BNL-103706-2014-TECH

AGS/RHIC/SN 081;BNL-103706-2014-IR

Calculation of the invariant spin field by adiabatically blowing up the beam with and rf dipole

A. Lehrach

August 1999

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

DISCLAIMER

Calculation of the invariant spin field by adiabatically blowing up the beam with an rf dipole[1]

A. Lehrach

Brookhaven National Laboratory, P.O. Box 5000, Upton, New York 11973, USA

Abstract: The Relativistic Heavy Ion Collider RHIC will collide polarized proton beams up to a maximum beam energy of 250 GeV [1]. The invariant spin field in a high energy ring can vary substantially across the beam. This decreases the amount of polarization provided to experiments and makes the polarization strongly dependent on the position in phase space. This paper describes a method to compute the invariant spin field by adiabatically blowing up the beam with an rf dipole. This method will also allow measuring the invariant spin field in RHIC.

1 Introduction

In order to investigate the motion of the spin during acceleration and storage of a polarized high energy beam, the invariant spin field has to be calculated. The invariant spin field $\vec{n}(\vec{z})$ depends on the position $\vec{z}$ of the particle in the six dimensional phase space. A particle with the initial spin $\vec{s}_i$ at the phase space position $\vec{z}_i$ has the final spin $\vec{s}_f$ and is transported to the phase space point $\vec{z}_f$ during one turn in a storage ring. If $T_{t=1}$ is the one turn spin transfer matrix, then for every phase space point $\vec{z}_i$ a spin field vector $\vec{n}(\vec{z}_i)$ exists such that

$$\vec{n}(\vec{z}_f) = T_{t=1} \cdot \vec{n}(\vec{z}_i). \tag{1}$$

The spin follows the invariant spin field if the motion of the spin is adiabatic. One method to calculate the invariant spin field is stroboscopic averaging [2], which is based on multi-turn tracking and averaging of the spin viewed stroboscopically from turn to turn at one position in the ring. The invariant spin field has also been studied using a method called adiabatic anti-damping [3], which is very similar to the method presented here. In this study the motion of the particle and spin is adiabatically excited with coherent betatron oscillations using an rf dipole. With respect to the amount of polarization that can be provided to experiments at RHIC, the invariant spin field is investigated at the interaction point of the PHENIX experiment. The RHIC lattice was taken without field errors and misalignments of the magnets. The spin and particle motion were calculated using the spin tracking program SPINK [4]. Coherent betatron oscillations have also recently been used for spin manipulation to overcome intrinsic spin resonances in the Alternating Gradient Synchrotron AGS [5].

[1] Work performed under the auspices of the U.S. Department of Energy

2 Calculation of the invariant spin field

A controlled betatron oscillation is introduced by an rf dipole to calculate the invariant spin field. The rf dipole is slowly energized to its final field and back to zero field in a way that the resulting particle and spin motion is adiabatic. Adiabaticity is achieved if the spin and particle arrive at their initial position after de-energizing this device. In order to get a large coherent amplitude Z_{coh}, the modulation tune f of the rf dipole, defined as the oscillation frequency of the rf dipole field divided by the revolution frequency of the particles in the accelerator, has to be close to the fractional betatron tune [5]:

$$Z_{coh} = \frac{1}{2}\,\beta_z\,\frac{B\;l}{B_\rho\;\rho}\,\frac{1}{2\pi\delta}, \tag{2}$$

In Eq. 2, β_z is the betatron amplitude at the location of the rf dipole, δ the difference between the modulation tune of the rf dipole field and the fractional betatron tune, l the length of the rf dipole, B the amplitude of the of dipole field, and $(B_\rho\;\rho)$ the magnetic rigidity of the beam. If the spin of the particle on the closed orbit is started parallel to the corresponding spin field vector[2], and the rf dipole excites the motion of the spin adiabatically, then the spin of the particle remains parallel to the invariant spin field during the excitation. In this case one particle spin tracking is sufficient to calculate the invariant spin field in the entire phase space. In the presented study the particle is excited vertically, and the vertical betatron tune is equal to the design betatron tune of RHIC ($\nu_y = 29.18$).

