0.1323E-03 & -0.4987E-10 & 0.0002 & -0.0009 & 0.9989
\end{bmatrix}$$

and T_2

$$\begin{bmatrix}
0.0000 & -0.0001 & -0.0001 & -0.1179E-03 & -0.5465E-03 & -0.1411E-05 & -0.0188 & 0.0177 & -0.0002 \\
-0.0001 & -0.0004 & 0.0002 & -0.5763E-03 & -0.2604E-02 & 0.2608E-05 & 0.0479 & 0.0105 & 0.0542 \\
0.0002 & 0.0010 & -0.0007 & 0.1546E-02 & 0.7099E-02 & -0.9001E-05 & -0.1125 & -0.0158 & -0.2107 \\
0.0000 & 0.0000 & 0.0000 & -0.8823E-18 & -0.2425E-17 & 0.5901E-20 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.9990E-19 & -0.7750E-18 & 0.5762E-21 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.2445E-19 & 0.7612E-18 & -0.5524E-21 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & -0.0003 & 0.0002 & -0.1245E-03 & -0.2025E-02 & 0.3073E-05 & 0.0203 & -0.0028 & 0.0916 \\
0.0005 & 0.0015 & -0.0017 & 0.3421E-02 & 0.1079E-01 & -0.2281E-04 & -0.2574 & -0.0370 & -0.4699 \\
-0.0001 & -0.0001 & 0.0001 & -0.4697E-03 & -0.8158E-03 & 0.1883E-05 & 0.0206 & 0.0167 & 0.0298
\end{bmatrix}$$

for **Sextupole**, transport matrix:

$$\begin{bmatrix}
0.9993 & -0.0009 & -0.0001 & -0.5751E-02 & -0.7563E-02 & -0.7422E-06 & 0.0393 & -0.0411 & 0.0386 \\
-0.0010 & 0.9998 & 0.0000 & -0.8263E-02 & -0.1860E-02 & -0.4375E-06 & 0.0266 & -0.0327 & 0.0140 \\
0.0002 & 0.0001 & 1.0000 & 0.1803E-02 & 0.1091E-02 & 0.2412E-06 & -0.0024 & 0.0010 & -0.0189 \\
0.0000 & 0.0000 & 0.0000 & 0.1672E-07 & 0.2996E-11 & 0.7889E-15 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.8112E-10 & 0.1663E-07 & -0.1222E-13 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.5967E-10 & 0.4695E-10 & 0.3339E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & -0.0001 & 0.0000 & 0.3109E-03 & -0.1149E-02 & 0.5563E-07 & 1.0020 & 0.0016 & -0.0070 \\
-0.0009 & -0.0006 & 0.0000 & -0.7776E-02 & -0.5382E-02 & -0.5760E-06 & 0.0337 & 0.9537 & 0.0325 \\
-0.0004 & -0.0003 & 0.0000 & -0.3196E-02 & -0.2281E-02 & -0.6220E-06 & 0.0093 & 0.0078 & 1.0229
\end{bmatrix}$$

and two order of matrix, T_1 is

$$\begin{bmatrix}
1.0011 & 0.0003 & 0.0002 & 0.8976E-02 & 0.2714E-02 & 0.3063E-05 & 0.0306 & -0.0049 & 0.0030 \\
0.0017 & 1.0012 & 0.0001 & 0.1475E-01 & 0.1047E-01 & 0.8855E-06 & -0.0210 & -0.0394 & 0.0055 \\
-0.0012 & -0.0002 & 0.9998 & -0.9927E-02 & -0.1919E-02 & -0.3060E-05 & -0.0105 & 0.0122 & -0.0387 \\
0.0000 & 0.0000 & 0.0000 & 0.1691E-07 & -0.4717E-10 & 0.9361E-14 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.7997E-10 & 0.1663E-07 & -0.2347E-14 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.1389E-09 & -0.7252E-10 & 0.3339E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0006 & 0.0000 & -0.0001 & 0.5097E-02 & -0.1481E-03 & -0.8601E-06 & 1.0097 & 0.0001 & -0.0066 \\
-0.0013 & -0.0002 & -0.0001 & -0.1085E-01 & -0.1902E-02 & -0.1757E-05 & -0.0182 & 1.0488 & 0.0114 \\
-0.0002 & 0.0002 & 0.0001 & -0.1926E-02 & 0.1607E-02 & 0.1054E-05 & -0.0177 & -0.0019 & 1.0208
\end{bmatrix}$$

and T_2

$$\begin{bmatrix}
0.0679 & 0.0393 & -0.0001 & 0.4873E+00 & 0.2806E+00 & -0.8412E-06 & 0.0005 & 0.0007 & 0.0006 \\
-0.1409 & 0.0585 & -0.0003 & -0.1015E+01 & 0.4163E+00 & -0.4868E-05 & -0.0007 & 0.0010 & 0.0023 \\
0.2194 & -0.0445 & 0.0009 & 0.1598E+01 & -0.3129E+00 & 0.1454E-04 & 0.0017 & -0.0021 & -0.0081 \\
0.0000 & 0.0000 & 0.0000 & -0.6070E-15 & -0.5910E-15 & -0.5356E-20 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.6624E-16 & 0.1923E-15 & -0.1456E-20 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.7969E-16 & -0.2154E-15 & 0.1533E-20 & 0.0000 & 0.0000 & 0.0000 \\
-0.0446 & 0.0597 & -0.0002 & -0.3285E+00 & 0.4241E+00 & -0.3672E-05 & -0.0004 & 0.0002 & 0.0020 \\
0.2556 & 0.1664 & 0.0015 & 0.1893E+01 & 0.1200E+01 & 0.2386E-04 & 0.0042 & -0.0029 & -0.0137 \\
-0.0225 & -0.0566 & -0.0001 & -0.1663E+00 & -0.4044E+00 & -0.2119E-05 & -0.0005 & 0.0004 & 0.0012
\end{bmatrix}$$

