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Combined Action of Two Helical Spin Rotators in RHIC

A. Luccio

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Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

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Alternating Gradient Synchrotron Department
Relativistic Heavy Ion Collider Project
BROOKHAVEN NATIONAL LABORATORY
Upton, New York 11973

Spin Note

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Spin Rotators in RHIC are installed in pairs, on both sides of an interaction region. The first rotator turns the spin from the vertical position to the horizontal, preferably in the longitudinal direction. The second rotator, after the interaction brings back the direction of the spin to the vertical. Since in RHIC the main axis in the interaction is not collinear with the adjacent ring section where either rotator is located, but forms an angle $\theta = 10.19$ deg with it, in order to obtain longitudinal polarization at the interaction point (IP), the spin should emerge from the rotator with an angle $G\gamma\theta$ with respect to the rotator axis, so that it will perform an additional rotation in the following bend[1]. This is shown in Fig. 1.

The effect of a pair of rotators has been investigated by tracking orbits and spin with the code Snig[1]. The spin rotation from the "up" orientation to the "longitudinal forward" direction at the IP is obtained with rotators with helicity

R L R L - R L R L

(R=Right handed, L=Left handed)

and currents in the coils as shown in Table 1, for the proton injection energy corresponding to γ = 27. The total angle of helicity is 345° in each helix to compensate for the effects of the fringe field, as explained in Ref. [1]. This angle is model dependent. The field starts from the horizontal in each helix. The starting angle of the field with the vertical is shown in the Table. The distance between the rotator and the anti rotator is arbitrary in the present calculation. For the calculation of the spin precession this distance doesn't matter.

The three components of the magnetic field, the behavior of the three components of the spin and the two components of the orbit are shown in the figures 2a, 2b and 2c, respectively. At the exit of the first rotator (and at the entrance of the anti rotator) the angle of the spin in the horizontal plane is

$$G\gamma\theta = ar\tan\left(\frac{S_x}{S_z}\right) = 10.2 \text{ deg}$$

as it should. This is apparent from Fig. 2b.

[1] A.Luccio. Numerical Studies of Siberian Snakes and Spin Rotators for RHIC. BNL Report No. 52461, April 17, 1995.

If the original spin is "down", the rotator, with the same values of the field given in Table 1, rotates it to the longitudinal "backward" position at the IP, as shown in Fig. 3.

The described pair of rotators can also be used as a Siberian Snake with longitudinal precession axis. This is accomplished with different values of the field in the two pair of helices and by reversing the currents in the anti rotator, as shown in Table 2 and Fig. 4. In this case, the field in pair 1-4 is lower than for the rotator case and the field in pair 2-3 is higher. Also the direction of the spin emerging from the rotator is strictly longitudinal and therefore it is not longitudinal at the IP, as shown in Fig. 5.

It is very important that the rotators are installed with the correct helicity sequence. If this is not the case and we have for instance the sequence LRLR, the second integral of the horizontal field, that controls the vertical component of the orbit, does *not* vanish. Let's recall that the angle of the trajectory at the exit of the magnet is equal to the angle at the entrance if the *first* field integral vanishes

$$I_1(L) \equiv \int_L \frac{1}{\rho(z)} dz = 0$$

(where ρ is the curvature radius of the trajectory) but the displacement at the exit is equal to the displacement at the entrance if the *second* integral vanishes (see Appendix.)

$$I_2(L) \equiv \int_L \frac{I_1(z)}{\sqrt{1 - I_1^2(z)}} dz = 0.$$

If the LRLR case conidered here, the effect on the orbits can be very bad, as shown in Fig. 7. The four orbit patterns correspond to the anti rotator set also to LRLR or RLRL, for the two possible relative signs of the excitation currents.

