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Computation of the Harmonics in a Helically Wound Multipole Magnet (4/95)

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Computation of the Harmonics in a Helically Wound Multipole Magnet

G.H. Morgan

The field of a beam transport magnet having spiral or helical windings has been examined by several authors [1,2,3]. The present treatment follows Caspi's [2] most closely, but uses the potential form of Ptitsin [3].

In regions where there is no current or iron, the field can be obtained from a scalar potential V which is assumed to be periodic in z, since the winding is also periodic in z with a pitch length L. Thus, $V = F(r)\, e^{in(\theta-kz)}$ in cylindrical coordinates, where $k = 2\pi/L$, and F(r) is to be determined. V satisfies LaPlace's equation, $\nabla^2 V = 0$. Substituting V into this, the differential equation for F is found to be a form of Bessel's equation:

$$r^2 \frac{\partial^2 F}{\partial r^2} + r\frac{\partial F}{\partial r} - (n^2k^2r^2 + n^2)F = 0.$$

This has the solution [4]

$$F_n(inkr) = C_n\, I_n(nkr) + D_n\, K_n(nkr),$$

where I_n is the modified Bessel function of the first kind of order n and K_n is the modified Bessel function of the second kind of order n. Of the two, I_n is finite at r = 0 and increases with radius, and K_n is infinite at r = 0 and decreases with radius. For solutions nearer the axis than the conductors, the potential thus has the form

$$V = C_n\, I_n(nkr)\, e^{in(\theta-kz)},$$

and outside the conductors, the potential has the form

$$V = D_n\, K_n(nkr)\, e^{in(\theta-kz)}.$$

At small values of it's argument, retaining only the leading terms in it's series expansion, $I_n(u) = 2^{-n} u^n/n!$. In order for the expansion of V and it's derivatives to resemble, in the limit as $k\to 0$, the expansion of a non-spiral magnet, it is necessary to include in the form for V the term $(nkr_0)^{-n}$, where r_0 is a reference radius. The interior potential is then

$$V = \Sigma\, C_n\, I_n(nkr)/(nkr_0)^n\, e^{in(\theta-kz)},$$

where the sum is from n = 1 to ∞.

The asymptotic form for this as $k\to 0$ is $V = \Sigma\, C_n/(2^n n!)\, (r/r_0)^n\, e^{in\theta}$. The field is given by $\mathbf{B} = -\nabla V$; the asymptotic V gives $B_r = -\Sigma\, nC_n/(2^n n! r_0)\, (r/r_0)^{n-1}\, e^{in\theta}$ and $B_\theta = -\Sigma\, in\, C_n/(2^n n! r_0)\, (r/r_0)^{n-1}\, e^{in\theta} = iB_r$. Then $B_\theta + iB_r = -\Sigma\, iC_n/(2^{n-1}(n-1)! r_0)\, (r/r_0)^{n-1}\, e^{in\theta}$, where the sum is from n=1 to ∞. The usual harmonic coefficients for an untwisted magnet can be given in Cartesian coordinates as $B_y + iB_x = \Sigma\, (B_n + iA_n)\, e^{in\theta}\, (r/r_0)^n$, where the sum is from n = 0 to ∞. In polar coordinates, this becomes $B_\theta + iB_r = \Sigma\, (B_n + iA_n)\, e^{in(\theta+1)}\, (r/r_0)^n$. Comparing this with the preceding, it is apparent that they are the same if $B_{n-1} + iA_{n-1} = -iC_n/(2^{n-1}(n-1)! r_0)$.