for **Octupole** , the transport matrix is

$$\begin{bmatrix}
1.0741 & -0.1053 & 0.0000 & 0.5308E+00 & -0.7485E+00 & 0.2083E-09 & -0.0001 & -0.0007 & 0.0000 \\
0.0283 & 0.9434 & 0.0000 & 0.2024E+00 & -0.4016E+00 & 0.1740E-09 & 0.0002 & -0.0006 & 0.0000 \\
0.0082 & 0.0256 & 1.0000 & 0.5782E-01 & 0.1822E+00 & -0.5057E-10 & -0.0001 & 0.0002 & 0.0001 \\
0.0000 & 0.0000 & 0.0000 & 0.1539E-07 & 0.1132E-08 & -0.4214E-18 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.2617E-08 & 0.9509E-08 & 0.2004E-17 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.3518E-08 & 0.6626E-08 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0299 & -0.0098 & 0.0000 & 0.2135E+00 & -0.7012E-01 & 0.1158E-10 & 0.9999 & 0.0001 & 0.0000 \\
0.0418 & -0.0873 & 0.0000 & 0.2987E+00 & -0.6202E+00 & 0.1871E-09 & 0.0001 & 0.9993 & 0.0001 \\
0.0150 & -0.0478 & 0.0000 & 0.1089E+00 & -0.3410E+00 & 0.1135E-09 & 0.0000 & -0.0002 & 0.9997
\end{bmatrix}$$

and two order of matrix, T_1 is

$$\begin{bmatrix}
0.9808 & -0.0298 & 0.0000 & -0.1439E+00 & -0.2172E+00 & 0.7798E-09 & 0.0007 & 0.0003 & 0.0002 \\
0.1510 & 0.9195 & 0.0000 & 0.1091E+01 & -0.5543E+00 & 0.3195E-07 & 0.0078 & 0.0105 & -0.0201 \\
-0.1550 & 0.1190 & 1.0000 & -0.1107E+01 & 0.8428E+00 & -0.1825E-07 & -0.0052 & -0.0082 & 0.0118 \\
0.0000 & 0.0000 & 0.0000 & 0.1908E-07 & -0.3685E-08 & 0.3974E-15 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.7446E-08 & 0.3175E-08 & -0.4872E-16 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.1824E-07 & -0.1905E-07 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0312 & 0.0421 & 0.0000 & 0.2288E+00 & 0.3006E+00 & 0.3328E-08 & 1.0007 & 0.0031 & 0.0003 \\
-0.1933 & -0.0381 & 0.0000 & -0.1386E+01 & -0.2784E+00 & -0.2903E-07 & -0.0087 & 0.9899 & 0.0164 \\
-0.0170 & -0.0848 & 0.0000 & -0.1249E+00 & -0.6020E+00 & 0.8794E-12 & -0.0003 & -0.0015 & 0.9974
\end{bmatrix}$$

and T_2

$$\begin{bmatrix}
-0.0002 & 0.0000 & -0.0001 & -0.1677E-02 & 0.2626E-03 & -0.1115E-05 & 0.0598 & -0.0192 & 0.0465 \\
-0.0009 & -0.0001 & 0.0002 & -0.6629E-02 & -0.4428E-03 & 0.6997E-06 & 0.1323 & -0.0187 & 0.2814 \\
0.0024 & 0.0001 & -0.0002 & 0.1724E-01 & 0.9937E-03 & 0.1838E-05 & -0.3721 & 0.0065 & -0.6789 \\
0.0000 & 0.0000 & 0.0000 & -0.6417E-17 & -0.3877E-18 & 0.4007E-21 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.1873E-17 & 0.7382E-19 & -0.4007E-21 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.2175E-17 & 0.3665E-18 & 0.1450E-20 & 0.0000 & 0.0000 & 0.0000 \\
-0.0007 & 0.0001 & -0.0001 & -0.4791E-02 & 0.3564E-03 & -0.2802E-05 & 0.1048 & 0.0183 & 0.1232 \\
0.0041 & 0.0000 & -0.0007 & 0.2915E-01 & 0.5168E-03 & -0.2864E-05 & -0.5846 & 0.0152 & -1.1717 \\
-0.0003 & -0.0001 & 0.0002 & -0.2343E-02 & -0.5070E-03 & 0.2120E-05 & 0.0405 & -0.0156 & 0.1400
\end{bmatrix}$$