Appendix

In a y-z motion (z the longitudinal coordinate) in a magnetic field, the curvature of the trajectory is given by

$$\frac{1}{\rho} = \frac{eB}{pc} = \frac{y''}{\left(1 + y'^2\right)^{3/2}}$$

where ρ and B are prescribed functions of x,z. After an integration

$$\frac{y'}{\left(1+y'^2\right)^{1/2}}-\frac{y'_0}{\left(1+y'^2_0\right)^{1/2}}=I_1(z)=\int_0^z\frac{1}{\rho(z)}dz$$

This result shows that the final angle of the trajectory is equal to the initial angle if the *first* field integral vanishes.

Express y' as a function of this first integral and integrate again

$$y(z)-y_0=I_2(z)=\int_0^z\frac{I_1(z)+A}{\sqrt{1-\left(I_1(z)+A\right)^2}}dz,\quad A=\frac{y'_0}{\sqrt{1+y'^2_0}}$$

This is the *second* field integral. The final displacement is equal to the initial displacement if the second integral vanishes.

Table 1. Pair of Spin Rotators

#	Center [m]	Length [m]	Start Angle [deg]	Helix [deg]	Field [T]	Field Integrals [T-m]		
1	1.92	2.40	97.5	345 (R)	2.047	3.266	3.266	0.110
2	4.64	2.40	82.5	-345 (L)	2.655	4.238	4.238	0.167
3	7.36	2.40	97.5	345 (R)	2.655	4.238	4.238	0.167
4	10.08	2.40	82.5	-345 (L)	2.047	3.266	3.266	0.110
5	15.92	2.40	97.5	345 (R)	-2.047	3.266	3.266	0.110
6	18.64	2.40	82.5	-345 (L)	-2.655	4.238	4.238	0.167
7	21.36	2.40	97.5	345 (R)	-2.655	4.238	4.238	0.167
8	24.08	2.40	82.5	-345 (L)	-2.047	3.266	3.266	0.110
Total Field Integrals [T-m]					30.011, 30.021, 1.117			
Trajectory Lengthening [mm]					2.504			
Max. Orbit excursion (x,y) [mm]					24.8889, 10.258			

Table 2. Pair of Spin Rotators as Siberian Snake

#	Center [m]	Length [m]	Start Angle [deg]	Helix [deg]	Field [T]	Field Integrals [T-m]		
1	1.92	2.40	97.5	345 (R)	1.773	2.828	2.829	0.083
2	4.64	2.40	82.5	-345 (L)	2.801	4.468	4.470	0.186
3	7.36	2.40	97.5	345 (R)	2.801	4.468	4.470	0.186
4	10.08	2.40	82.5	-345 (L)	1.773	2.828	2.829	0.083
5	15.92	2.40	97.5	345 (R)	1.773	2.828	2.829	0.083
6	18.64	2.40	82.5	-345 (L)	2.801	4.468	4.470	0.186
7	21.36	2.40	97.5	345 (R)	2.801	4.468	4.470	0.186
8	24.08	2.40	82.5	-345 (L)	1.773	2.828	2.829	0.083
Total Field Integrals [T-m]					29.189, 29.201, 1.064			
Trajectory Lengthening [mm]					2.447			
Max. Orbit excursion (x,y) [mm]					20.868, 10.570			