2.1 Invariant spin field away from spin resonances

Beam energies corresponding to half integer γG values were chosen to investigate the invariant spin field away from spin resonances. In Fig. 1 the excitation of the particle and spin caused

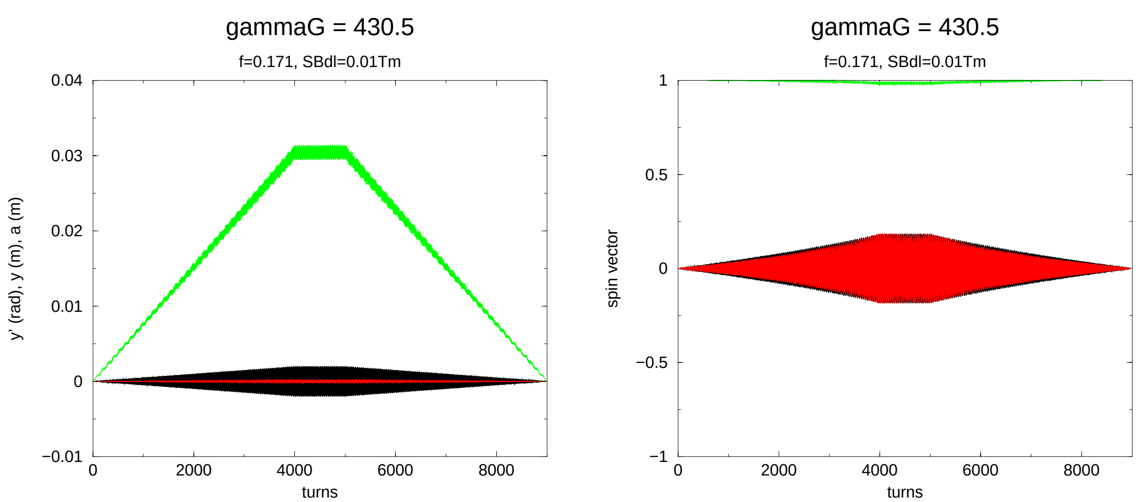

Figure 1: *Simulated excitation of the particle (left; upper curve: a, lower curves: (y, y')) and spin (right; upper curve: s_y, lower curves: s_x and s_s) by an rf dipole in RHIC.*

by an rf dipole with a modulation tune of $f = 0.171$ at a beam energy corresponding to

[2] called $\vec{n}_0$ axis on the closed orbit, calculated using the method of stroboscopic averaging [2]

$\gamma G = 430.5$ is shown. The coordinates (y, y') are the position and angle of the particle in the vertical phase space, and a the normalized vertical amplitude, given by the square root of the vertical betatron amplitude[3] and normalized vertical beam emittance. The amplitude of the rf dipole field was increased to an integral field strength of $\int B\, dl = 0.01\,\mathrm{Tm}$ and decreased back to zero amplitude in 9000 turns. The particle and the spin return adiabatically to their initial position after 9000 turns. In Fig. 2 the components of the invariant spin field are plotted versus the vertical position and angle at the interaction point of PHENIX. The deviation of

