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Spin Note

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Abstract

In a polarized proton beam experiment at RHIC, care must be taken to preserve the polarization of the proton beam during its transfer from AGS to RHIC via the AtR line [1]. The energy dependance of the stable spin direction of a polarized proton beam at the exit of the W-line [1], which is a subsection of the AtR line, has been studied earlier for a vertically polarized proton beam at AGS, and results have been reported[2]. In his technical note, the stable beam direction at the RHIC injection point which is defined below, is studied as a function of the proton beam energy and initial beam polarization at AGS, and results are presented.

Description of the Sections of the AtR line which affect the Stable Spin Direction of a Polarized Proton Beam

The AGS to RHIC transfer line (AtR)[1] will be used to transfer beam bunches from AGS to RHIC. A schematic diagram of the AtR beam transfer line is shown in Fig. 1. In the AtR transfer line there are two sections of the beam line where the beam direction is below the horizontal level by 12.5 mrad and 3.0 mrad respectively; outside these sections the beam direction coincides with the horizontal. In this note these two sections will be called the "12.5 mrad" and "3.0 mrad" vertical bends respectively. A schematic diagram of the 12.5 mrad vertical bend is shown in Fig 2. This section consists of a -12.5 mrad vertically bending magnet which is located after the second horizontally bending magnets of the W-line[1] and is followed by six combined function dipoles which bend the beam horizontally to the right by 15^0. This vertical bend ends with a 12.51 mrad vertically bending magnet which restores the beam direction to the horizontal plane.

The 3 mrad vertical bend is located at the end of the AtR line (Fig. 3) and consists of a -3 mrad vertical bend, followed by a 38 mrad horizontally bending (to the right) Lambertson septum magnet[3], a RHIC quadrupole, a RHIC dipole, another RHIC quadrupole and finally the injection kicker which restores the injected beam to the RHIC plane. The initial vertical bend of -3 mrad is partially compensated by the two RHIC quadrupoles (Fig. 3) because the beam is injected off axis with respect to the axes of the RHIC quadrupoles. In this technote,the exit point of the 1.73 mrad vertically deflecting kicker is defined as "the RHIC injection point". It is the arrangement of the bending magnets in these two vertical bends, (vertical bend followed by horizontal and then vertical) that affects the stable spin direction of a polarized proton beam. In the next section, the method to calculate the effect of the vertical bends on the stable spin direction will be discussed.

Procedure to Calculate the Stable Spin Direction at the RHIC Injection Point

The stable spin direction, of an initially vertically polarized proton beam, at the exit of the 12.5 mrad vertical bend has been calculated before and the results appear in (Ref. 2). In those calculations[2] the effect of the 3 mrad vertical bend (defined above) was not included for the mere fact that the last section of the AtR line was designed after the publication of Ref. 2. Moreover the direction of the stable spin direction at the origin of the AtR line, was assumed to be vertical since no provisions were made for a partial snake in AGS[4]. However, with the insertion of the partial snake in AGS[4], the stable spin direction at AGS is a function of the beam energy and azimuthal location in AGS. Therefore in order to obtain more complete results about the stable spin direction at the RHIC injection point one should include in the calculations both, the stable spin direction at AGS as given in (Ref. 4). and the 3 mrad vertical bend. One more item which is included in the present calculations, is the effect, on the stable spin direction, of the AGS and AtR quadrupole fields which act on a non-central trajectory. A comparison of the stable spin direction at the RHIC injection point, between the central and a non-central trajectory will provide an indication of the directional spread of the stable spin direction at the location of the RHIC injection point.

The calculation were based on the equations 1 and 2 below which were numerically integrated using the Runge Kutta integration routine.

$$\frac{d^2\boldsymbol{r}}{dt^2}=\left(\frac{e}{m_0\gamma}\right)\left[\boldsymbol{v}\times\boldsymbol{B}+\boldsymbol{E}-\frac{(\boldsymbol{v}\cdot\boldsymbol{E})\,\boldsymbol{v}}{c^2}\right] \qquad (1)$$

$$\frac{d\boldsymbol{s}}{dt}=\left(-\frac{e}{m_0\gamma}\right)\left[(G\gamma+1)\,\boldsymbol{B}\times\boldsymbol{s}-(\gamma-1)\,G\frac{(\boldsymbol{v}\cdot\boldsymbol{B})\,(\boldsymbol{v}\times\boldsymbol{s})}{(v^2)}+\left(G\gamma+\frac{1}{\left(1+\frac{1}{\gamma}\right)}\right)\frac{(\boldsymbol{E}\times\boldsymbol{v})\times\boldsymbol{s}}{c^2}\right] \qquad (2)$$

Equations 1 and 2 are the differential equations for the equation of motion (r(t)) and magnetic moment (s) respectively of a particle with rest mass (m_0) charge (e) and

G=(g-2)/2 (g=gyromagnetic ratio of the proton) in a magnetic field (B) and an electric field (E). In equations 1 and 2 v=dr/dt is the velocity of the particle, $\gamma=E_{tot}/c^2$ ($E_{tot}=m_0c^2+K$) and c=velocity of light. The simultaneous integration of the equations above, yields both the equation of motion and the directional cosines of the spin. This integration was performed numerically using a modified version[5] of the computer code RAYTRACE[6]. The RAYTRACE code was modified to include the integration of the equation 2 above and also to incorporate the experimentally measured fields of the AGS magnets. The initial conditions (particle coordinates and stable spin direction at the AGS) which are required for the integration of the equations 1 and 2 above are discussed below.