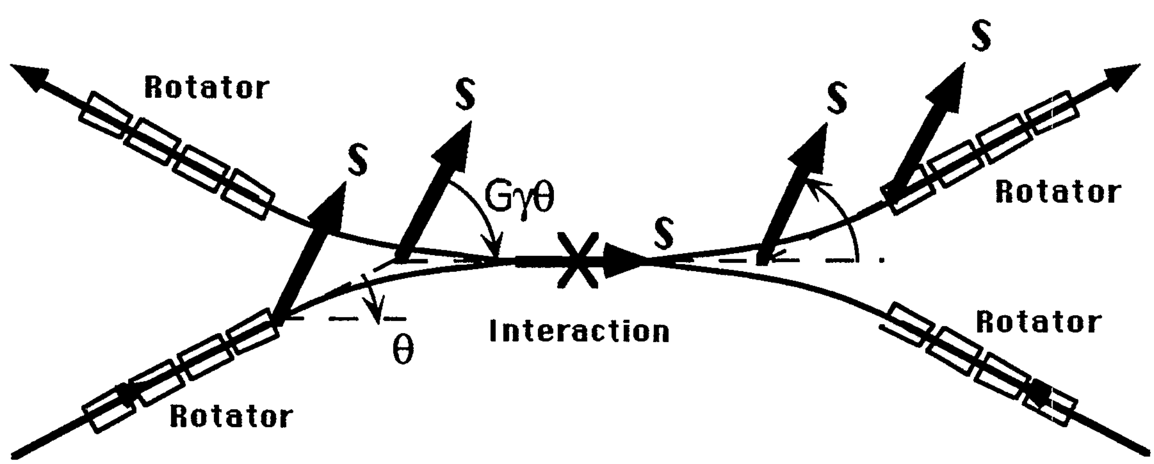

Fig. 1. Rotation of spin to obtain longitudinal polarization at the interaction point. The figure is not to scale.

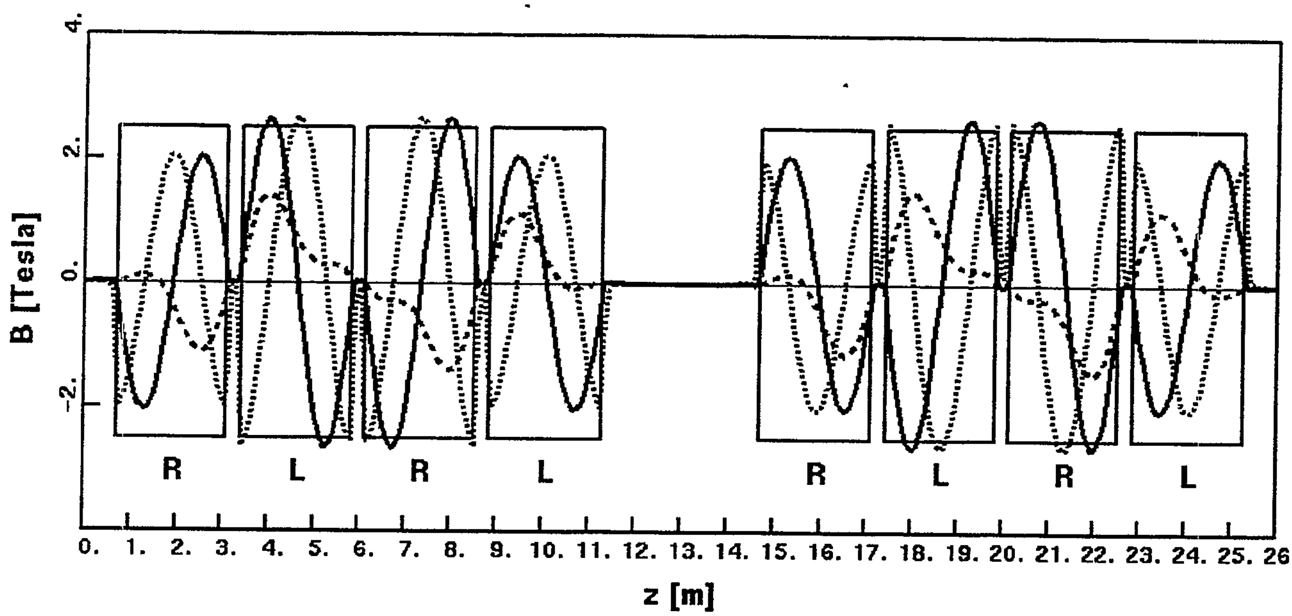

Fig. 2a. Magnetic field components for a pair of spin rotators on both sides of a RHIC interaction region. The longitudinal component (calculated along the orbit) is shown magnified x 10.