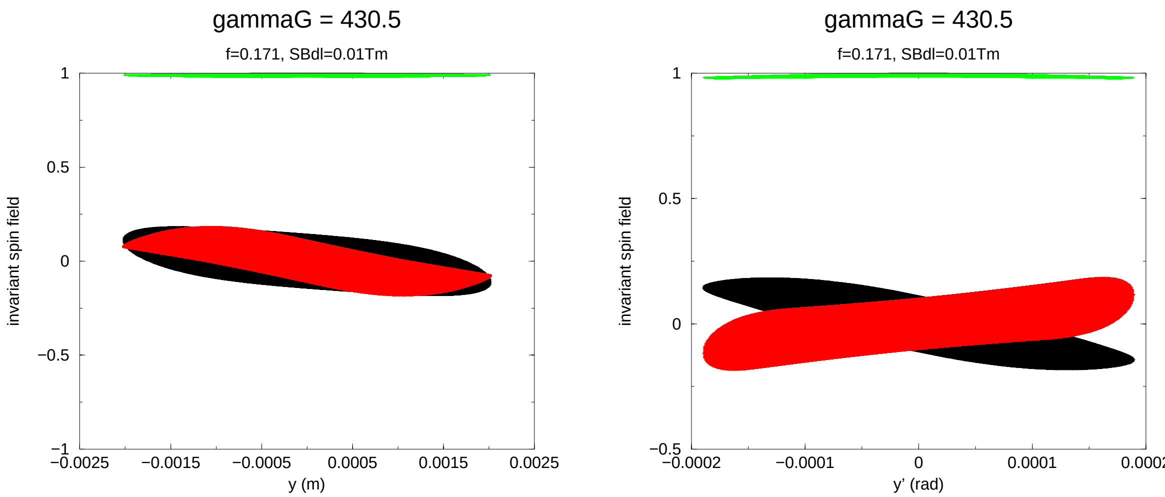

Figure 2: *Components of the invariant spin field versus vertical position (left) and vertical angle (right). The upper curve is n_y, the lower curves are n_x and n_s.*

the invariant spin field from the vertical axis for an excitation of y=2 mm or y'=0.2 mrad is less than 15°. The components of the invariant spin field as a function of the normalized vertical amplitude can be seen in Fig. 3. A normalized vertical beam emittance of 20π mm mrad, which

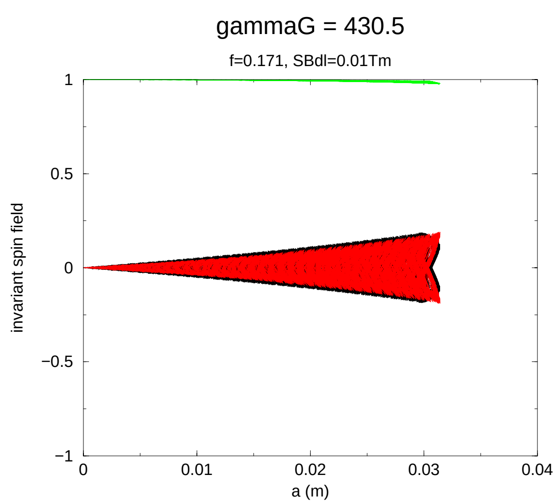

Figure 3: *Components of the invariant spin field versus normalized vertical amplitude. The upper curve is n_y, the lower curves are n_x and n_s.*

corresponds to a normalized vertical amplitude of a=0.015 m at the interaction point, includes

[3]about 10 m at the interaction point of PHENIX

95% of the phase space of the RHIC beam. Even for particles on the surface of the beam the deviation of the invariant spin field from the vertical direction away from any spin resonance is very small.

2.2 Invariant spin field at the strongest intrinsic spin resonance

The behavior of the invariant spin field at the resonance energy of the strongest intrinsic spin resonance at RHIC, corresponding to $\gamma G = 422.18$, has also been investigated. In order to keep the motion of the spin adiabatic, the amplitude of the rf dipole has to be increased in three million turns to $\int B\, dl = 0.004\,\mathrm{Tm}$ (Fig. 4). After reaching the maximum modulation field, the