Stable Spin Direction at AGS

The stable spin direction in AGS machine is normally in the vertical direction; however in order to accelerate polarized proton beams at energies closer to the full energy provided by the AGS, machine, the insertion of a partial snake[4] was necessary. The AGS partial snake has eliminated all the imperfection resonances which occur at proton energies with $G\gamma$=integer. With the insertion of the partial snake, the stable spin direction in AGS becomes a function of the proton energy as well as the azimuthal angle of the proton in AGS. In particular at the location of the AGS straight section C10, located 180^0 from the location of the straight section I10 where the partial snake is installed, the stable spin direction is given by:

$$n_{rad}=0$$
$$n_{vert}=(1/N)\sin(\pi G\gamma)\cos(\delta/2) \qquad (3)$$
$$n_{long}=(1/N)\sin(\delta/2)$$

where:

$N=[1-\cos^2(\pi G\gamma)\cos^2(\delta/2)]^{1/2}$

n_{rad}, n_{vert}, and n_{long} are the directional cosines of the radial, vertical and longitudinal spin direction respectively and δ is the spin rotation angle introduced by the partial snake.

The energy dependance of the directional cosine (n_{vert}) of the stable spin direction at the AGS straight section (SS-C10) which is located 180^0 from the partial snake ($\delta=9^0$) is shown in Fig. 4. At $G\gamma$=an integer, the snake acts as an imperfection resonance which flips the spin when the holding field of the AGS is ramped adiabatically over the imperfection resonance. Thus the vertical spin direction is flipped at energies corresponding to $G\gamma$=an integer. When the partial snake is off, ($\delta=0^0$) the directional cosine $n_{vert}=1$ or $n_{vert}=-1$ except at energies coinciding with the various depolarizing resonances of AGS where the stable spin axis becomes undefined and the beam looses its polarization. The results of the stable spin direction at the RHIC injection point will be presented for the two cases:

a) partial snake off.
b) partial snake on.

Closed Orbit Beam Coordinates and Parameters at the Middle of the Straight Section C10

In the pervious section the initial conditions for the stable spin direction at the middle of the straight section C10 of AGS were discussed. In this section an outline of the method to determine the initial coordinates of the closed central trajectory and the beam parameters at the middle of the straight section C10 of AGS will be discussed.

The position coordinates (at C10) of the central closed orbit were obtained by calculating the closed orbit trajectory of the central orbit in AGS using a modified version of the computer code BEAM (Ref.5). In this version of the code the experimentally measured magnetic fields of the AGS magnets were implemented[5]. A detailed description of how the experimentally measured fields of the AGS magnets were introduced into the BEAM code as well as how the code can be used to obtain the coordinates of the central closed orbit and the beam parameters at a given point along the AGS is discussed in Ref.5.

The result from the calculations appear in Table I. For each magnetic field map corresponding to 20.55, 25.63, 27.47, and 29.23 GeV/c momentum, the closed orbit was calculated. The high field quadrupoles of the AGS were adjusted so that the horizontal (ν_h) and vertical (ν_v) tunes acquire the values =8.78 and 8.72 respectively. (Each unit of the high field quadrupole strength in Table I, corresponds to an integrated gradient $\int G \cdot ds$=4.749 Gauss). The extraction bumps[6] were also adjusted in order to suppress any residual oscillations of the closed orbit outside the region of the bumps. The relative strength of the individual bumps (Bump_1a,_1b,_1c) and (Bump_2a,_2b,_2c) appear on the Table I. The chromaticicities (ξ_h ξ_v), the beam parameters (α_x, β_x, α_y, β_y) and the central orbit coordinates (x, θ), which are defined in the beam axis system[5,8], also appear in Table I. Looking at the results of Table I, only the chromaticicities ξ_h , ξ_v and the spatial and angular dispersions η_x, η'_x are varying with the beam momentum, with the rest of the beam parameters remainining almost constant over the beam momentum range of 20.5 GeV/c to 29.2 GeV/c. In the present study the beam momentum range is from 22.5 GeV/c to 25.1 GeV/c or ($G\gamma$=42 to $G\gamma$=48). Therefore the beam coordinates and parameter used in this study were obtained from the third column of Table I corresponding to the momentum 25.632 GeV/c.

With the initial conditions of the particle coordinates and spin orientations determined at the middle of the AGS straight section C10, the particles were tracked from C10, to the extraction point H13, and then to the RHIC injection point (via the AtR line) where the orientation of the stable spin direction

was recorded. The particle tracking from C10 to the extraction point H13, includes the fast extraction kicker at G10, the extraction pumps, and the extraction septum at H10. The required strengths of these extraction magnets were obtained from the third column of Table I. A description of the fast extraction system (FEB) can be found in Refs. 5 and 6.
The particle tracking through the AGS magnets utilized the measured magnetic fields maps of the AGS magnets[5]. These experimental fields maps were implemented into the RAYTRACE code[5,6] and were used in these calculations. The fields of the magnetic elements of the AtR line were assumed to be those of ideal dipoles, quadrupoles and combined function magnets without fringing fields. (sharp cut-off approximation at the entrance and exit of boundaries).

Stable Spin Direction at the RHIC Injection Point (AGS Partial Snake off); Results:

The directional cosine of the stable spin direction with the vertical, at the RHIC injection point, as a function of $G\gamma$, is shown in Fig. 5. Since the partial snake in AGS is turned off, the initial spin direction at AGS is vertical. The filled squares, which are joined by the continuous curve, are the directional cosines of the AGS central orbit particles which were extracted from AGS and transferred to the RHIC injection point. For each central orbit particle, the directional cosine of a non-central orbit particle (non-filled squares in Fig. 5) having the same momentum as the central orbit particle, is also plotted on Fig. 5 and the points are joined with a noncontinuous curve.

