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Magnetic Field Error Coefficients for Helical Dipoles (1/16/96)

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1 Introduction

The aim of this paper is to give a notation for the magnetic field error coefficients of helical dipoles. These coefficients shall be the magnetic multipole coefficients of ordinary dipoles when the helical wave length tends to infinity. Such a notation is different from Ref. [1].

For comparison, the magnetic field error notation for ordinary dipoles will be presented first. The notation for helical dipoles is given thereafter.

2 Magnetic Field Errors of Ordinary Dipoles

In a current free region in vacuum where the electrical field $\vec{E}$ is constant, the magnetic field $\vec{B}$ can be derived from a scalar potential ψ as

$$\vec{B} = -\nabla\psi. \tag{1}$$

We will use a Cartesian coordinate system (x, y, z) and a cylindrical coordinate system (r, θ, z). Here, x denotes the horizontal, y the vertical and z the longitudinal direction. Furthermore we have

$$\begin{aligned} x &= r \ \cos\theta, \\ y &= r \ \sin\theta. \end{aligned} \tag{2}$$

We consider a dipole of infinite length, thus neglecting fringe fields. The symmetry condition of such an element reads

$$\psi(r, \theta, z) = \psi(r, \theta, z + \Delta z) \tag{3}$$

where Δz is arbitrary. Therefore, the potential ψ is independent of z:

$$\psi(r, \theta, z) = \psi(r, \theta). \tag{4}$$

Having a main field B_0 in y-direction, the solution of the Laplace equation $\Delta\psi = 0$ can be written in cylindrical coordinates as

$$\psi(r,\theta) = -B_0\Bigg\{r\sin\theta + \\ +\sum_{n=0}^{\infty}\frac{1}{n+1}\frac{r^{n+1}}{r_0^n}\left[a_n\cos\left((n+1)\theta\right) + b_n\sin\left((n+1)\theta\right)\right]\Bigg\}. \tag{5}$$

The term $-B_0 r\sin\theta$ gives the main field and the coefficients a_n and b_n denote deviations from the main field. The b_n are called "normal" and the a_n "skew" multipole coefficients. Here, the subscript "0" denotes a dipole, "1" a quadrupole etc. r_0 is a reference radius. For the RHIC dipoles $r_0 = \frac{5}{8}r_{coil}$ is used with $r_{coil} = 40$ mm.

From equations (1) and (5) the magnetic field can be obtained in cylindrical coordinates. We have

$$\begin{aligned} B_r &= B_0\left\{\sin\theta + \sum_{n=0}^{\infty}\left(\frac{r}{r_0}\right)^n\left[a_n\cos\left((n+1)\theta\right) + b_n\sin\left((n+1)\theta\right)\right]\right\}, \\ B_\theta &= B_0\left\{\cos\theta + \sum_{n=0}^{\infty}\left(\frac{r}{r_0}\right)^n\left[b_n\cos\left((n+1)\theta\right) - a_n\sin\left((n+1)\theta\right)\right]\right\}, \\ B_z &= 0. \end{aligned} \tag{6}$$

The Cartesian components of $\vec{B}$ can be written as

$$\begin{aligned} B_x &= B_0\left\{\sum_{n=0}^{\infty}\left(\frac{r}{r_0}\right)^n\left[a_n\cos(n\theta) + b_n\sin(n\theta)\right]\right\}, \\ B_y &= B_0\left\{1 + \sum_{n=0}^{\infty}\left(\frac{r}{r_0}\right)^n\left[b_n\cos(n\theta) - a_n\sin(n\theta)\right]\right\}, \\ B_z &= 0, \end{aligned} \tag{7}$$

which can also be expressed as

$$B_y + iB_x = B_0\left[1 + \sum_{n=0}^{\infty}(b_n + ia_n)\left(\frac{x+iy}{r_0}\right)^n\right]. \tag{8}$$