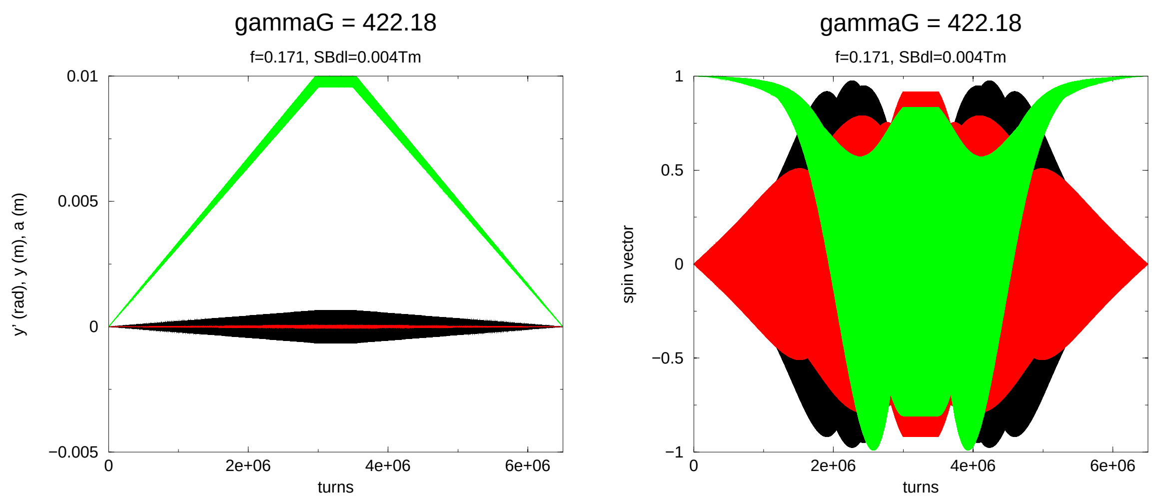

Figure 4: *Simulated excitation of the particle (left; upper curve: a, lower curves: (y, y')) and spin (right; upper curve: s_y, lower curves: s_x and s_s) by an rf dipole in RHIC.*

amplitude of the rf dipole field was kept constant for half a million turns and decreased to zero amplitude in another three million turns. During energizing and de-energizing of the rf dipole, interference structures of the spin motion, caused by the overlap between the intrinsic spin resonance and the spin resonance excited by the rf dipole, can be observed (Fig. 4). To reduce the effect of the rf dipole on the spin, the modulation tune has to be moved closer to the vertical betatron tune. At the same time the amplitude of the rf dipole was reduced to keep the coherent amplitude constant. The components of the invariant spin field for different modulation tunes and amplitudes of the rf dipole are plotted in Fig. 5 versus the normalized vertical amplitude up to a=0.01 m, which corresponds to a normalized vertical beam emittance of 10π mm mrad. The interference structure of the invariant spin field disappears as the modulation tune of the rf dipole is moved closer to the vertical betatron tune. The oscillation of the invariant spin field around the vertical axis at a normalized vertical beam emittance of 10π mm mrad is very large. On the surface of the beam the deviation of the invariant spin field from the vertical direction varies between 30° and more than 90°. The behavior of the invariant spin field versus vertical position and angle at the interaction point of PHENIX is shown in Fig. 6. In the range from $y = \pm 0.7$ mm or $y' = \pm 0.07$ mrad the vertical component of the invariant spin field changes from one to negative values. This shows that it is essential to calculate the invariant spin field before providing a polarized beam to an experiment at a certain energy.

