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Examples of Spin Rotators and Siberian Snakes for RHIC (6/93)

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Spin Note

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(Prepared for the June 21-22, 1993 Review)

Alfredo Luccio

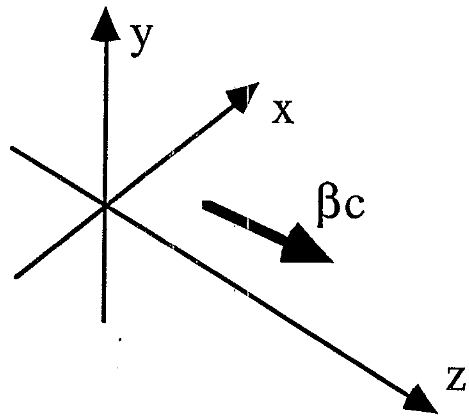

Fig.1. Reference coordinates.

A transverse field spin rotator or snake consists of a sequence of magnetic dipoles with their field perpendicular to the main beam axis z In a typical configuration, the field direction is alternatively vertical (y) and horizontal (x) in successive magnets.

Computation of transverse field Spin Rotators and Steffen Snakes for RHIC. Code SNIG

Code SNIG has been written to:

1- Integrates the equations of motion of the particle beam in the three dimensional magnetic field;

2- Integrates the BMT equation for the spin precession;

3- Calculates the equivalent transfer matrix of the device by an analysis of the behavior of an ensemble of particle in phase space.

Equation of motion

$$\frac{d\beta}{dz} = -\Omega \times \beta$$

BMT equation for the spin s

$$\frac{ds}{dz} = \Omega^{(s)} \times s$$

with the definitions

$$\begin{cases} \Omega_x^{(s)} = \dfrac{1}{\beta_z} C_1 \Omega_x + C_2 x' \, \Omega \cdot \beta \\ \Omega_y^{(s)} = \dfrac{1}{\beta_z} C_1 \Omega_y + C_2 y' \, \Omega \cdot \beta \\ \Omega_z^{(s)} = \dfrac{1}{\beta_z} C_1 \Omega_z + C_2 \, \Omega \cdot \beta \end{cases}$$

$$\Omega = \frac{e}{m\gamma} \mathbf{B}$$

$$C_1 = 1 + G\gamma \quad , \quad C_2 = -\frac{G\gamma^2}{1+\gamma}$$

x' and y' horizontal and vertical beam angles

For protons

$$G = 1.7928 \quad ; \quad \frac{e}{m} = 9.58\,10^7 \sec^{-1} \mathrm{T}^{-1} \quad ; \quad \gamma = \frac{E[GeV]}{0.938}$$

Magnetic Compensation

A spin rotator of length L is magnetically compensated when its first and second field integrals vanish

$$\int_L B\,dz = 0, \quad \int_L dz \int_L B\,dz = 0$$

The first condition implies that the angle of the trajectory with respect to the z axis at the exit of the magnet is equal to the angle at the entrance. The second, that the displacement of the trajectory is the same at the entrance and at the exit.

Transfer Matrix

Magnetically compensated rotator, with no coupling between x and y, (matrix of a drift)

$$\begin{pmatrix} 1 & L_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_y \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Complete, approximate transfer matrix

$$\begin{pmatrix} 1+a_x & L(1+b_x) & e_x & f_x \\ c_x & 1+d_x & g_x & h_x \\ e_y & f_y & 1+a_y & L(1+b_y) \\ g_y & h_y & c_y & 1+d_y \end{pmatrix}$$

$a, b, c, d, e, f, g, h,$ quantities small compared to 1.

$a, b, c, d,$ contain the information on focusing.

$e, f, g, h,$ contain the information on linear x-y coupling.

SNIG Calculates the parameters of the matrix by tracking an ensemble of particle and performing averages.

Quadrupole Doublet

Focusing and coupling properties of a dipole sequence by a skew quadrupole doublet

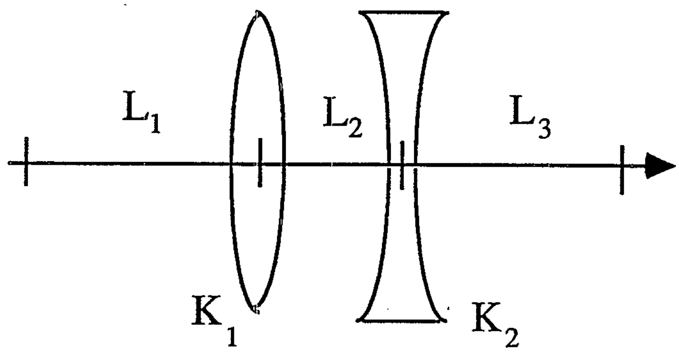

Fig.2. Equivalent Quadrupole Doublet.

Parameters of the doublet written in terms of the elements of the matrix

$$L_1 = \frac{a_x + a_y}{c_x + c_y} \quad , \quad L_3 = \frac{d_x + d_y}{c_x + c_y} \quad , \quad L_2 = L - (L_1 + L_3)$$

$$K_1 = -\frac{1}{L_2}\left[a_x - c_x(L_1 + \tfrac{1}{2}L_2)\right] \quad , \quad K_2 = \frac{1}{L_2}\left[a_x - c_x L_1\right]$$

$$\alpha_1 = \frac{e_x + e_y - (g_x + g_y)(L_1 + L_2)}{2K_1\left[K_2 L_1 L_2 + K_1(L_1 + L_2)\right]} \quad , \quad \alpha_2 = -\frac{e_x + e_y - (g_x + g_y)K_2 L_1 L_2}{2K_2\left[K_2 L_1 L_2 + K_1(L_1 + L_2)\right]}$$

K_1 and K_2 , inverse focal lengths of the (thin) quadrupoles

a_1 and a_2 , skew angles

Calculation Strategy

<u>SNIG reads an input file containing:</u>

(i) an initial geometrical description of the device: the longitudinal position of the center, the length and the angle of each dipole, and (ii) an initial value of the dipole field. The field in each dipole is multiplied by a corresponding tuning parameter, initially set to 1.