Using the complete interior potential, the field is

$$B_r = -\Sigma\, n\, k\, C_n\, I_n'(nkr)/(nkr_0)^n\, e^{in(\theta-kz)},$$
$$B_\theta = -i\, \Sigma\, n\, C_n\, I_n(nkr)/(r(nkr_0)^n)\, e^{in(\theta-kz)}, \text{ and}$$
$$B_z = i\, k\, \Sigma\, n\, C_n\, I_n(nkr)/(nkr_0)^n\, e^{in(\theta-kz)},$$

where the prime denotes the derivative of I_n wrt it's argument. The exterior field is

$$B_r = -\Sigma\, n\, k\, D_n\, K_n'(nkr)\, e^{in(\theta-kz)},$$
$$B_\theta = -i\, \Sigma\, n\, D_n\, K_n(nkr)/r\, e^{in(\theta-kz)}, \text{ and}$$
$$B_z = i\, k\, \Sigma\, n\, D_n\, K_n(nkr)\, e^{in(\theta-kz)}.$$

The relation between C_n and D_n is found by assuming a current element at radius R given by **J**

R$\Delta\theta\Delta$R, where **J**(R,θ,z) = $J_\theta\, \boldsymbol{\theta_1} + J_z\, \mathbf{z_1}$ is the current density in A/m^2 ($\boldsymbol{\theta_1}$ is a unit vector in the θ direction, etc.). At r = R, B_r is continuous, so equating the inner and outer B_r's gives $C_n\, I_n'(nkR)/(nkr_0)^n = D_n\, K_n'(nkR)$. Then the exterior field becomes

$$B_r = -\Sigma\, n\, k\, C_n\, I_n'(nkR)/[K_n'(nkR)(nkr_0)^n)]\, K_n'(nkr)\, e^{in(\theta-kz)},$$
$$B_\theta = -i\, \Sigma\, n\, C_n\, I_n'(nkR)/[K_n'(nkR)(nkr_0)^n)]\, K_n(nkr)/r\, e^{in(\theta-kz)}, \text{ and}$$
$$B_z = i\, k\, \Sigma\, n\, C_n\, I_n'(nkR)/[K_n'(nkR)(nkr_0)^n)]\, K_n(nkr)\, e^{in(\theta-kz)}.$$

The constants C_n can be determined for the current element by considering a closed path in the z = constant plane enclosing the element at radius R. Applying Ampere's circuital law gives $(B_{\theta,out} - B_{\theta,in})\,|_{\,r=R} = \mu_0\, J_z\, \Delta R$, or

$$J_z = -i/(\mu_0\, kR^2\, \Delta R)\ \Sigma C_n\, e^{in(\theta-kz)}/[(nkr_0)^n\, K_n'(nkR)],$$

where use has been made of the Wronskian $I_n(u)K_n'(u)-K_n(u)I_n'(u) = -1/u$. Similarly, considering a closed path at constant radius enclosing the current element, $(B_{z,in} - B_{z,out})\,|_{\,r=R} = \mu_0\, J_\theta\, \Delta R$, so

$$J_\theta = -i/(\mu_0\, R\, \Delta R)\ \Sigma C_n\, e^{in(\theta-kz)}/[(nkr_0)^n\, K_n'(nkR)].$$

It may be noted that $J_\theta/J_z = kR = 2\pi R/L$, which says that **J** is in the direction of the helix; the helix angle is $\tan^{-1}(2\pi R/L)$.

The C_n may be obtained in terms of either current density component; if the equation for J_z is multiplied through by $e^{-im\theta}$ and integrated wrt θ over 2π, $\int e^{i(n-m)\theta} d\theta$ is zero if $n \neq m$, and 2π if n=m. This gives

$$C_n = i(\mu_0/2\pi) kR^2 (nkr_0)^n\, K_n'(nkR)\Delta R \int J_z\, e^{-in(\theta-kz)}\, d\theta.$$

If the winding is thick, this may be integrated wrt R over the radial extent of the winding.

Caspi's treatment [2] allows for a sum over submultiples of the pitch length, i.e., $k_m = 2\pi m/L$, m = 1 to ∞; this complete solution permits one to give a correct description of the field in the ends of a helical magnet. For the present purpose, it is sufficient to integrate the C_n expression above wrt z throughout the end, giving a form of "unit-meters"; from this the effective length of a harmonic is obtained by dividing by the corresponding C_n for the helical section of the magnet.

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