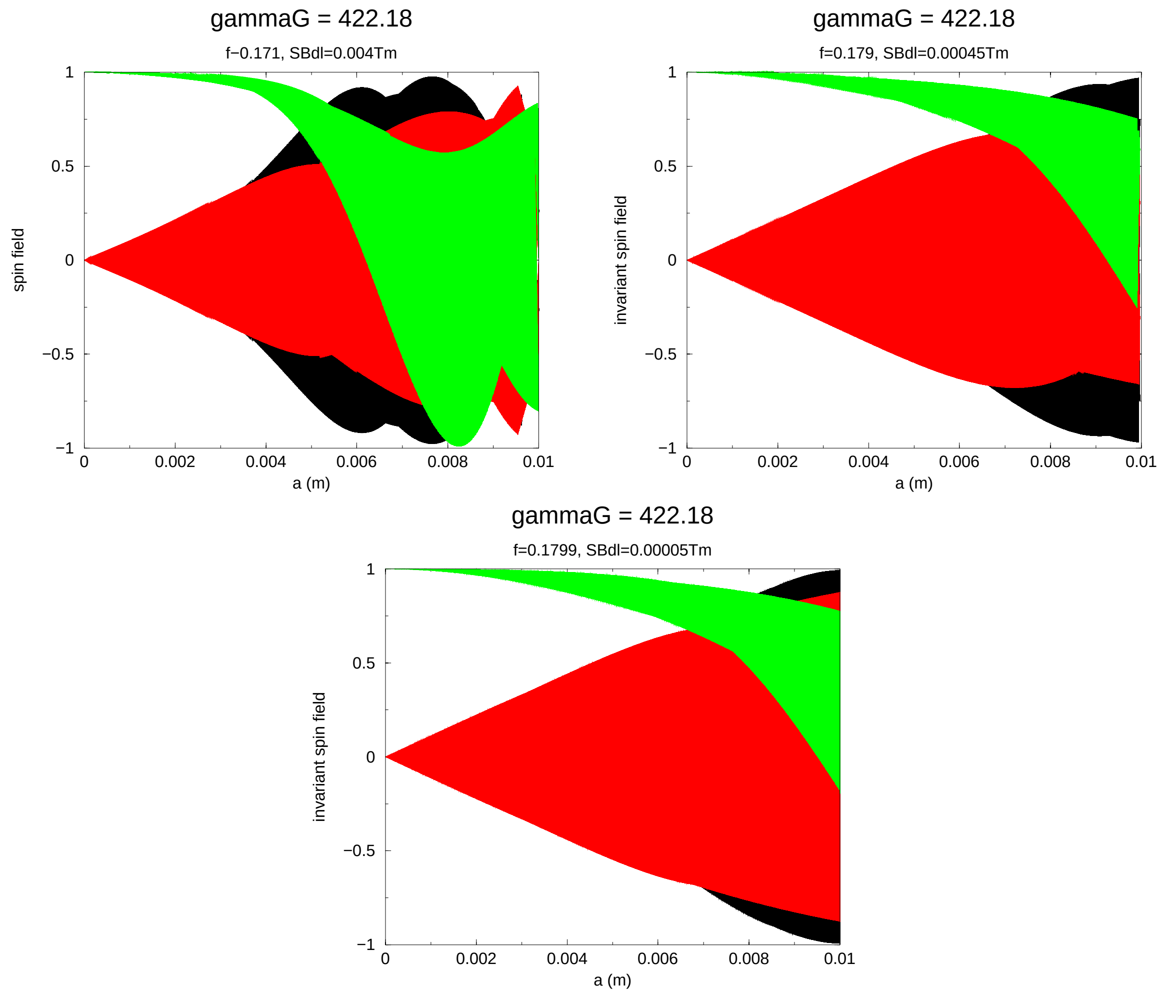

Figure 5: *Components of the invariant spin field versus normalized vertical amplitude for different modulation tunes and amplitudes of the rf dipole. The upper curve is n_y, the lower curves are n_x and n_s.*