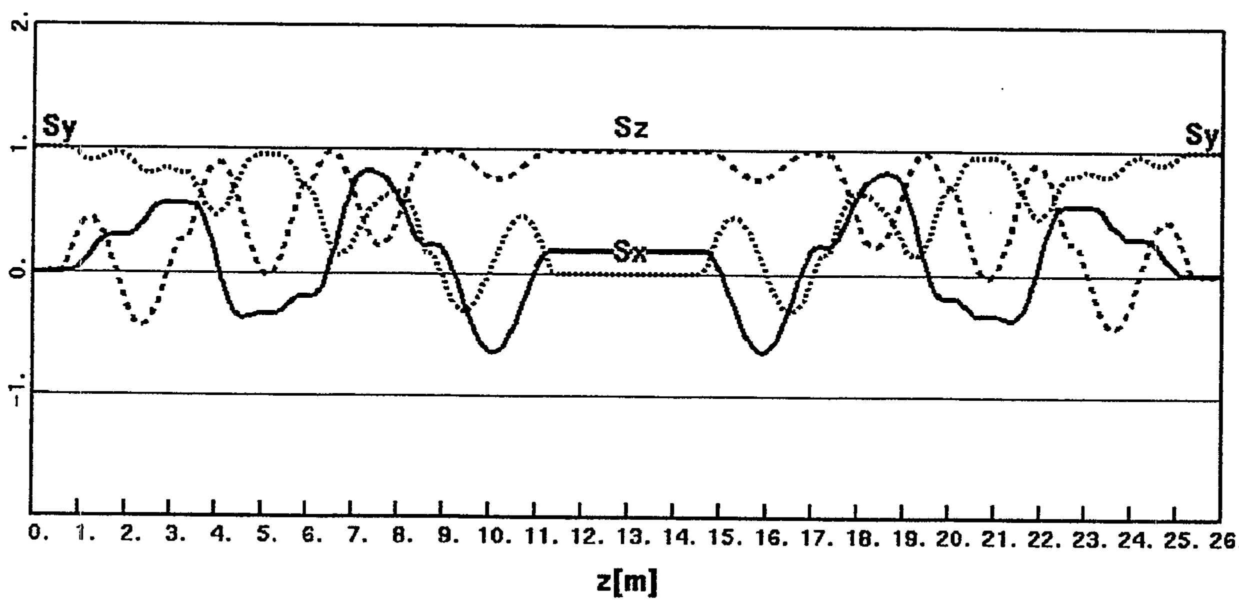

Fig. 2b. Spin precession in the pair of spin rotators of Fig. 2a. The spin is rotated from the "up" direction, to the longitudinal "forward" direction at the IP, and back to "up" after the second magnet.