<u>Magnetic compensation</u>

SNIG starts with one particle on axis. Then, the equations of motion are integrated, and the final beam coordinates are shown. The calculation is repeated by fine tuning the x and y dipole lengths until the particle lands on axis again.

<u>Spin rotation</u>

SNIG reads the initial components of the spin vector for the one particle on axis, e.g. s = (0,1,0) and show the final values.

Desired values for the spin are obtained by slowly varying the field tune coefficients, uniformly applied to the field of groups of dipoles, not to interfere in the previously found magnetic compensation.

Spin Rotator

Inserted in pairs on both sides of an interaction region in RHIC. Two magnetic configuration according if it is used to rotate the spin from vertical to radial (Type 1, easier), or from vertical to longitudinal (Type 2, more difficult).

We have two solutions: (i) a system with two dipole triplets for a total of six dipoles, using two independent main power supplies, and (ii) a system with twelve dipoles and three independent main power supplies. Rotator (ii) is more complex, but produces a much smaller orbit distortion.

Rotator (i), (table 1 and figures 3-6), requires a magnetic gap of 20 cm, same as in other dipoles of RHIC and necessary to accommodate large variation of the trajectory for proton relative energies between 25 and 250. This gap is very large compared to the length of the magnets. The fringe field is expected to extend considerably beyond the dipoles and will play a major role in the optical properties and in the spin precession. The particles will experience a large integral of the longitudinal component of the magnetic field that will affect both focusing and precession. An extended fringe field of a dipole may affect the field of the neighboring dipole. Lacking a detailed design of the magnets, we assumed that the fields add up linearly. In this system, spin rotation is achieved using two independent field tune parameters, for each triplet respectively. The transfer matrix for this rotator, calculated by tracking 128 random particles is shown in table 2.

Due to the smaller orbit distortion, rotator (ii), (table 3 and figures 7-10), has been designed with a gap of only 10 cm. In this system, spin rotation is achieved with three independent field tune parameters.

Table 1. Spin Rotator (i)

No of dipoles				6	
No independent power supplies				2 (A,B)	
Total length [m]				11	
Magnetic gap [mm]				200	
Dipoles				Field [T]	
	center [m]	length [m]	orient.	Type 1 ($s_y \to s_x$)	Type 2 ($s_y \to s_z$)
1	1.000	0.260	V	1.843	1.843
2	3.000	0.660	-H	1.176	3.605
3	4.400	0.900	-2V	2.000	2.000
4	6.200	1.700	2H	1.240	3.800
5	7.800	0.260	V	1.843	1.843
6	9.400	0.660	-H	1.176	3.605
Max orbit excursion @ γ=200 [mm]				6 (x) 7 (y)	18 (x) 7 (y)
Total field integrals [T-m]				5.599 (x) 5.456 (y) 0.030 (z)	5.599 (x) 16.724 (y) 0.060 (z)
Orbit lengthening [mm]				0.024	0.112

Table 2. Optical Matrix for the Spin Rotator (i), Type 2,
calculated by tracking 128 random particles

$$\left\| \begin{matrix} 1-.005866 & 10.987676 & -0.000460 & 0.001052 \\ -0.000789 & 1-.004435 & 0.000043 & -0.001042 \\ -0.002271 & 0.001254 & 1-.015658 & 10.926036 \\ -0.000180 & -0.002096 & -0.003108 & 1-.019227 \end{matrix} \right\|$$

This Spin Rotator is optically represented by a doublet of skew quadrupoles

$$L_1 = 5.523 \text{ m}$$
$$L_2 = 0.405 \text{ m}$$
$$L_3 = 6.072 \text{ m}$$
$$K_1 = 0.00489 \text{ m}^{-1}$$
$$K_2 = -0.00373 \text{ m}^{-1}$$
$$a_1 = 20.80^\circ$$
$$a_2 = 20.75^\circ$$

Table 3. Spin Rotator (ii)

No of dipoles				12	
No independent power supplies				3 (A,B,C)	
Total length [m]				9	
Magnetic gap [mm]				101	
Dipoles				Field [T]	
	center [m]	length [m]	orient.*	Type 1 ($s_y \to s_x$)	Type 2 ($s_y \to s_z$)
1	0.715	0.358	-H (AB)	2.922-0.225	2.923+0.941
2	1.430	0.250	-V (C)	1.868	2.620
3	2.145	0.447	H (A)	4.074	4.074
4	2.860	0.447	V (C)	2.992	4.196
5	3.575	0.447	±H (B)	0.944	3.944
6	4.290	0.447	-V (C)	2.992	4.196
7	5.005	0.447	±H (B)	0.944	3.944
8	5.720	0.447	V (C)	2.992	4.196
9	6.435	0.447	-H (A)	4.074	4.074
10	7.150	0.447	-V (C)	2.992	4.196
11	7.865	0.358	H (BA)	2.922-0.225	0.941+2.923
12	8.580	0.250	V (C)	1.868	2.620
Max orbit excursion @ γ=200 [mm]				2.5 (x) 0.9 (y)	3.9 (x) 1.9 (y)
Total field integrals [T-m]				7.855 (x) 7.959 (y) 0.049 (z)	11.017 (x) 12.309 (y) 0.124 (z)
Orbit lengthening [mm]				0.012	0.031

* In brackets, the power supply.