For **Decapole**, the transport matrix is:

$$\begin{bmatrix}
0.9992 & -0.0010 & -0.0001 & -0.6346E-02 & -0.8067E-02 & -0.8092E-06 & 0.0106 & -0.0125 & 0.0739 \\
-0.0010 & 0.9997 & 0.0000 & -0.8233E-02 & -0.2630E-02 & -0.7038E-06 & 0.0121 & -0.0133 & 0.0392 \\
0.0003 & 0.0001 & 1.0000 & 0.2164E-02 & 0.1124E-02 & 0.2673E-06 & 0.0024 & 0.0047 & -0.0178 \\
0.0000 & 0.0000 & 0.0000 & 0.1671E-07 & 0.3588E-11 & 0.1136E-14 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.8621E-10 & 0.1663E-07 & -0.8379E-14 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.5020E-10 & 0.5298E-10 & 0.3339E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0001 & -0.0001 & 0.0000 & 0.8449E-03 & -0.1215E-02 & 0.6910E-07 & 1.0005 & 0.0019 & 0.0038 \\
-0.0009 & -0.0006 & -0.0001 & -0.7791E-02 & -0.5389E-02 & -0.7668E-06 & 0.0155 & 0.9885 & 0.0607 \\
-0.0003 & -0.0003 & 0.0000 & -0.2905E-02 & -0.2693E-02 & -0.4708E-06 & -0.0130 & -0.0117 & 1.0257
\end{bmatrix}$$

and two order of matrix, T_1 is

$$\begin{bmatrix} 1.0053 & 0.0029 & 0.0002 & 0.4502E-01 & 0.2482E-01 & 0.2896E-05 & 0.0115 & -0.0140 & 0.0474 \\ 0.0496 & 1.0388 & -0.0004 & 0.4182E+00 & 0.3256E+00 & -0.6272E-05 & -0.1565 & 0.0782 & -0.0886 \\ -0.0332 & -0.0211 & 1.0000 & -0.2803E+00 & -0.1770E+00 & 0.4441E-06 & 0.1122 & -0.0148 & 0.0093 \\ 0.0000 & 0.0000 & 0.0000 & 0.2239E-07 & 0.3054E-08 & -0.7026E-13 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.1415E-08 & 0.1571E-07 & 0.8581E-14 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.4643E-08 & -0.5121E-08 & 0.3348E-10 & 0.0000 & 0.0000 & 0.0000 \\ 0.0077 & 0.0020 & -0.0001 & 0.6461E-01 & 0.1639E-01 & -0.1472E-05 & 0.9804 & 0.0083 & -0.0178 \\ -0.0386 & -0.0314 & 0.0004 & -0.3259E+00 & -0.2638E+00 & 0.7260E-05 & 0.1960 & 0.9267 & 0.1234 \\ 0.0007 & 0.0024 & 0.0002 & 0.5925E-02 & 0.2041E-01 & 0.2491E-05 & 0.0230 & -0.0115 & 1.0250 \end{bmatrix}$$

and T_2

$$\begin{bmatrix} 0.1769 & -0.1997 & -0.0004 & 0.1257E+01 & -0.1424E+01 & -0.6488E-05 & 0.0030 & 0.0044 & 0.0005 \\ 0.6435 & -0.8735 & -0.0027 & 0.4525E+01 & -0.6258E+01 & -0.4299E-04 & -0.0054 & 0.0070 & 0.0071 \\ -1.4914 & 2.3014 & 0.0074 & -0.1047E+02 & 0.1649E+02 & 0.1177E-03 & 0.0151 & -0.0167 & -0.0321 \\ 0.0000 & 0.0000 & 0.0000 & 0.3851E-14 & -0.6351E-14 & -0.4542E-19 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.1062E-14 & -0.1561E-14 & -0.1182E-19 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.1388E-14 & 0.1500E-14 & 0.1475E-19 & 0.0000 & 0.0000 & 0.0000 \\ 0.3668 & -0.4740 & -0.0019 & 0.2582E+01 & -0.3402E+01 & -0.3008E-04 & -0.0024 & 0.0012 & 0.0048 \\ -2.4279 & 3.9710 & 0.0124 & -0.1701E+02 & 0.2845E+02 & 0.1970E-03 & 0.0331 & -0.0225 & -0.0547 \\ 0.2258 & -0.4362 & -0.0011 & 0.1576E+01 & -0.3119E+01 & -0.1764E-04 & -0.0045 & 0.0037 & 0.0070 \end{bmatrix}$$

For **Dodecapole**, the transport matrix is:

$$\begin{bmatrix} 0.9625 & -0.0303 & 0.0000 & -0.2643E+00 & -0.2198E+00 & 0.8072E-10 & -0.0002 & -0.0003 & 0.0000 \\ -0.0360 & 0.9449 & 0.0000 & -0.2546E+00 & -0.3957E+00 & 0.6254E-10 & -0.0002 & -0.0002 & 0.0000 \\ 0.0185 & 0.0148 & 1.0000 & 0.1312E+00 & 0.1057E+00 & -0.2251E-10 & 0.0001 & 0.0001 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.1689E-07 & 0.1146E-08 & -0.1258E-18 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.4309E-08 & 0.1301E-07 & 0.7799E-18 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.1158E-08 & 0.2574E-08 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\ 0.0096 & 0.0073 & 0.0000 & 0.6816E-01 & 0.5186E-01 & 0.8378E-12 & 1.0000 & 0.0000 & 0.0000 \\ -0.0457 & -0.0447 & 0.0000 & -0.3229E+00 & -0.3227E+00 & 0.7100E-10 & -0.0002 & 0.9997 & 0.0001 \\ -0.0109 & -0.0207 & 0.0000 & -0.7581E-01 & -0.1483E+00 & 0.4508E-10 & -0.0001 & -0.0001 & 0.9999 \end{bmatrix}$$