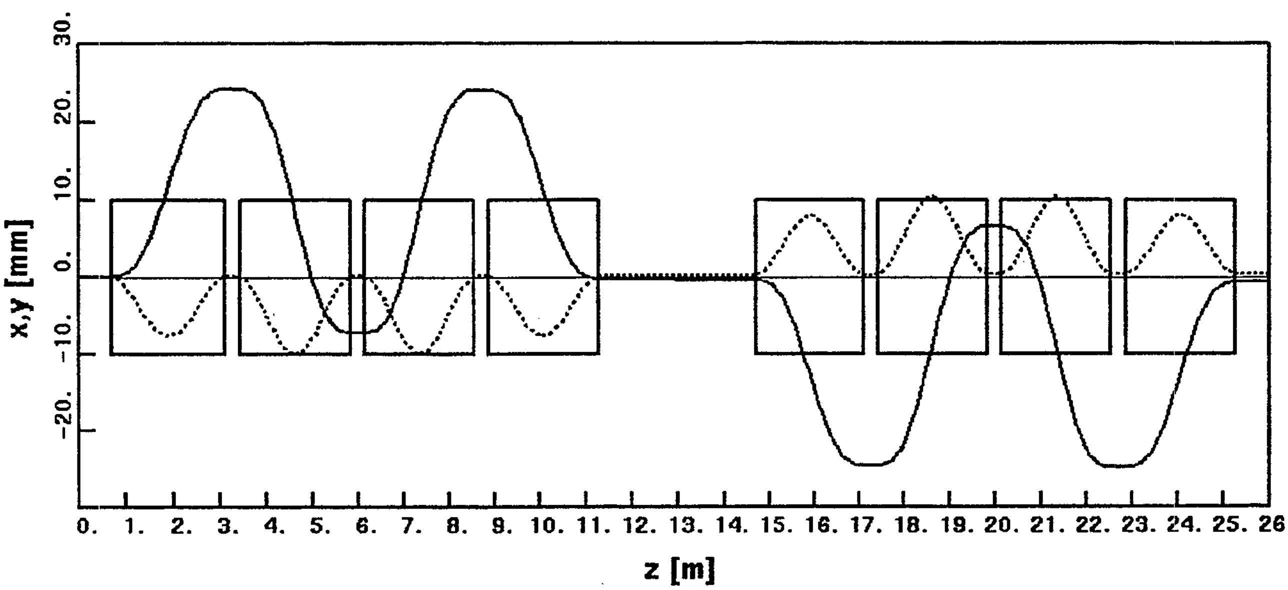

Fig. 2c. Horizontal (solid line) and vertical components of the trajectory for the pair of fig. 2a., for $\gamma = 27$.

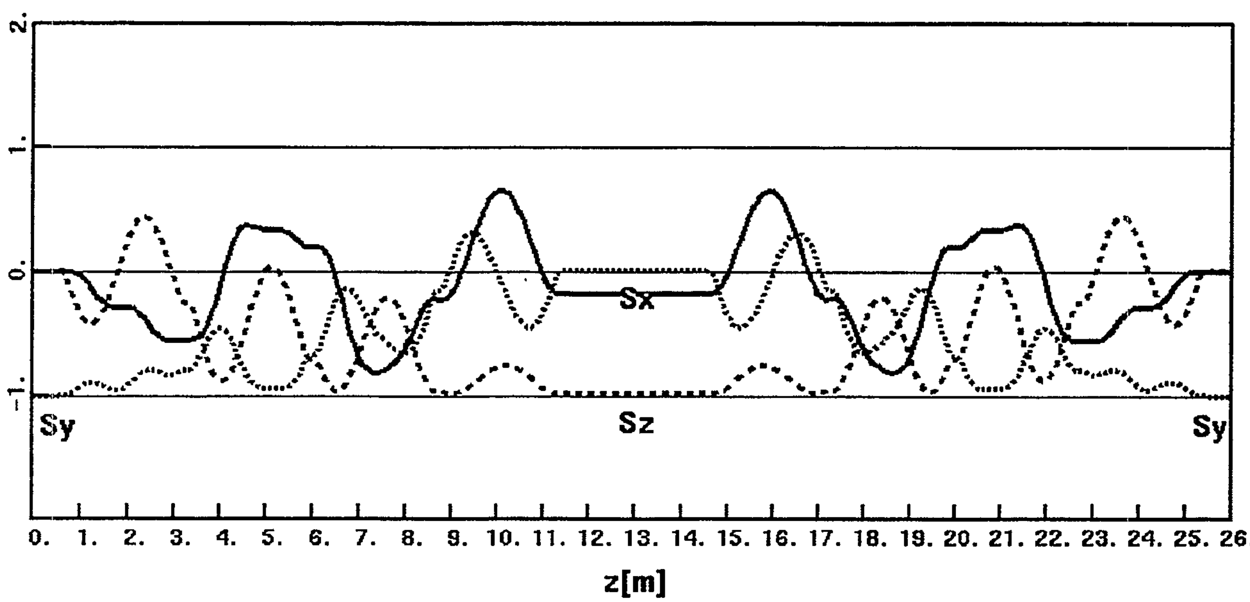

Fig. 3. Spin precession in the pair of spin rotators of Fig. 2 with reverted fields. The spin is rotated from the "down" direction, to the longitudinal "backward" direction at the IP, and back to "down" after the second magnet.