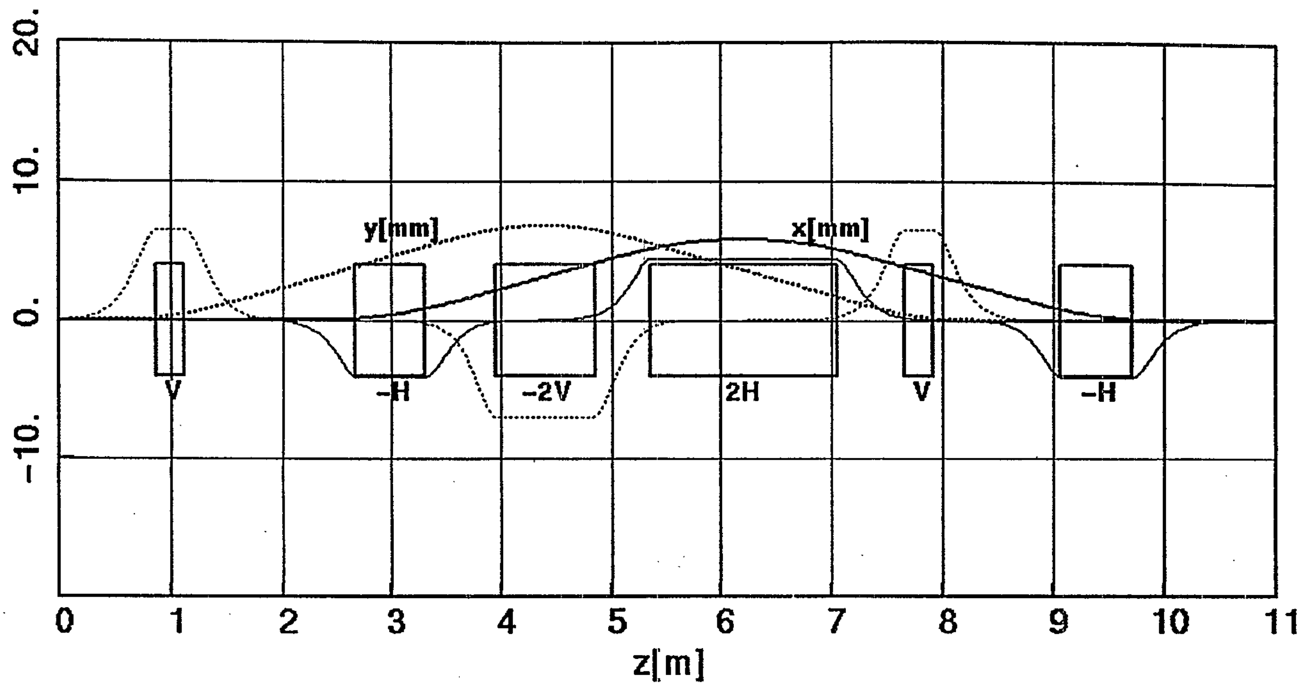

Fig.3. Spin Rotator (i). Type 1. Field and trajectory.

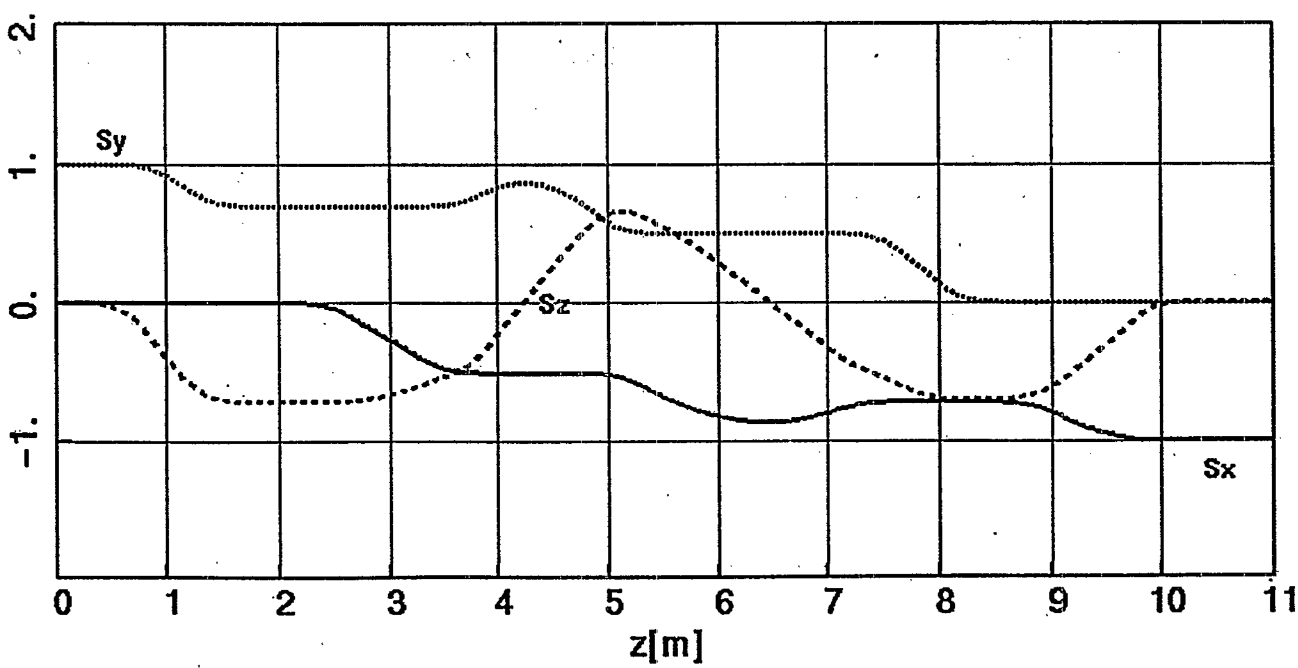

Fig.4. Spin Rotator(i). Type 1. Precession of the three spin components.