and two order of matrix, T_1 is

$$\begin{bmatrix} 0.8705 & 0.0309 & 0.0000 & -0.9255E+00 & 0.2270E+00 & 0.1941E-08 & 0.0028 & -0.0005 & -0.0006 \\ 0.8317 & 0.3347 & 0.0000 & 0.6122E+01 & -0.4526E+01 & 0.1028E-06 & 0.0529 & -0.0017 & -0.1436 \\ -0.4518 & 0.6064 & 1.0000 & -0.3335E+01 & 0.4193E+01 & -0.5185E-07 & -0.0241 & -0.0134 & 0.0874 \\ 0.0000 & 0.0000 & 0.0000 & 0.1130E-06 & -0.6223E-07 & 0.1227E-14 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.4120E-07 & -0.2110E-08 & -0.4966E-15 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.5238E-07 & 0.5087E-07 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\ 0.0284 & -0.1153 & 0.0000 & 0.2363E+00 & -0.8096E+00 & 0.8431E-08 & 1.0029 & 0.0062 & -0.0064 \\ -0.4779 & 0.6029 & 0.0000 & -0.3560E+01 & 0.4073E+01 & -0.1048E-06 & -0.0729 & 1.0123 & 0.1217 \\ 0.1137 & -0.0204 & 0.0000 & 0.8023E+00 & -0.1466E+00 & -0.2945E-08 & -0.0070 & 0.0006 & 0.9899 \end{bmatrix}$$

and T_2

$$\begin{bmatrix} 0.0000 & 0.0001 & -0.0013 & 0.2195E-03 & 0.5992E-03 & -0.1942E-04 & -0.1613 & -0.0693 & -0.1139 \\ 0.0001 & 0.0002 & -0.0066 & 0.3777E-03 & -0.3354E-03 & -0.1028E-03 & -0.4617 & -0.2091 & -0.1085 \\ -0.0004 & -0.0003 & 0.0175 & -0.2567E-02 & 0.1957E-02 & 0.2742E-03 & 1.2695 & 0.6324 & -0.0164 \\ 0.0000 & 0.0000 & 0.0000 & 0.1378E-17 & -0.1034E-17 & -0.1041E-18 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.1521E-18 & 0.2898E-18 & -0.2699E-19 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.1470E-17 & -0.5202E-18 & 0.3606E-19 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0002 & -0.0042 & -0.5123E-03 & 0.3653E-03 & -0.6505E-04 & -0.3422 & -0.2231 & -0.0945 \\ -0.0009 & -0.0008 & 0.0286 & -0.5987E-02 & 0.1678E-02 & 0.4500E-03 & 2.1384 & 1.1512 & -0.2019 \\ 0.0002 & 0.0000 & -0.0028 & 0.1276E-02 & -0.6452E-03 & -0.4384E-04 & -0.1759 & -0.0558 & 0.0950 \end{bmatrix}$$

6. Polarized Proton in Dipole Magnets

Dipole magnet is very important magnet, it is used in almost of all accelerator and beam transport system. In store ring liking RHIC, the high energy beam bend by a lot of dipole magnet, and with same setup direction (upright or down). The field of central part is very simple $B_y = constant, B_x, B_z$ equal zero, and normal case, the value of B_y is big, the fringe field is very important for spin trace calculation. Any fringe field can change the Z direction spin little that Bz, Bx not equal zero.

Similar with multipole magnetic fringe field, we deduce dipole fringe field, that is dependent not only with z/D but also with x and y. If the curve surface of magnetic field, that is the surface near edge of magnetic core and outside of the maximum field with the maximum field, is written as

$$s_0 + s_1x^2 + s_2y^2 + s_3x^2y^2 + s_4x^4 + s_5y^4 + s_6x^6 + s_7y^6 + z = 0$$

with a set coefficients $s_0, s_1, s_2 ... s_7$. According to the symmetry, the odd term equal zero, of course, one can add the odd term for asymmetric field. The effective distance for exponential decay should be minimum distance from the curve surface to the point (x, y) of calculation, the point of the curve surface is $P(x_p, z_p)$. There are relationship:

$$(\frac{\partial z}{\partial x})_P = -\frac{x - x_p}{z - z_p}.$$

$$(\frac{\partial z}{\partial y})_P = -\frac{y - y_p}{z - z_p}.$$

We used the repeated instead to close the point P

$$z_p = -(s_0 + s_1x^2 + s_2y^2 + s_3x^2y^2 + s_4x^4 + s_5y^4 + s_6x^6 + s_7y^6)$$

$$x_p = \frac{\frac{\partial z}{\partial x}(z - z_p + \frac{\partial z}{\partial x}x_p) + x}{1 + (\frac{\partial z}{\partial x})^2}$$

$$y_p = \frac{\frac{\partial z}{\partial y}(z - z_p + \frac{\partial z}{\partial y}y_p) + y}{1 + (\frac{\partial z}{\partial y})^2}$$

with first value $x_p = x, y_p = y$ and second step value $x_p = (3x_p + x)/4, y_p = (3y_p + y)/4$. The distance function f and the field value are obtained with set of parameters c_0, c_1, c_2, c_3, c_4,