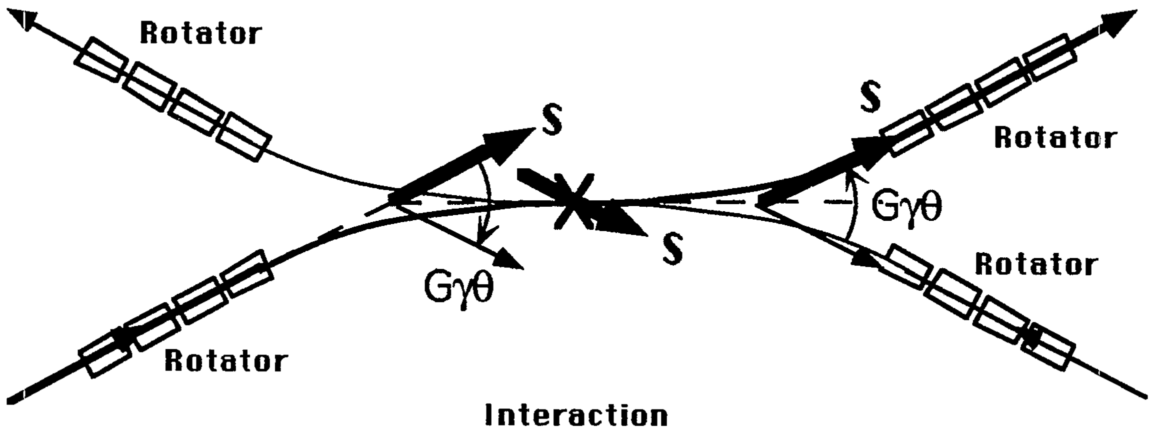

Fig. 5. Pair of Spin Rotators used as a Snake.