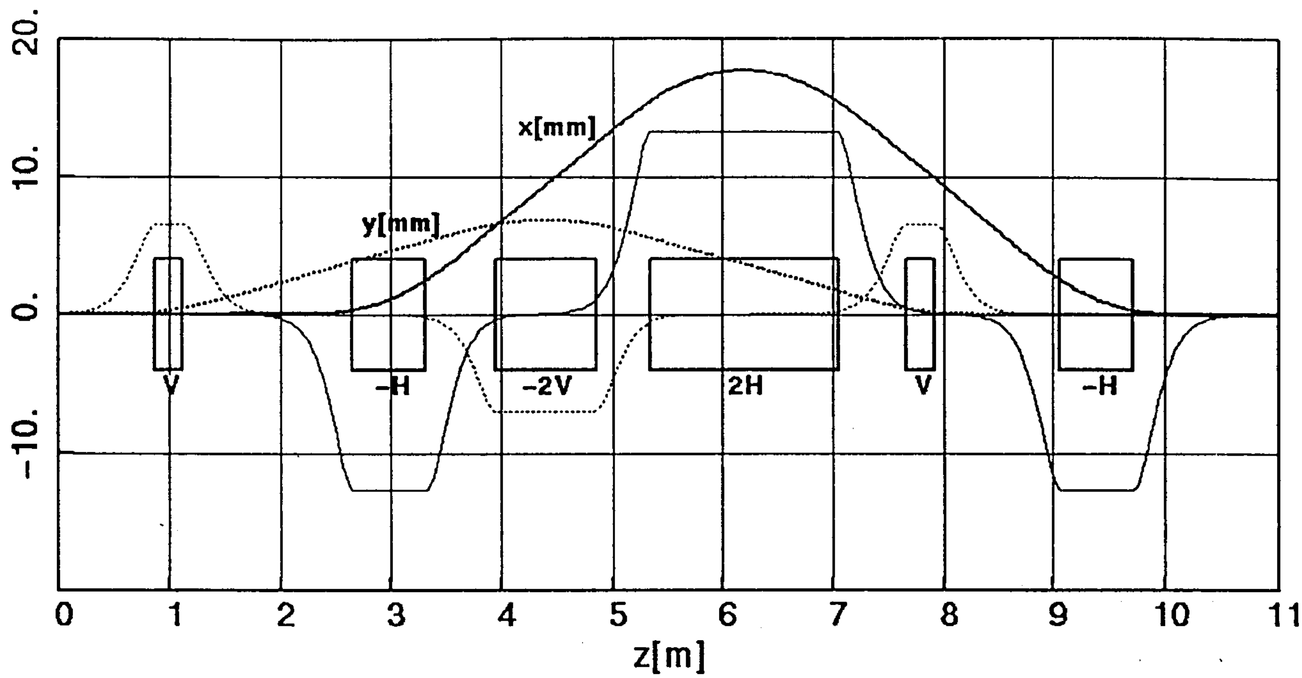

Fig.5. Spin Rotator (i). Type 2. Field and trajectory.

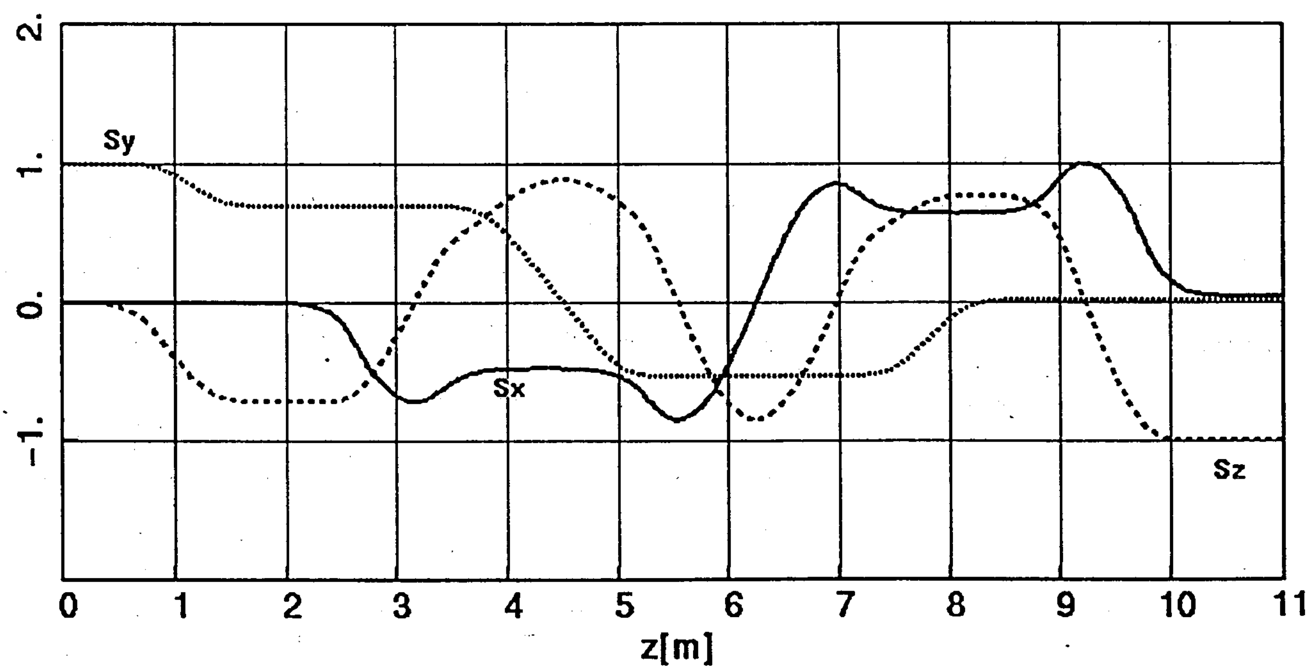

Fig.6. Spin Rotator (i). Type 2 Precession of the three spin components.