$$f = c_1 + c_2X + c_3X^2 + c_3X^3 + c_4X^4$$

$$X = \frac{\sqrt{(x-x_p)^2 + (z-z_p)^2 + (y-y_p)^2}}{D}$$

$$B = \frac{2B_0}{1 + exp(f)}$$

For a high order to calculation, we use the express as ref. 3) that is approximately independent with y,

$$B_1 = B(x, y, z + \Delta) \qquad B_2 = B(x, y, z + 2\Delta)$$

$$B_3 = B(x + \Delta, y, z + \Delta) \qquad B_4 = B(x - \Delta, y, z + \Delta)$$

$$B_5 = B(x + \Delta, y, z) \qquad B_6 = B(x + 2\Delta, y, z)$$

$$B_7 = B(x - \Delta, y, z) \qquad B_8 = B(x - 2\Delta, y, z)$$

$$B_9 = B(x, y, z - \Delta) \qquad B_{10} = B(x, y, z - 2\Delta)$$

$$B_{11} = B(x + \Delta, y, z - \Delta) \qquad B_{12} = B(x - \Delta, y, z - \Delta)$$

to deduce B_x, B_y, B_z as:

$$
\begin{aligned}
B_x = \; & \tfrac{y}{\Delta}(\tfrac{2}{3}(B_5 - B_7) - \tfrac{1}{12}(B_6 - B_8)) + (\tfrac{y}{\Delta})^3(\tfrac{1}{6}(B_5 - B_7) - \tfrac{1}{12}(B_6 - B_8) \\
& -\tfrac{1}{12}(B_3 + B_{11} - B_4 - B_{12} - 2B_5 + 2B_7)) \\
B_y = \; & B_0 - (\tfrac{y}{\Delta})^2(\tfrac{2}{3}(B_1 + B_9 + B_5 + B_7 - 4B_0) - \tfrac{1}{24}(B_2 + B_{10} + B_6 + B_8 - 4B_0)) \\
& +(\tfrac{y}{\Delta})^4(-\tfrac{1}{6}(B_1 + B_9 + B_5 + B_7 - 4B_0) + \tfrac{1}{24}(B_2 + B_{10} + B_6 + B_8 - 4B_0)+ \\
& \tfrac{1}{12}(B_3 + B_{11} + B_4 + B_{12} - 2B_1 - 2B_9 - 2B_5 - 2B_7 + 4B_0)) \\
B_z = \; & \tfrac{y}{\Delta}(\tfrac{2}{3}(B_1 - B_9) - \tfrac{1}{12}(B_2 - B_{10})) + (\tfrac{y}{\Delta})^3(\tfrac{1}{6}(B_1 - B_9) - \tfrac{1}{12}(B_2 - B_{10}) \\
& -\tfrac{1}{12}(B_4 + B_4 - B_{11} - B_{12} - 2B_1 + 2B_9))
\end{aligned}
$$

The maximum field B_0 of dipole magnet is set at 1.44 $Tesla$, the distance of central part is 200cm, and the parameters $c_1 = 0.075501; c_1 = 1.961363; c_2 = -0.198532; c_3 = 0.453699; c_4 = -0.011455; c_5 = 0.039178, s_0 = 1.5, s_1 = 0.015, s_2 = 0.489796, s_3... = 0.0$ the differential distance $\Delta = 1/4$ of integral distance (0.01cm), the gap of two pole equal 3.5cm are set for example calculation. The matrixes are given as following for dipole, as well as helical snake dipole the gap of magnets is $32cm, s_0 = 2.0, s_1 = 0.222, s_2 = 0.222$.

The dipole magnet change very little for S_z, but large for S_x, S_y. The matrix is multiplied when several dipole magnets are used, shown that S_x and S_y exchange periodically. The helical snake with gap is not clear when change to no gap.

The transport matrix of 9×9 for dipole magnet is obtained

$$
\begin{bmatrix}
1.0000 & 0.0000 & 0.0000 & -0.7395E-04 & -0.1296E-04 & 0.7684E-08 & 0.0003 & 0.0002 & 0.0003 \\
0.0000 & 1.0000 & 0.0000 & 0.8054E-05 & -0.1466E-03 & -0.6381E-09 & 0.0000 & 0.0001 & -0.0004 \\
0.0000 & 0.0000 & 1.0000 & 0.4746E-05 & 0.6029E-06 & 0.7562E-09 & 0.0001 & 0.0000 & -0.0002 \\
0.0000 & 0.0000 & 0.0000 & 0.1669E-07 & -0.8782E-15 & 0.1345E-12 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.3873E-13 & 0.1669E-07 & 0.1200E-15 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.6733E-10 & 0.3907E-13 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.2132E-05 & -0.2369E-05 & 0.4143E-08 & 0.9807 & 0.0000 & -0.1967 \\
0.0000 & 0.0000 & 0.0000 & 0.4098E-05 & -0.7298E-05 & 0.7013E-08 & 0.0004 & 1.0002 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.2277E-05 & -0.2482E-05 & -0.4434E-08 & 0.1959 & 0.0000 & 0.9806
\end{bmatrix}
$$