The initial coordinates of the non-central orbit particle, at the straight section C10 of AGS, were calculated using the simple formulae.

$$x_{nco}=x_{co}+\Delta x \qquad \Delta x=(\epsilon\beta_x)^{1/2}$$
$$\theta_{nco}=\theta_{co}+\Delta\theta \qquad \Delta\theta=[\epsilon(1+\alpha_x^2)/\beta_x]^{1/2}$$
$$y_{nco}=y_{co}+\Delta y \qquad \Delta x=(\epsilon\beta_y)^{1/2}$$
$$\phi_{nco}=\phi_{co}+\Delta\phi \qquad \Delta\phi=[\epsilon(1+\alpha_y^2)/\beta_y]^{1/2}$$

where the subindices (co) and (nco) correspond to central and non-central orbit respectively, $\epsilon=\epsilon_N/(\beta\gamma)$, [$\epsilon_N=20\pi$ (mm) (mrad) $\gamma=25$ and $\beta=v/c\approx1.0$] The beam parameters α_x β_x α_y β_y and the horizontal and vertical coordinates [$x_{co}=0.32$ (cm), $\theta_{co}=0.02$ (mrad), $y_{co}=0.0$ (cm), $\phi_{co}=0.0$ (mrad)] of the central orbit at AGS C10 location were obtained from Table I.

The non-central orbit provides an indication of the spread of the stable spin axis at the RHIC injection point. This spread which is shown in Fig. 4 (vertical distance between a central and non-central point) is an upper limit of the spread of the stable spin axis. This is because the non-central particle initiates just outside the 95% population of the beam ellipsoid where the particle density is low. A more exact way of obtaining a distribution of the directional spread of the spin's stable axis at RHIC could have been achieved by sampling random particles from the beam ellipsoid (which is defined by the beam parameters at the straight section C10), and subsequently looking at the corresponding spin distribution at the RHIC injection point. At the energy range $46<G\gamma<48$ (see Fig. 5) this spread is insignificant whereas at other regions this spread is significant. This can be easily explained by the fact that the small differences of the stable spin direction between the central ray and off central ray which are caused by the quadrupole fields acting on the off central ray, are magnified by the vertical bends (especially the 12.5 mrad vertical bend) mentioned above. At $G\gamma=45$ there is a 10% loss in the polarization of the beam. (The RHIC transition energy occurs at $G\gamma\approx41.03$)

Stable Spin Direction at RHIC Injection Point (AGS Partial Snake on); Results:

The procedure to calculate and obtain the results for the stable spin direction at the RHIC injection point with the partial snake on, is similar to the procedure with snake off (see previous section) . When the AGS partial snake is turned on, the stable spin direction at AGS is energy dependent, especially at energies where $G\gamma$ is close to an integer (see formulae 3 above or Fig. 4). Therefore in each beam bunch at AGS there will be a stable spin direction associated with the central momentum particle p_0, as well as a distribution of stable spin directions associated with the off momentum particles. This distribution depends on the central momentum p_0 and the momentum spread of the beam bunch. In order to calculate how this distribution is modified at the RHIC injection point, during the transfer of the beam bunch from AGS(SS C10) to the RHIC injection point, a sets of five particles with momenta p_0, $(1\mp0.0005)p_0$ and $(1\mp0.001)p_0$ were selected and traced from the middle of the C10 straight section of AGS to the RHIC injection point.

The directional cosine of the stable spin axis with the vertical (non-filled squares), at the RHIC injection point, as a function of $G\gamma$ are shown in Fig. 6. On the same figure the corresponding directional cosines (filled squares) at the AGS straight section C10 are also shown. Each set of five squares (shown close together in Fig. 6) corresponds to the central momentum p_0 and the four off momentum trajectories mentioned above. The vertical distance of these five points shown in Fig. 6 is a measure of the directional cosine spread. The curves connecting the squares is to guide the eye. The vertical lines drawn in Fig. 6 at $G\gamma$=an integer are for visual separation of the energy regions where the stable spin direction is up (+ signs in Fig. 6) or down (- signs in Fig. 6).

The results appearing in Fig. 6 can be used as a basis of selecting the optimum momentum of a polarized proton beam which will yield a minimum polarization loss and minimum spread of the stable spin direction at the RHIC injection point. As an example a polarized proton beam with momentum corresponding to $G\gamma=46.4$ (see Fig. 6) has a stable spin direction almost vertical ($S_y=0.997$) and a directional cosine spread of 0.001. The corresponding directional cosine S_y of the stable spin axis at the RHIC injection point is 0.975 with a spread of 0.001. Obviously a polarized proton beam injected into RHIC with such a momentum is most desirable. The dependance of the polarization at the RHIC injection point on the initial stable spin direction can be shown by comparing Fig. 5 with Fig. 6. From Fig. 5 (initial stable spin direction at AGS is vertical, $S_y=1.0$) the directional cosine S_y of the stable spin direction at the RHIC injection point for $G\gamma=42.1$ is $S_y=0.825$. From Fig. 6 (initial stable spin direction at AGS has $S_y=0.975$) the directional cosine S_y of stable spin direction at the RHIC injection point for $G\gamma=42.1$ is 0.92 which is almost 10% better compared to that of Fig. 5. At energies where $G\gamma$ approaches an integer, the stable spin direction at AGS depends strongly on the beam momentum spread. This momentum dependance of the stable spin axis can be used to select the proton injection momentum which yields a small directional spread of the stable spin axis at the RHIC injection point. As an example (see Fig. 6) the beam with central momentum $p_0=23.48$ ($G\gamma=44.91$) and momentum spread $\delta p/p=0.1\%$ has a stable spin axis spread, in the directional cosine, of 0.1. However the corresponding spin axis spread, of the directional cosine at the RHIC injection point is only 0.02. In contrast the central momentum particle $p_0=23.59$ ($G\gamma=45.11$) has a stable spin axis spread of 0.05 at AGS, and the corresponding spread at RHIC is 0.12.