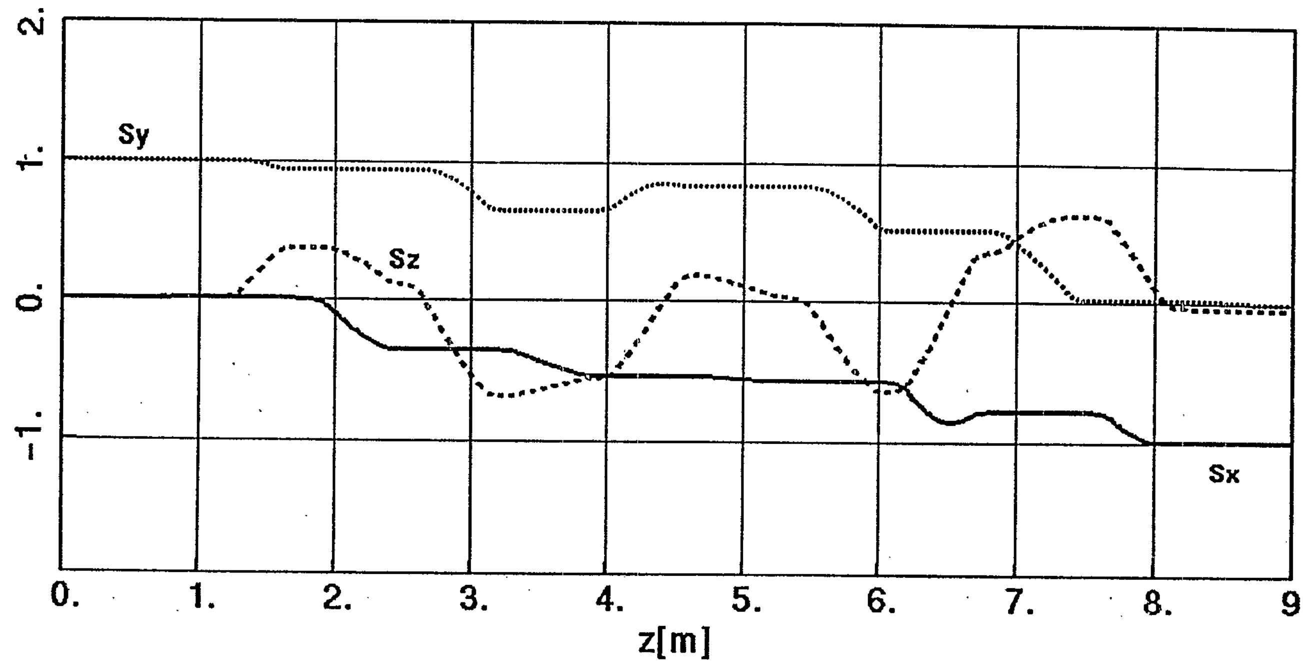

Fig.7. Spin Rotator (ii). Type 1. Field and trajectory.