and nonlinear of matrix, T_1 is

$$
\begin{bmatrix}
1.0000 & 0.0000 & 0.0000 & -0.7972E-04 & 0.5837E-05 & 0.9411E-07 & 0.0012 & 0.0000 & -0.0003 \\
0.0000 & 1.0000 & 0.0000 & -0.3361E-04 & -0.1049E-03 & -0.6665E-07 & -0.0028 & 0.0001 & -0.0010 \\
0.0000 & 0.0000 & 1.0000 & 0.2320E-04 & -0.1895E-04 & 0.3701E-07 & 0.0011 & 0.0000 & 0.0006 \\
0.0000 & 0.0000 & 0.0000 & 0.1669E-07 & 0.1309E-12 & 0.1325E-12 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.6480E-13 & 0.1669E-07 & -0.9894E-15 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.6637E-10 & -0.1168E-12 & 0.3338E-10 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.1340E-05 & 0.1833E-05 & 0.1540E-08 & 0.9814 & 0.0000 & -0.1968 \\
0.0000 & 0.0000 & 0.0000 & 0.1362E-04 & -0.1207E-04 & 0.2370E-07 & 0.0012 & 0.9999 & 0.0032 \\
0.0000 & 0.0000 & 0.0000 & -0.1072E-04 & 0.5562E-05 & -0.1403E-07 & 0.1952 & 0.0000 & 0.9814
\end{bmatrix}
$$

and T_2,

$$
\begin{bmatrix}
0.0016 & 0.0000 & 0.0000 & 0.2778E-01 & -0.6782E-03 & 0.2215E-06 & 0.0005 & -0.0004 & -0.0001 \\
0.0009 & 0.0000 & 0.0000 & 0.1275E-01 & -0.5679E-03 & 0.9936E-07 & 0.0014 & -0.0016 & -0.0002 \\
-0.0055 & 0.0000 & 0.0000 & -0.8606E-01 & -0.7291E-03 & -0.6793E-06 & -0.0044 & 0.0043 & 0.0005 \\
0.0000 & 0.0000 & 0.0000 & 0.2846E-16 & 0.2466E-17 & 0.2243E-21 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.1114E-16 & -0.4232E-18 & 0.8799E-22 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.1176E-16 & 0.4474E-18 & -0.9307E-22 & 0.0000 & 0.0000 & 0.0000 \\
0.0015 & 0.0000 & 0.0000 & 0.2335E-01 & -0.7964E-03 & 0.1837E-06 & 0.0013 & -0.0015 & -0.0002 \\
-0.0101 & -0.0004 & 0.0000 & -0.1560E+00 & -0.6609E-02 & -0.1232E-05 & -0.0082 & 0.0059 & 0.0010 \\
0.0010 & 0.0001 & 0.0000 & 0.1649E-01 & 0.1462E-02 & 0.1312E-06 & 0.0005 & -0.0003 & -0.0001
\end{bmatrix}
$$

and the transport matrix of 9×9 for gaped helical snake is obtained

0.9846	−0.0005	0.0000	−0.1395E − 01	−0.1509E − 03	−0.1652E − 08	0.0005	0.0029	−0.0002
0.0121	0.9502	−0.0001	0.3762E − 03	−0.4532E − 01	−0.5874E − 08	0.0003	0.0026	−0.0004
0.0000	0.0000	1.0000	−0.2069E − 04	0.1235E − 04	0.1217E − 09	0.0000	−0.0005	0.0000
0.0000	0.0000	0.0000	0.1643E − 07	0.1188E − 09	−0.2178E − 16	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	−0.3069E − 09	0.1586E − 07	0.1868E − 14	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	−0.3081E − 12	−0.1514E − 11	0.3338E − 10	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	−0.5927E − 05	−0.1250E − 04	−0.1736E − 09	0.1879	0.0330	0.9816
0.0001	0.0000	0.0000	0.8432E − 04	−0.7603E − 04	−0.1224E − 08	−0.0309	−0.9963	0.0393
0.0000	0.0000	0.0000	0.3975E − 04	−0.3069E − 04	−0.1974E − 09	0.9820	−0.0375	−0.1868

and nonlinear of matrix, T_1 is

0.9845	−0.0004	0.0000	−0.1412E − 01	−0.2353E − 03	−0.4400E − 08	−0.0021	0.0003	−0.0014
0.0121	0.9504	−0.0001	0.3829E − 03	−0.4499E − 01	0.2933E − 08	0.0047	0.0002	0.0014
−0.0001	−0.0001	1.0000	−0.7823E − 04	−0.1313E − 03	−0.2238E − 08	−0.0024	0.0005	−0.0009
0.0000	0.0000	0.0000	0.1643E − 07	0.1196E − 09	0.4473E − 16	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	−0.3078E − 09	0.1586E − 07	0.1964E − 14	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	−0.4558E − 12	−0.3667E − 11	0.3338E − 10	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	−0.4481E − 05	0.3335E − 04	0.2770E − 08	0.1891	0.0333	0.9816
−0.0001	−0.0001	0.0000	−0.7610E − 04	−0.2046E − 03	−0.4914E − 08	−0.0346	−0.9981	0.0386
0.0000	0.0000	0.0000	0.3171E − 04	−0.5065E − 04	−0.2903E − 08	0.9814	−0.0392	−0.1866