Conclusions

The stable spin direction of a polarized proton beam at the RHIC injection point has been calculated as a function of $G\gamma$ for two cases:

a) Partial snake off.

b) Partial snake on.

The results of the calculations can be used as a guide to choose the optimum proton injection energy which will yield the minimum polarization loss during the beam transfer from AGS into RHIC.

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Figure Captions

1. Schematic diagram of the AGS to RHIC (AtR) transfer line. The 12.5 mrad vertical bend is located at the region of the 20^0 bend. The 3 mrad vertical bend is located at the end of the RHIC injection section.

2. Schematic diagram of the 12.5 mrad vertical bend. A horizontal bend of 15^0 follows the first 12.5 mrad downward bend. The beam is restored to the horizontal by the second 12.5 mrad upward bend.

3. Schematic diagram of the 3 mrad vertical bend. The combination of the two RHIC quadrupoles reduces the vertical slope of the beam from 3 mrad down to 1,73 mrad. The RHIC injection kicker restores the beam to the horizontal direction. A 38 mrad bend to the right is accoplished by a Lambertson septum magnet. The RHIC dipole bends the beam to the left by 39.86 mrad. The horizontal bend change sign in the injection into the ring where the beam circulates clockwise (Blue ring).

4. The directional cosine of the stable spin direction with the vertical, as a function of $G\gamma$,at the middle of the straight section C10 of AGS.

5. Directional cosine of the stable spin direction with the vertical, as a function of $G\gamma$, at the RHIC injection point. The initial stable spin direction at AGS is along with the vertical (partial snake in AGS turned off). The points shown as solid squares correspond to the cosines of central trajectories. The non-solid squares correspond to the cosines of non-central trajectories.

6. Directional cosine of the stable spin direction with the vertical, as a function of $G\gamma$, at the RHIC injection point. The initial stable spin direction at AGS is defined by the partial snake which, in this case is on. The points shown as non-solid squares correspond to the cosines at the RHIC injection point. The solid squares correspond to the cosines at the middle of the straight section C10 of AGS.

TABLE I

CLOSED ORBIT BEAM PARAMETERS

AT

AGS STRAIGHT SECTION SS-50

P (GeV/c)	29.2305	27.4763	25.6320	20.5570
γ	31.1696	29.3010	27.3366	21.9322
$G\gamma$	55.8808	52.5308	49.0090	39.3201
HF_Q_h	2350	1350	800	400
HF_Q_v	-1380	-600	-150	100
ν_h	8.78355	8.78383	8.77696	8.78084
ν_v	8.72376	8.72356	8.72387	8.72942
ξ_h	-37.5	-27.5	-27.5	-20.0
ξ_v	17.5	9.0	9.0	0.0
β_x(m)	15.1	15.3	15.4	15.5
α_x	1.412	1.366	1.347	1.336
β_y(m)	16.3	16.0	15.8	15.7
α_y	-1.478	-1.402	-1.352	-1.325
η_x(m)	0.377	0.674	0.864	1.01
η'_x(rad)	0.046	0.012	-0.027	-0.047
x(cm)	-0.33	-0.33	-0.32	-0.31
θ(mrad)	0.0035	-0.009	-0.0197	-0.0275
Bump_1a	0.918	0.969	0.990	1.015
Bump_1b	0.918	0.969	0.990	1.015
Bump_1c	1.042	1.005	0.981	0.967
Bump_2a	0.871	0.837	0.830	0.796
Bump_2b	1.015	1.004	1.002	0.985
Bump_2c	1.015	1.004	1.002	0.985
G10(mrad)	-1.53	-1.65	-1.69	-1.77
H10(mrad)	21.32	20.9	20.90	20.9