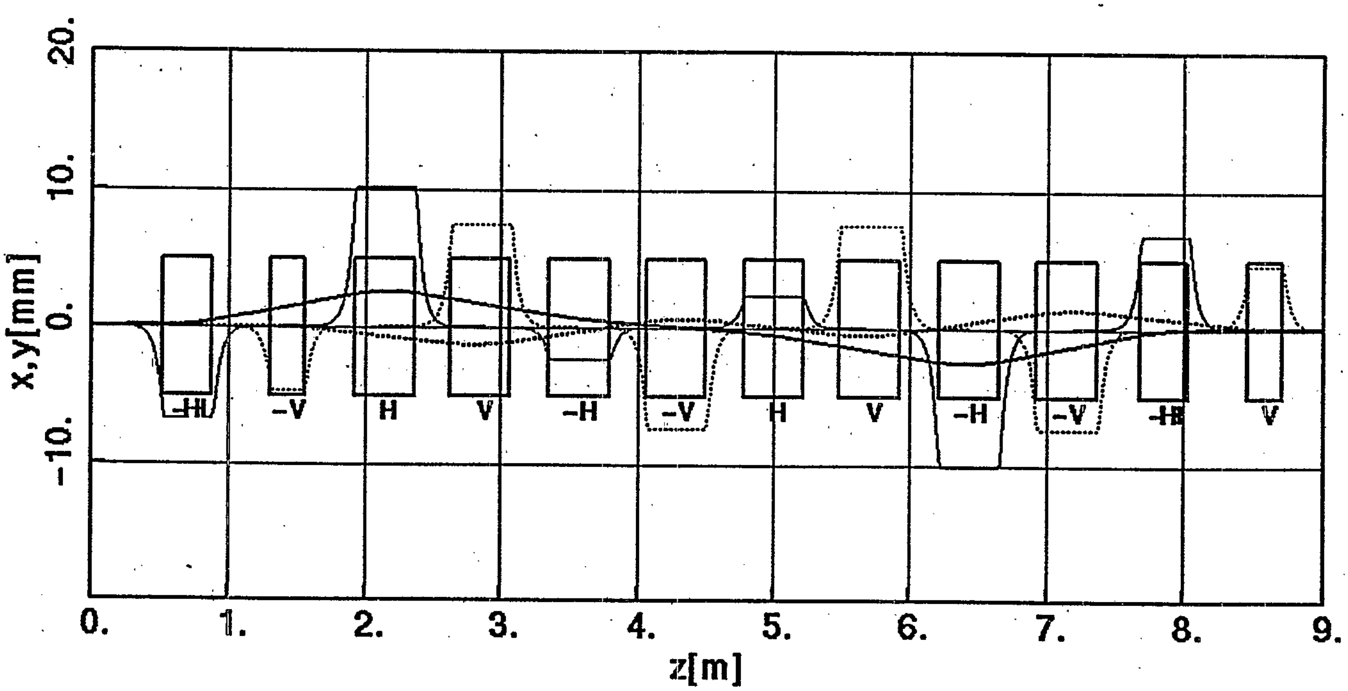

Fig.8. Spin Rotator(ii). Type 1. Precession of the three spin components.

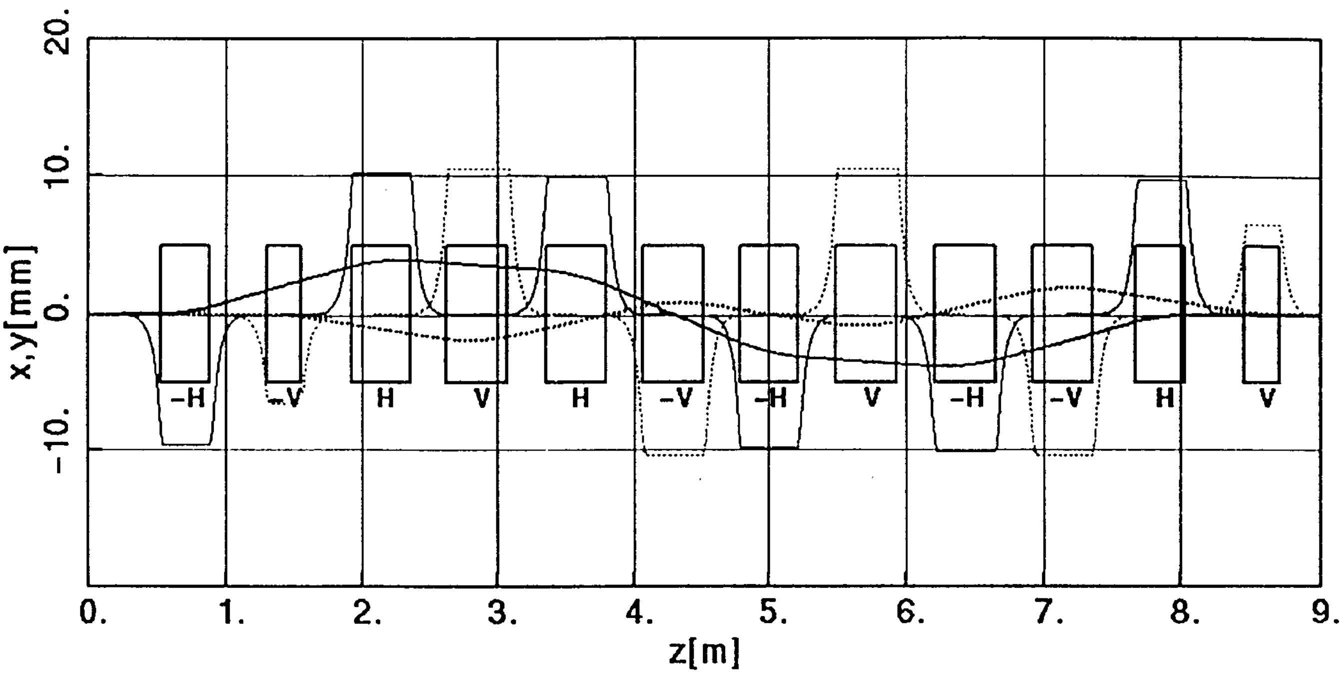

Fig.9. Spin Rotator (ii). Type 2. Field and trajectory.