and T_2

−0.0068	−0.0004	0.0001	−0.2582E − 02	−0.1463E − 03	−0.2705E − 06	0.0006	0.0002	−0.0018
−0.0133	−0.0014	−0.0003	−0.5516E − 02	−0.1458E − 03	−0.5262E − 06	0.0053	0.0013	−0.0031
0.0362	0.0057	0.0009	0.1642E − 01	0.6575E − 03	0.1540E − 05	−0.0125	−0.0028	0.0108
0.0000	0.0000	0.0000	−0.6692E − 17	−0.5459E − 18	−0.6080E − 21	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	−0.1899E − 17	0.3389E − 19	−0.1836E − 21	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.1725E − 17	0.1276E − 19	0.1675E − 21	0.0000	0.0000	0.0000
−0.0097	−0.0006	−0.0002	−0.4973E − 02	0.1156E − 03	−0.4844E − 06	0.0055	−0.0037	−0.0023
0.0609	0.0125	0.0017	0.3036E − 01	0.1277E − 02	0.2825E − 05	−0.0118	−0.0077	0.0197
−0.0063	−0.0022	−0.0002	−0.2680E − 02	−0.4022E − 03	−0.2339E − 06	−0.0001	0.0018	−0.0023

7. Magnets 'Depolarization'

The position and the velocity of beam at one point are Gauss distribution for each right angle of components, (Fig. 7). Figure 8 shows the position distribution in the quadrupole magnet field, the position A, B, and center point C have different Lorentz force and rotation strength. It means the spin with different beam condition have little change regularly, but it is depolarization for a single magnet.

The polarization can be changed little by little when the number of enter the magnet tend to infinite that transport matrix is the matrix powered by same number, little bit of various of polarization result in a big trouble in the storage ring. But the matrixes of our calculations are upright setup for the magnetic field, the various angular of setup in a real beam line can decease the speed of this kinds of depolarization. And for some matrix, the polarized direction can change from one to another then to initial one in increasing the transport times, the total matrix of RHIC for one circle have or not this property need to calculation that matrix.

The setup of magnets is very important, if the setup of two of Siberian snakes in same sequential and same helicitys of four helical magnets at one side and the other side of the ring (neglect the others magnet), the transport matrix is power two of one Siberian snake when transport one circle. The depolarization is increased increasing the number of circles, because there are no prefect magnetic field to change $S_z = 1$ to $S_z = -1$, and because the ring need keep all energy from boost energy to maximum value, the matrix is dependent with energy. If one Siberian snake is reverse of sequential, helicitys and magnet field of another one, the matrix of one is reverse matrix of another one, the two matrix times into one unit, it is prefect restore to initial spin direction, although the Siberian snake is not work so well (Fig.9). Liking the periodic structure of 'DOFO', if all quadrupoles work as 'F' or 'D', the beam will lost all.

The closed orbit trace means there are some period structure in the ring, the actions by force and rotation strength at two side of one focal point are reverse, the matrixes are nearly reverse matrix. There are lots of periods structure dependent with magnet setup and magnet field as whole. If the beam keep in the ring, the 'polarization' will partly lost and restore periodically, it is difficult to estimate polarization at one energy point.

Acknowledgments

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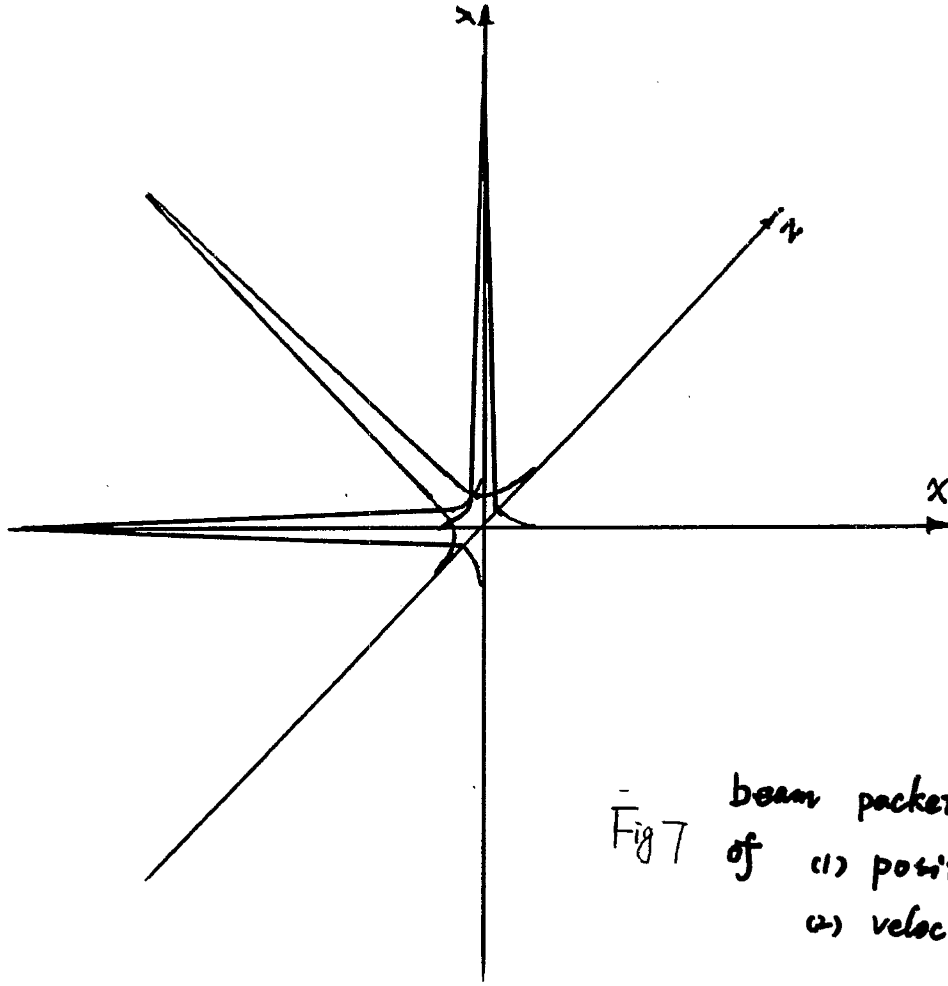