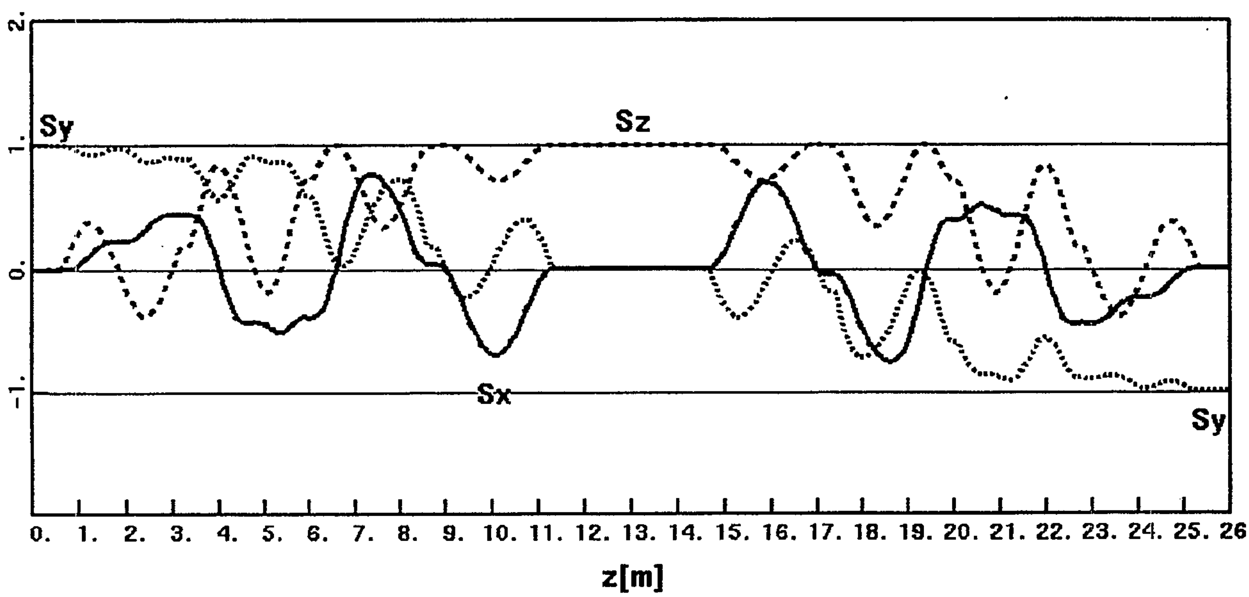

Fig. 6. Spin precession in the pair of spin rotators of Fig. 2 acting as a Snake. The spin is rotated from the "up" direction to "down". The axis of precession is longitudinal.

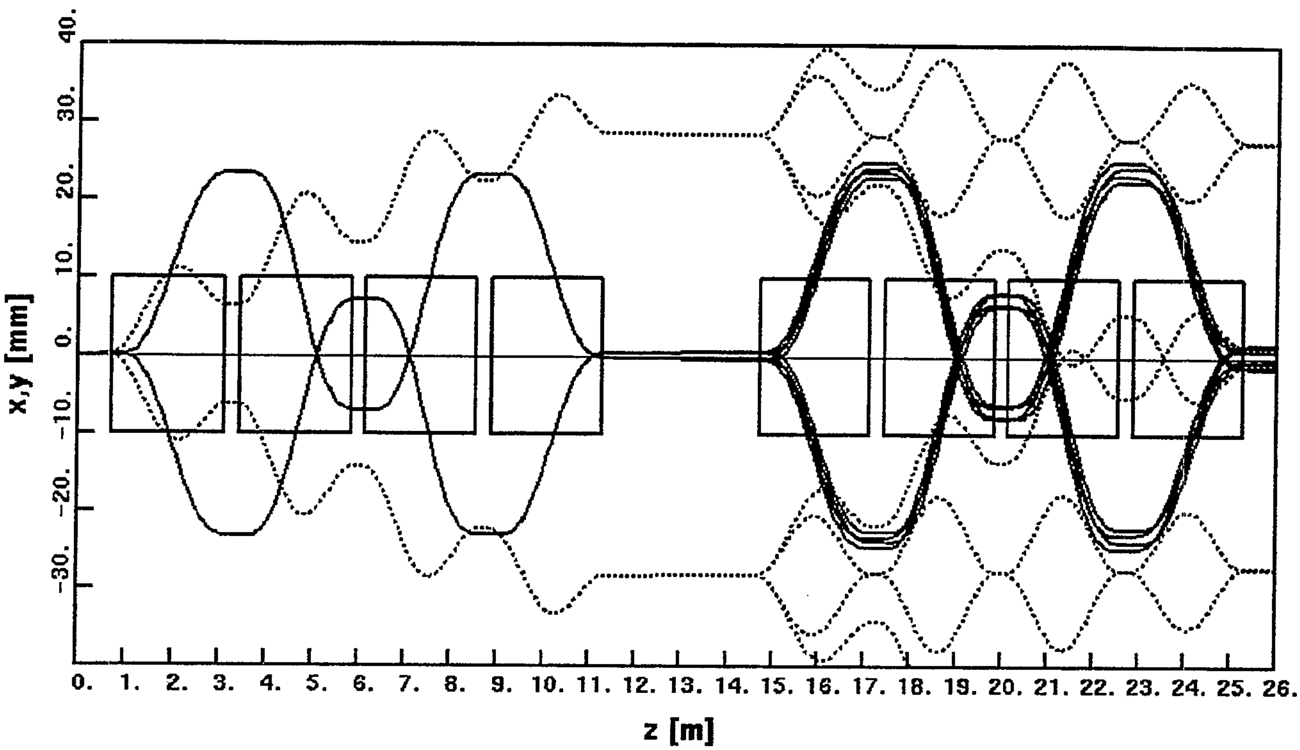

Fig. 7. Trajectories in a pair of spin rotators installed with the wrong sequence of helicities in the first magnet.