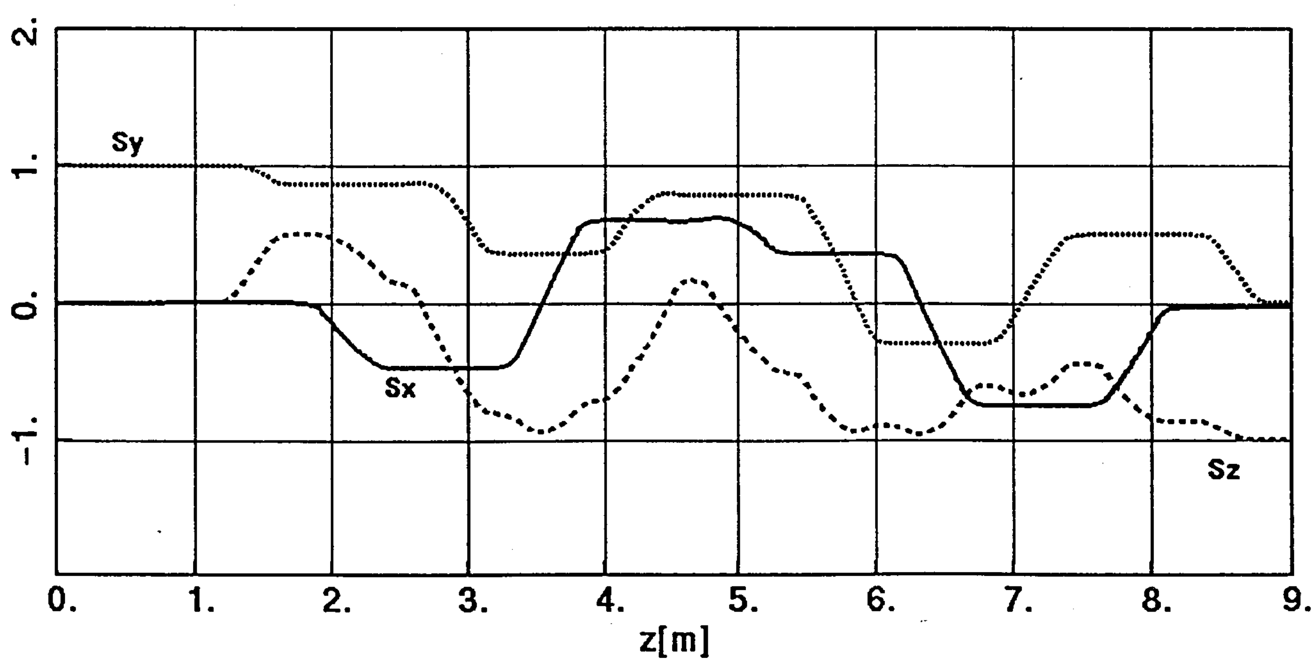

Fig.10. Spin Rotator (ii). Type 2 Precession of the three spin components.

Siberian Snakes

Two Siberian Snakes to flip the spin vertically have been proposed for RHIC: in each the axis of precession should lay in the horizontal plane, at 45° with the z axis. This condition can be found tracking a particle with spin originally at 45° emerging with the same spin orientation.

As a Siberian Snake, we have studied a Steffen Snake, composed by 7 dipoles with magnetic gap of 20 cm, a vertical triplet and an horizontal quartet, interlaced. Spin precession and rotation axis angle are found by finely varying the field in the triplet vs. the quartet. This snake requires two main power supplies plus two correction power supplies. The Steffen Snake parameters to obtain a spin flip around an axis at 45° with z are given in table 4. Field pattern, orbits and spin precession are shown in the figures 11 and 12. Figure 13 shows the fine tuning of the magnetic field that allows one to change the angle of the precession axis around 45°.

The machine optics transfer matrix for this snake calculated by tracking 128 random particles is shown in table 5

Table 4. Steffen Snake.

No of dipoles				7
No independent power supplies				2 (A,B)
Total length [m]				7
Magnetic gap [mm]				200
Dipoles				Field [T]
	center [m]	length [m]	orient.	(s_y ->-s_y)
1	0.956	0.184	H	3.144
2	1.596	0.528	-V	4.084
3	2.404	0.528	-H	3.842
4	3.500	1.266	2V	4.055
5	4.596	0.528	H	3.842
6	5.404	0.528	-V	4.084
7	6.044	0.184	-H	3.144
Max orbit excursion @ γ=200 [mm]				2.4 (x) 7.3 (y)
Total field integrals [T-m]				11.872 (x) 7.995 (y) 0.083 (z)
Orbit lengthening [mm]				0.038

Table 5. Optical Matrix for the Steffen Snake of table 4, calculated by tracking 128 random particles

1-.000518	7-.001038	0.000140	-0.000364
-0.000104	1-.000350	-0.000003	-0.000078
0.000305	0.000047	1-.000190	7-.000364
0.000001	0.000079	-0.000063	1-.000195

No simple optical interpretation

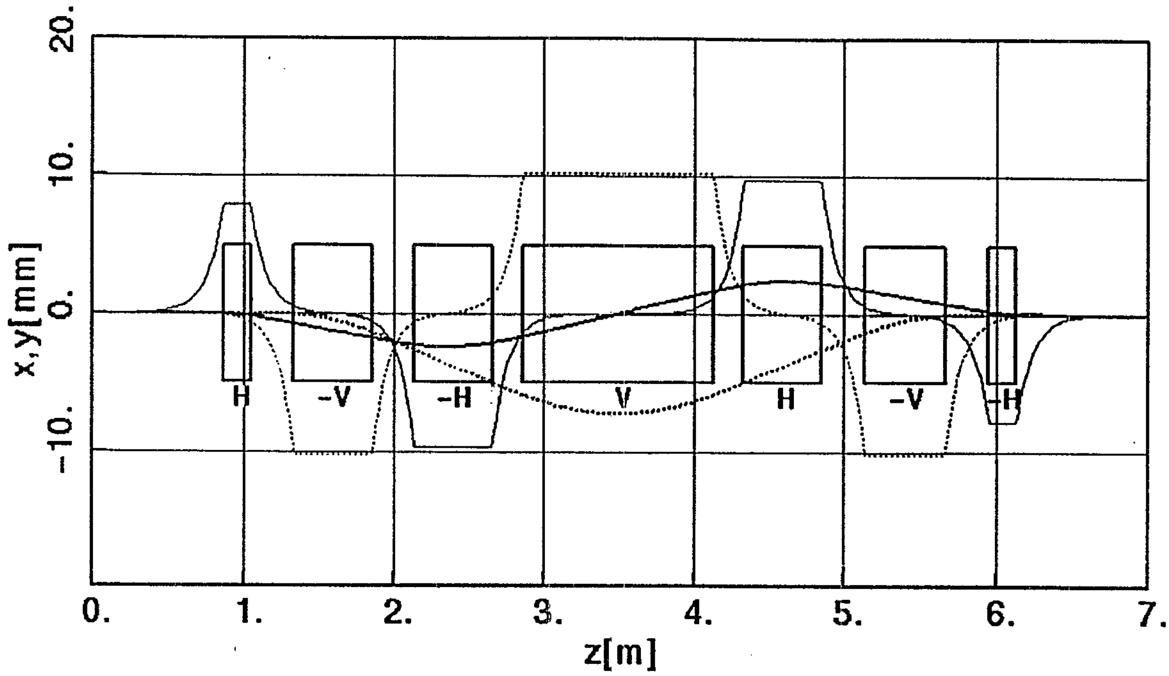

Fig.11. Steffen Snake. Field and trajectory.