Fig 7 beam packet distribution of (1) position (2) velocity

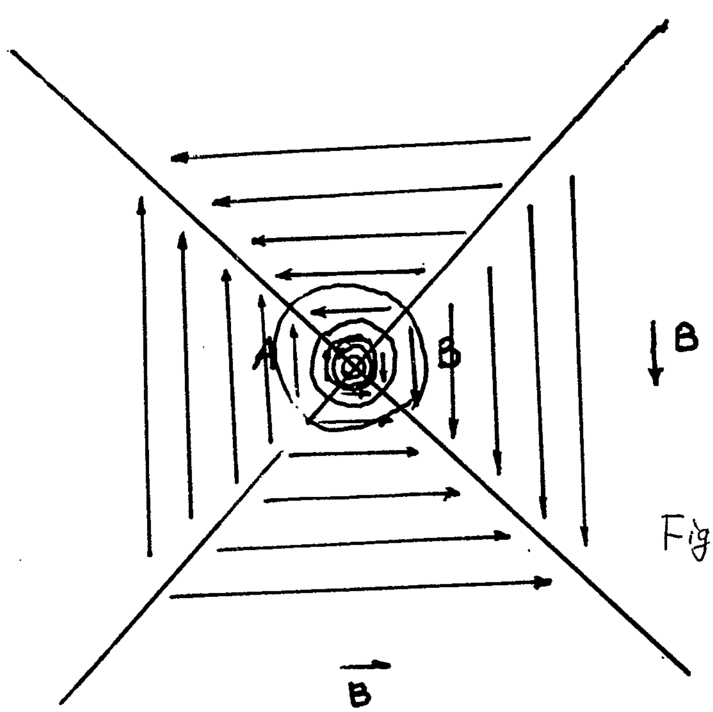

Fig 8 beam in quadrupe; transverse surface

- the spin shift is different between A and B recommand.

two snake setup: $\text{Matrix 1} = [\text{Matrix 2}]^{-1}$

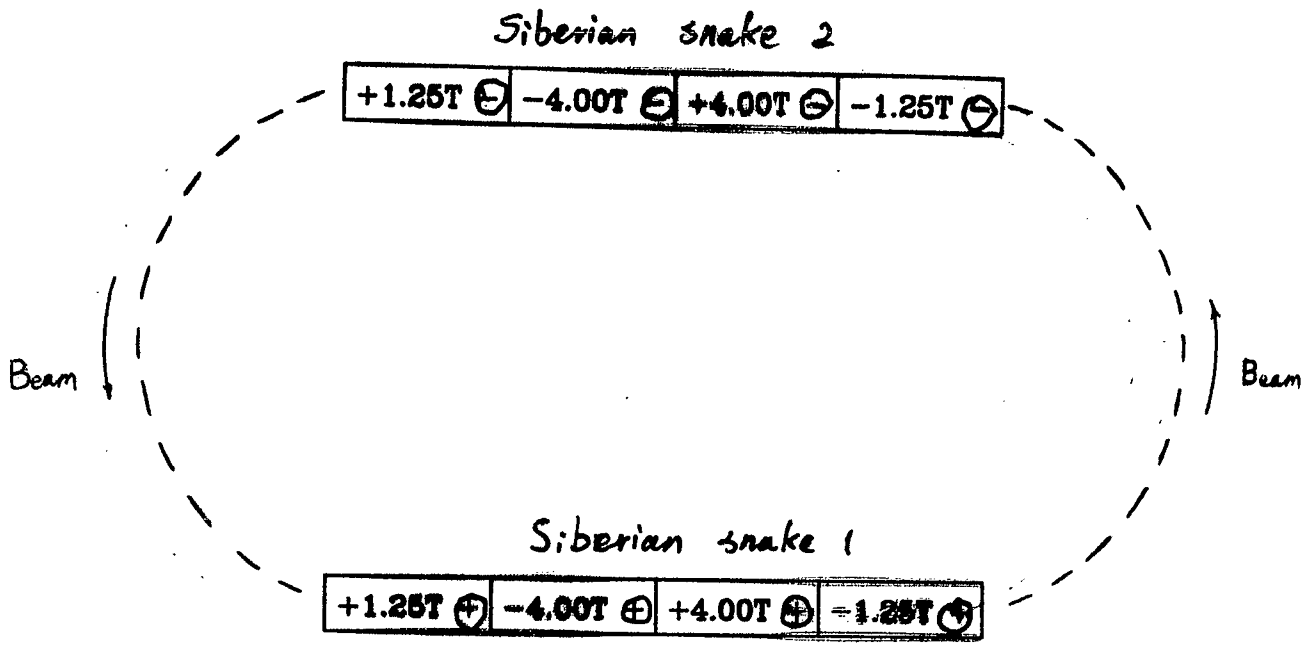

Fig 9

Matrix dependant E beam ; B field

Fig 10.

transport periodic structure:

- position
- spin