Note that the European notation (see for example Ref. [2]) differs from the American one presented here. The transformation is

$$b_n(American) = b_{n+1}(European), \tag{9}$$

$$a_n(American) = -a_{n+1}(European). \tag{10}$$

3 Magnetic Field Errors of Helical Dipoles

We consider again a magnet of infinite length, thus neglecting fringe fields. The symmetry condition for a helical dipole is

$$\psi(r,\theta,z) = \psi(r,\theta - k\Delta z, z + \Delta z), \tag{11}$$

where Δz is arbitrary. In other words, $\theta - kz = const.$ $k = 2\pi/\lambda$ is the wave number and λ the wave length of the helix. k shall have the positive sign for right-handed and the negative sign for left-handed helices. Introducing the new variable

$$\tilde{\theta} = \theta - kz, \tag{12}$$

the symmetry condition (11) leads to a potential ψ which is only dependent on r and $\tilde{\theta}$:

$$\psi(r,\theta,z) = \psi(r,\tilde{\theta}). \tag{13}$$

The tilde shall remind the reader of the fact that $\tilde{\theta}$ in a helix is similar to θ in a ordinary dipole. Using $(r,\tilde{\theta})$ as coordinates and having a transverse helical main Field B_0 a solution of the Laplace equation $\Delta\psi = 0$ is (cf. Eq. (5) and Ref. [1])

$$\begin{aligned}\psi(r,\tilde{\theta}) = -B_0 \Bigg\{ & \frac{4}{k} I_1(kr)\sin\tilde{\theta} + \\ & + \sum_{n=0}^{\infty} \frac{2^{n+1}(n+2)!}{(n+1)^{n+2}} \frac{1}{r_0^n k^{n+1}} I_{n+1}((n+1)kr) \times \\ & \times \left[\tilde{a}_n \cos((n+1)\tilde{\theta}) + \tilde{b}_n \sin((n+1)\tilde{\theta})\right]\Bigg\}\end{aligned} \tag{14}$$

where I_n are modified Bessel functions. Similar to the ordinary dipole case, the term $-B_0 \frac{4}{k} I_1(kr)\sin\tilde{\theta}$ yields the main field and the coefficients $\tilde{b}_n$, $\tilde{a}_n$ the deviations thereof. Here, the $\tilde{b}_n$ are called "normal" and the $\tilde{a}_n$ "skew" helical multipole coefficients (with respect to the direction of the main field B_0). The subscript "0" denotes a helical dipole, the subscript "1" a helical quadrupole etc. r_0 is again a reference radius.

The factors in (14) are chosen in such a way as to obtain the potential (5) when the helical wave length tends to infinity. In this case $k \to 0$, $\tilde{\theta} \to \theta$ and the Bessel function can be approximated by (cf. Ref. [3])

$$I_n(z) \approx \frac{1}{2^n} \frac{z^n}{(n+1)!}. \tag{15}$$

Now, the magnetic field can be computed as (cf. Ref. [1])

$$
\begin{aligned}
B_r &= B_0 \Bigg\{ 4I_1'(kr)\sin\tilde{\theta} + \\
&\quad + \sum_{n=0}^{\infty} \frac{2^{n+1}(n+2)!}{(n+1)^{n+1}} \frac{1}{r_0^n k^n} I_{n+1}'((n+1)kr) \times \\
&\quad \times \left[\tilde{a}_n \cos((n+1)\tilde{\theta}) + \tilde{b}_n \sin((n+1)\tilde{\theta}) \right] \Bigg\}, \\
B_\theta &= -\frac{1}{kr} B_z, \\
B_z &= -B_0 \Bigg\{ 4I_1(kr)\cos\tilde{\theta} + \\
&\quad + \sum_{n=0}^{\infty} \frac{2^{n+1}(n+2)!}{(n+1)^{n+1}} \frac{1}{r_0^n k^n} I_{n+1}((n+1)kr) \times \\
&\quad \times \left[\tilde{b}_n \cos((n+1)\tilde{\theta}) - \tilde{a}_n \sin((n+1)\tilde{\theta}) \right] \Bigg\},
\end{aligned}
\tag{16}
$$

where I_n' denotes the derivative with respect to the argument of the Bessel function.

Since the Bessel function is nonlinear, the magnetic field of a helical dipole is nonlinear too, even the main field given by B_0. Close to the magnet axis we have $r \to 0$ and the field can be approximated by

$$
\begin{aligned}
B_x &= -B_0 \sin(kz), \\
B_y &= B_0 \cos(kz), \\
B_z &= -B_0 k \left[x\cos(kz) + y\sin(kz) \right],
\end{aligned}
\tag{17}
$$

i.e. even close to the magnet axis there is a longitudinal field component that will lead to coupling.

References

[1] V. Ptitsin, "Notes on the helical field", RHIC/AP/41 (1994).

[2] J. Rossbach and P. Schmüser, "Basic course on accelerator optics", Fifth General Accelerator Course, University of Jyväskylä, Finland, CERN 94-01 (1994).

[3] M. Abramowitz and I. Stegun, "Handbook of Mathematical Functions", Dover, New York (1972).