Fig. 1

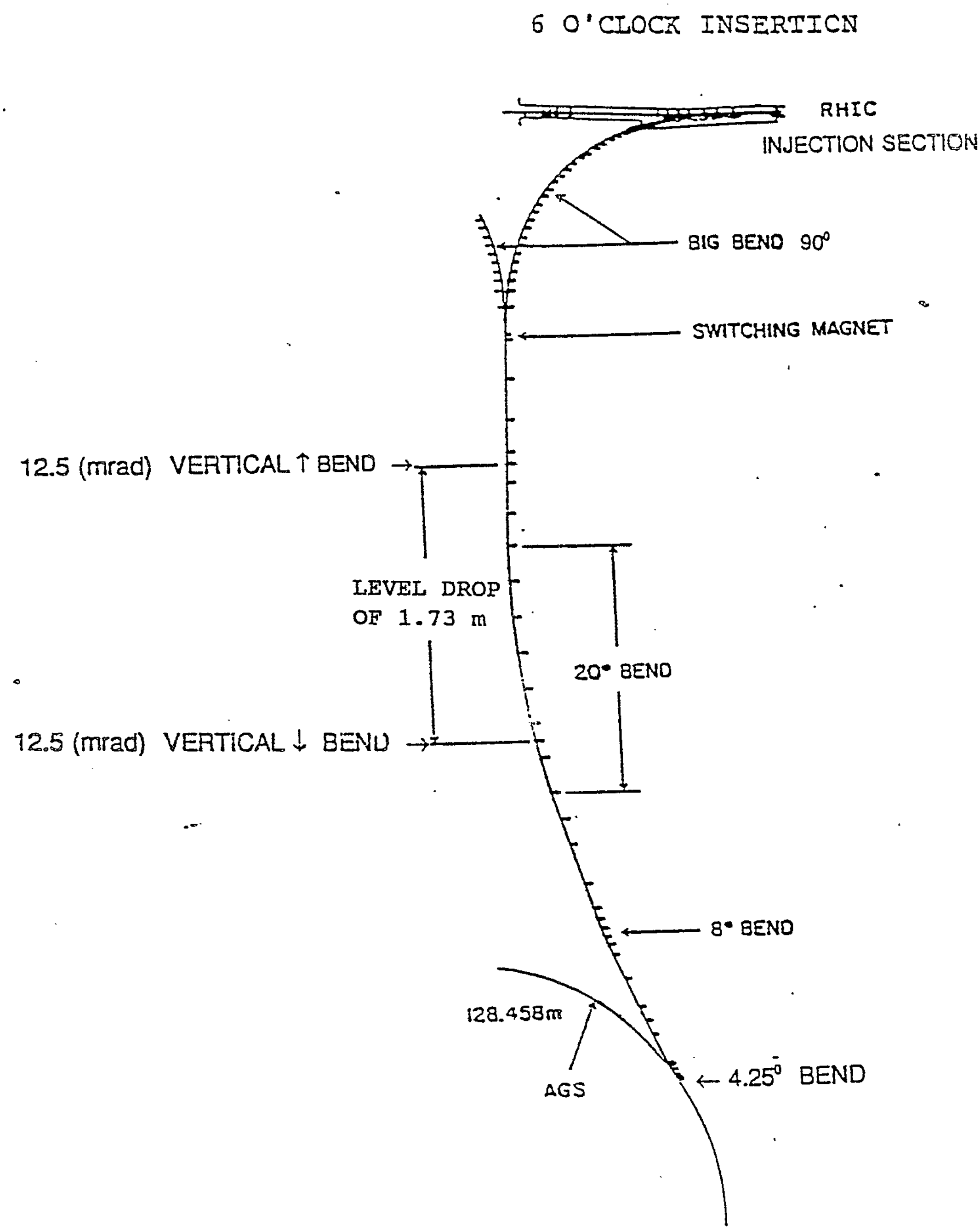

Figure. 1

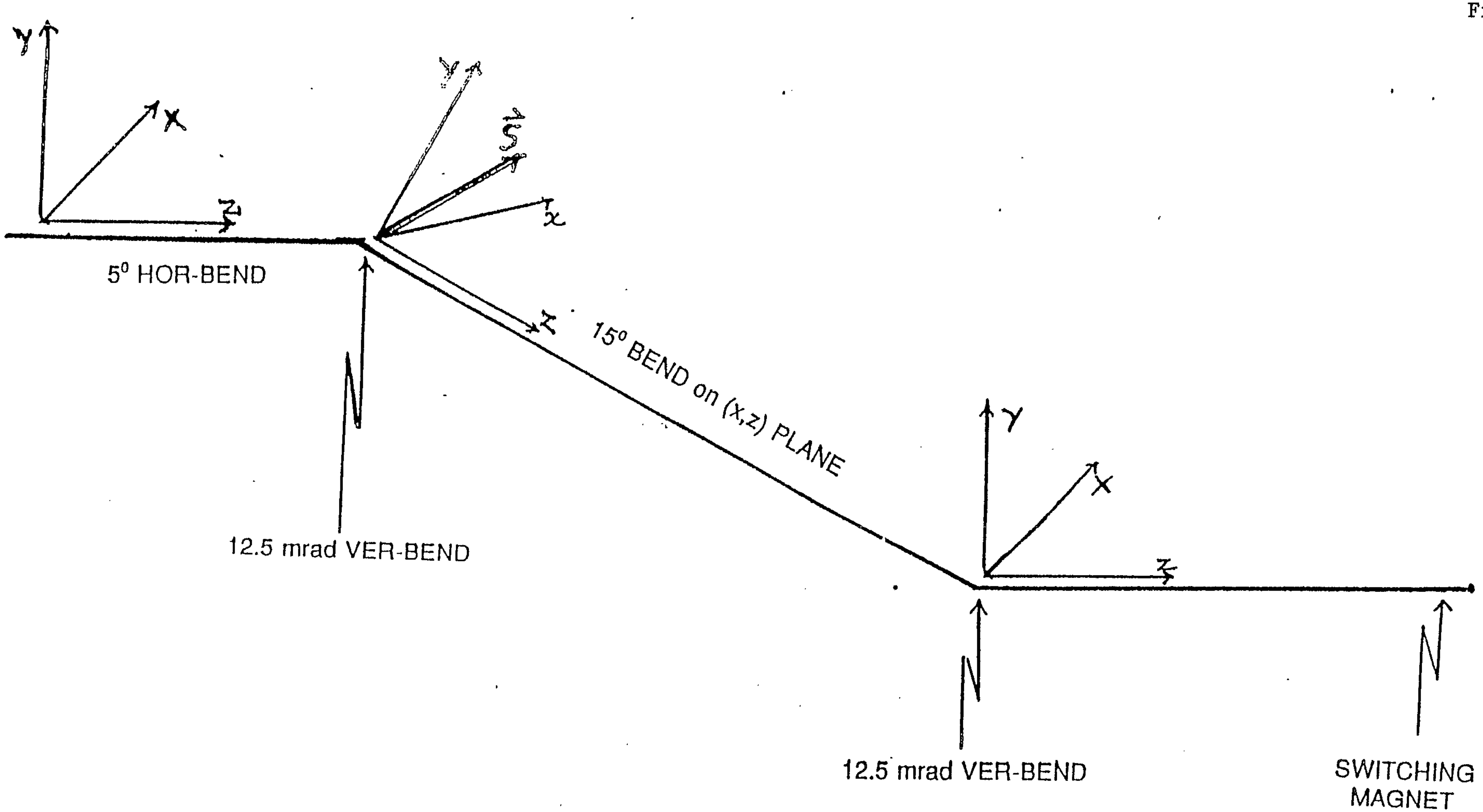
Y
X
Z
ζ
x
5⁰ HOR-BEND
15⁰ BEND on (x,z) PLANE
12.5 mrad VER-BEND
12.5 mrad VER-BEND
SWITCHING
MAGNET