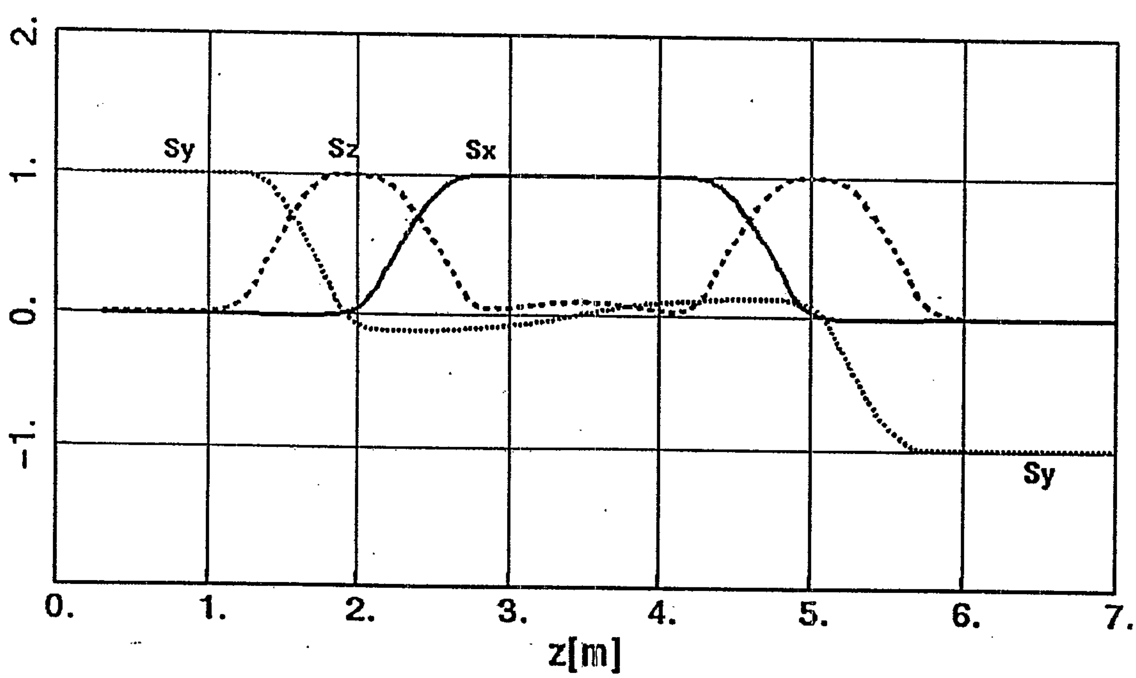

Fig.12. Steffen Snake. Precession of the three spin components.

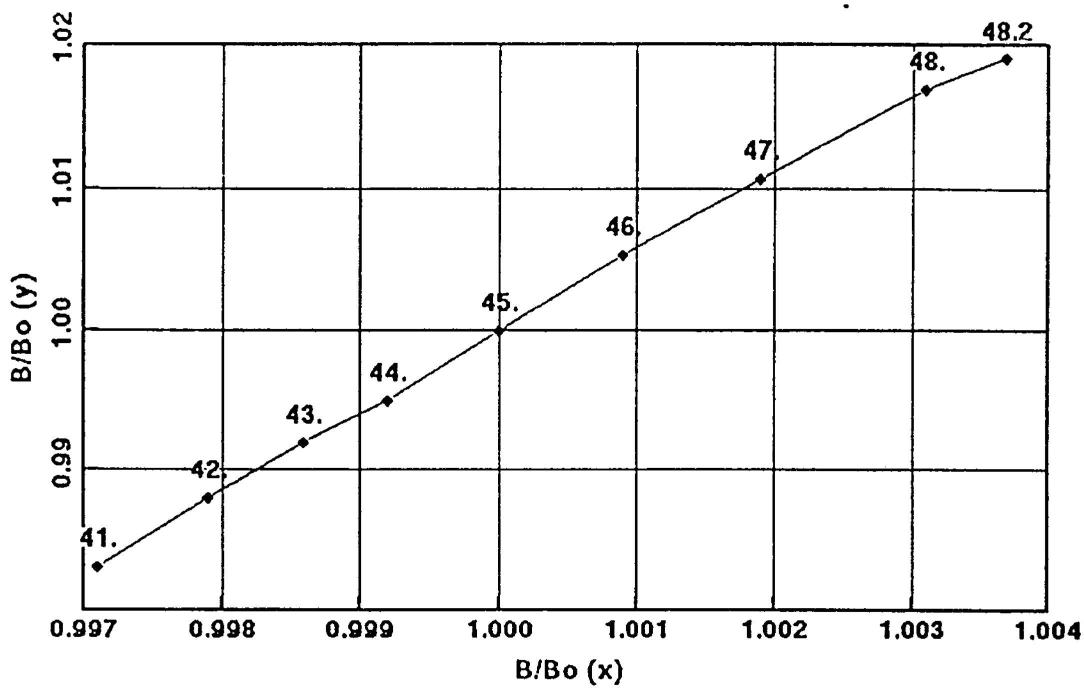

Fig.13. Spin Rotator. Fine tuning of the magnet triplet vs. the quartet to obtain different orientations of the spin rotation axis, around 45°.