Figure. 2

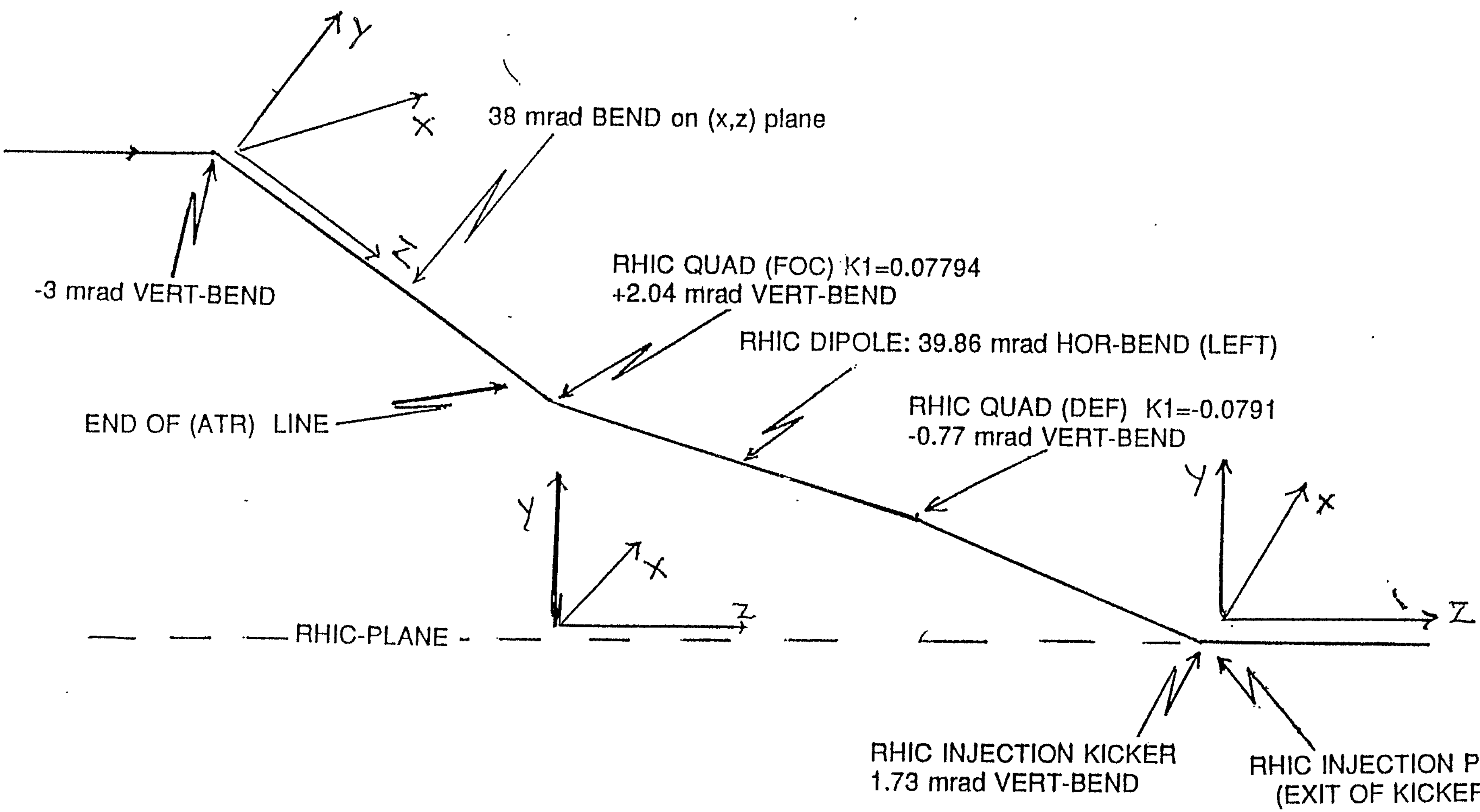
Y
X
Z
38 mrad BEND on (x,z) plane
-3 mrad VERT-BEND
RHIC QUAD (FOC) K1=0.07794
+2.04 mrad VERT-BEND
RHIC DIPOLE: 39.86 mrad HOR-BEND (LEFT)
END OF (ATR) LINE
RHIC QUAD (DEF) K1=-0.0791
-0.77 mrad VERT-BEND
Y
X
Z
Y
X
Z
RHIC-PLANE
RHIC INJECTION KICKER
1.73 mrad VERT-BEND
RHIC INJECTION POINT
(EXIT OF KICKER)

Figure. 3

Fig. 4

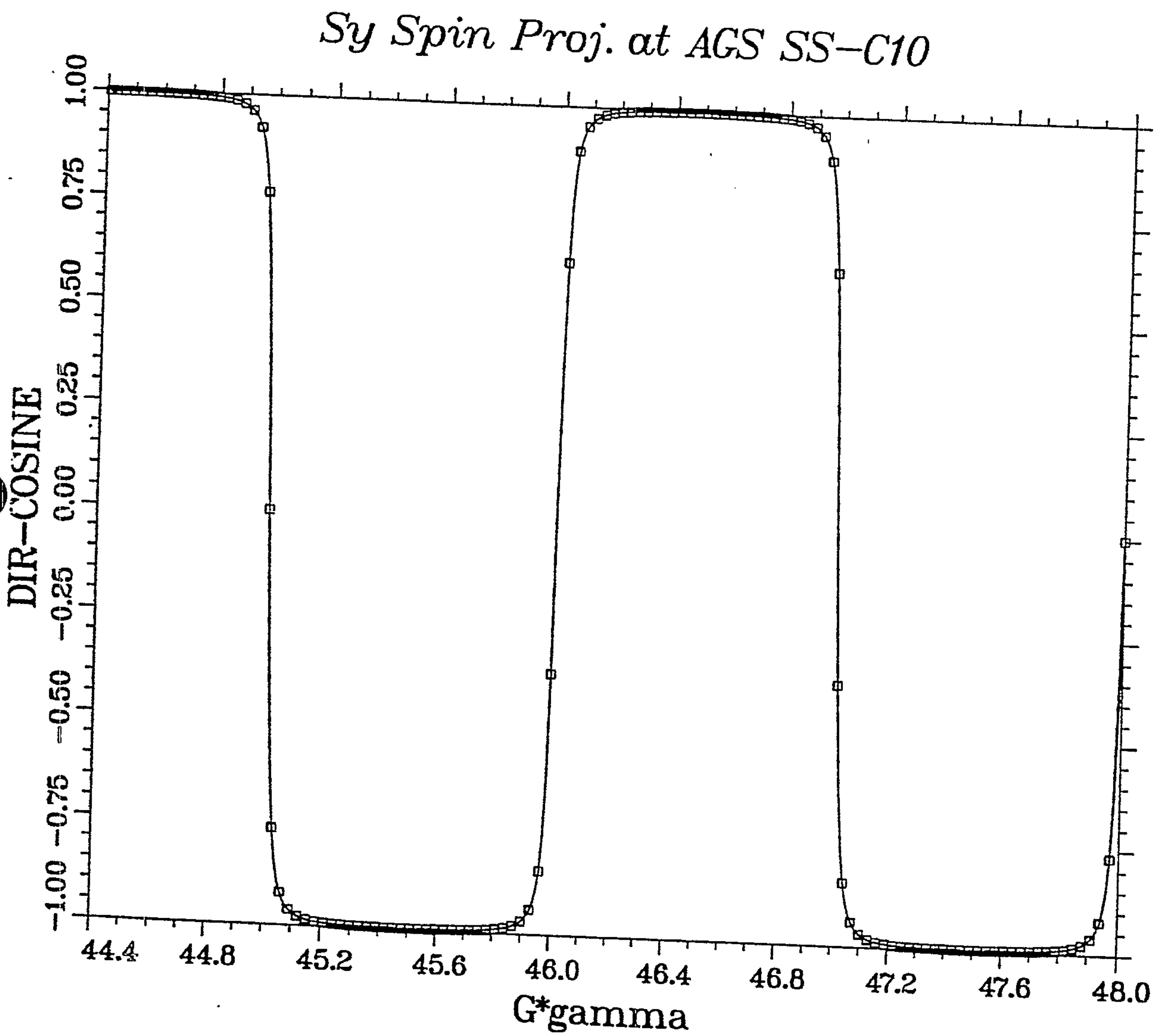

Figure. 4

Fig. 5

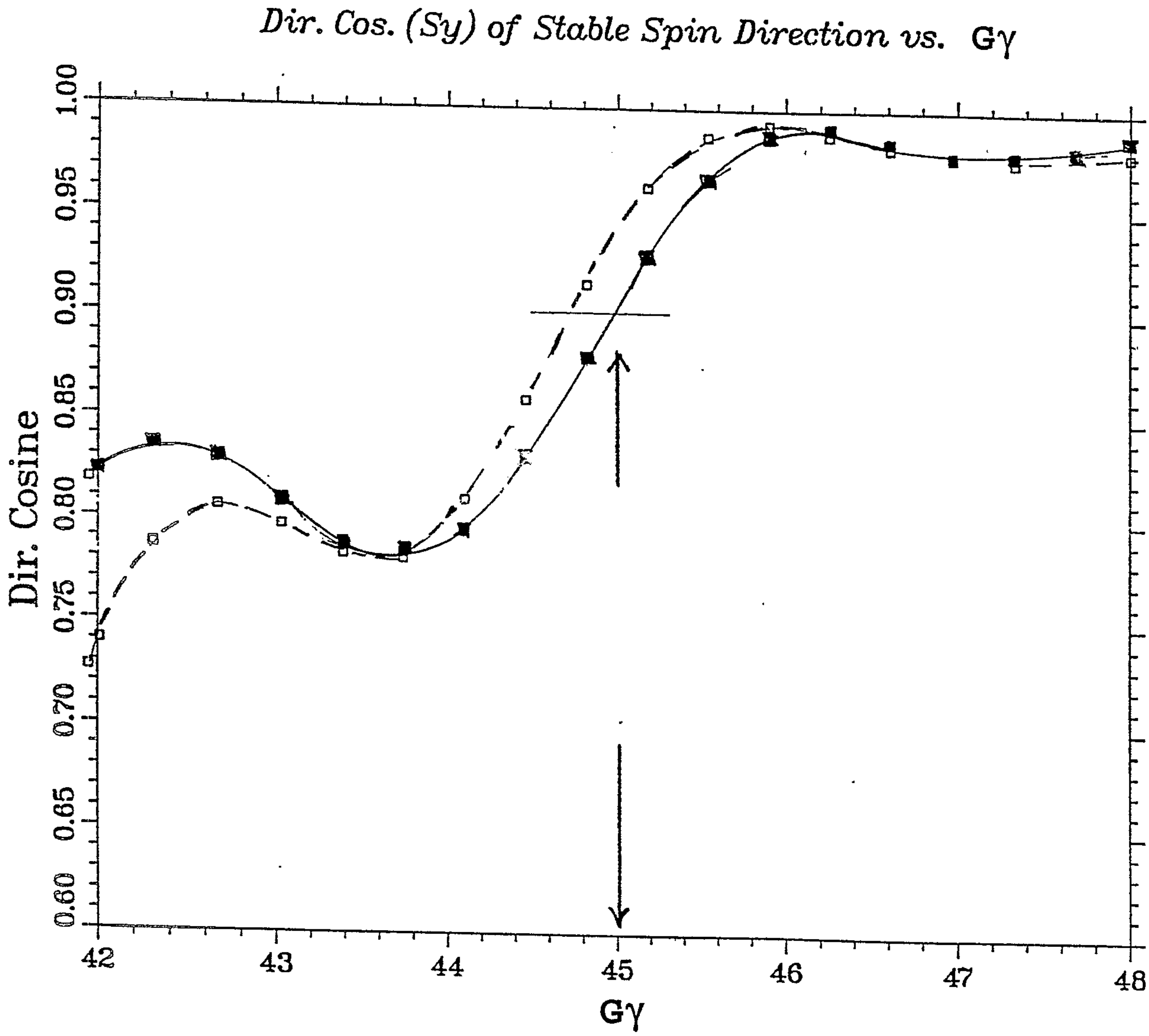

Figure. 5

Fig. 6

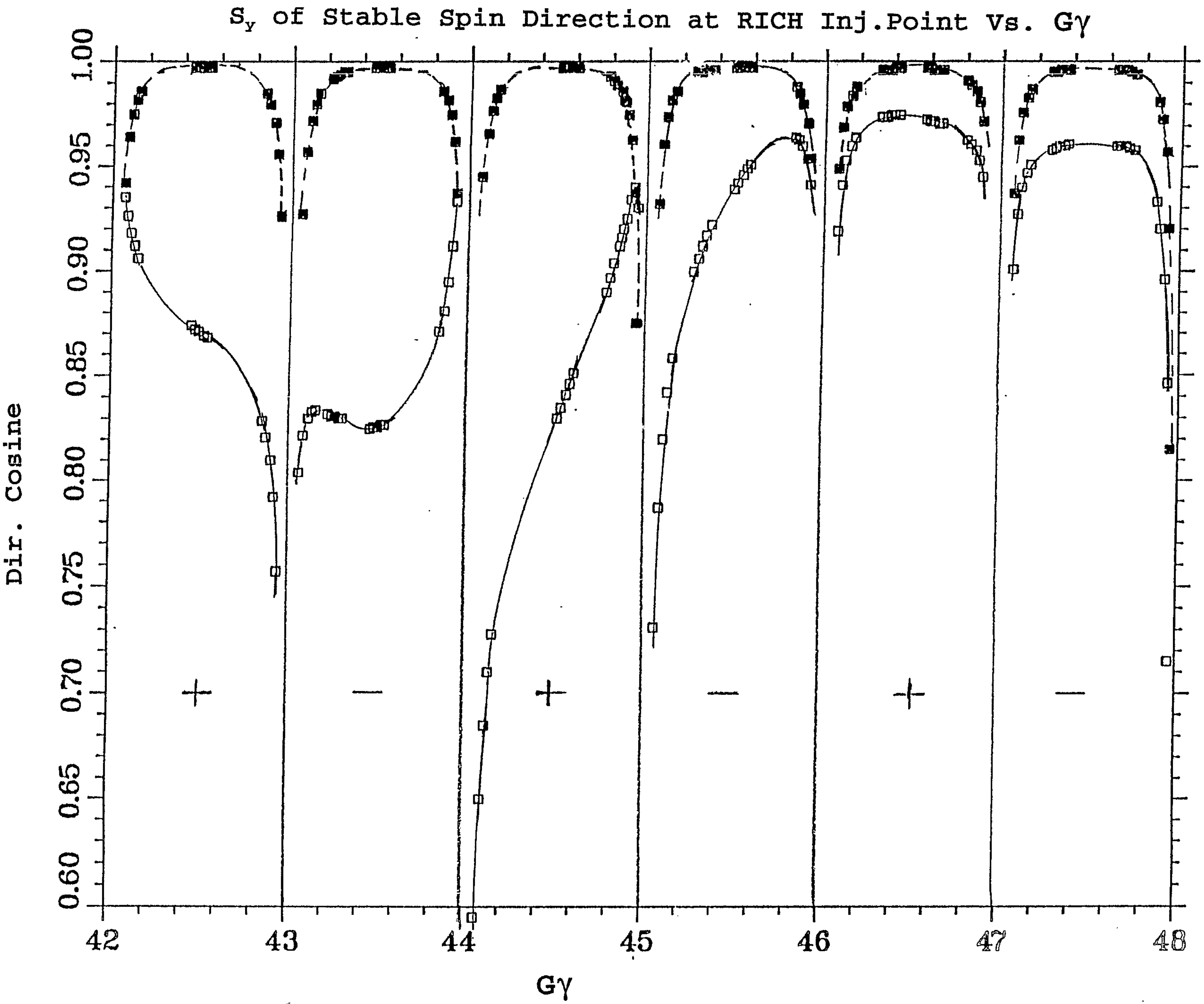

Figure. 6