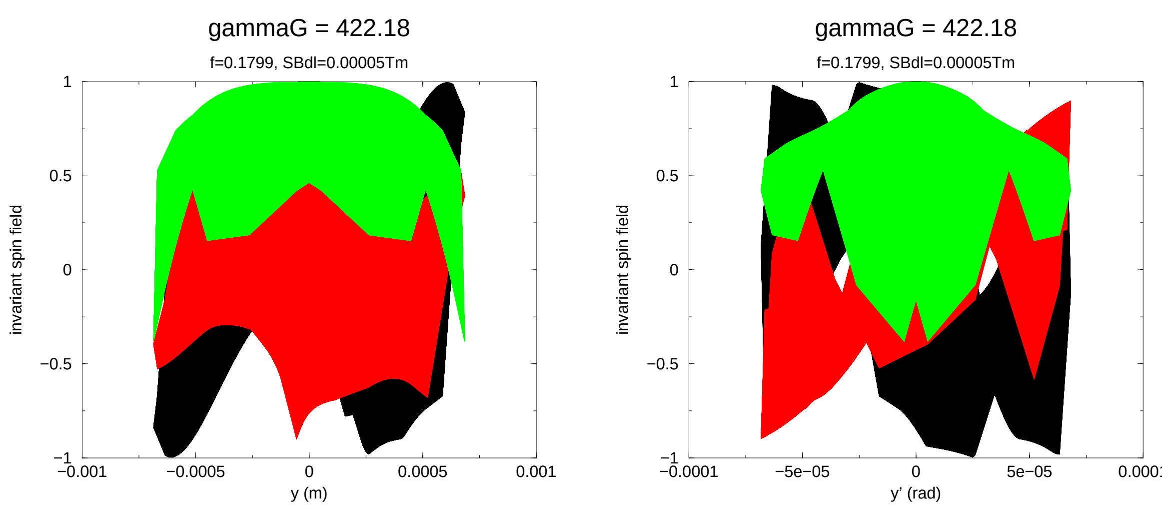

Figure 6: *Components of the invariant spin field versus vertical position (left) and vertical angle (right). The upper curve is n_y, the lower curves are n_x and n_s.*

3 Conclusion

Coherent betatron oscillations cannot only be used for spin manipulation, but also for spin diagnostics. The particle motion has been excited with an rf dipole to calculate the invariant spin field at various energies in RHIC. The calculation of the invariant spin field close to an intrinsic spin resonance is delicate, because the influence of the rf dipole on the spin motion is not negligible. The modulation tune of the rf dipole has to be moved very close to the betatron frequency in order to keep the effect of the rf dipole on the spin motion small. This is hard to achieve if one wants to use this method to measure the invariant spin field close to an intrinsic spin resonance because the chromaticity of the accelerator has to be corrected very precisely to keep the betatron tune spread small enough. It is much easier to measure the invariant spin field including the interference structure and then compare it to the simulation to get an estimation for the excitation of the invariant spin field by intrinsic spin resonances. In the next step this calculation will be extended including field errors and misalignments of the RHIC magnets. For the experimental proof of this calculation, a polarimeter has been proposed which is able to measure sideways polarization [6]. Not only for spin dynamics calculation but also from the experiments' point of view, it is important to calculate and measure the invariant spin field. This effect will likely exclude certain energy ranges in the vicinity of spin resonances from being used for spin experiments at RHIC.

4 Acknowledgment

Many have contributed to the work presented here. In particular, T. Roser had the idea, A.U. Luccio developed most parts of the spin tracking program SPINK, and N. Tsoupas contributed many fruitful discussions.

References

[1] Design Manual - Polarized Proton Collider at RHIC, Brookhaven National Laboratory (1998), http://www.ags.bnl.gov/rhicspin;
T. Roser, Acceleration of Polarized Protons to High Energies, Proceedings of the Particle Accelerator Conference PAC 99, New York, 26 (1999).

[2] G.H. Hoffstätter et al., A tracking algorithm for the stable spin polarization field in storage rings using stroboscopic averaging, *Phys. Rev.* E **54**, 4240 (1996).

[3] M. Vogt et al., The amplitude dependent spin tune and the invariant spin field in high energy proton accelerator, Proceedings of the European Particle Accelerator Conference EPAC 98, Stockholm, 1362 (1998).

[4] A.U. Luccio et al., Development of the spin tracking program SPINK, Proceedings of the Particle Accelerator Conference PAC 99, New York, 1578 (1999).

[5] M. Bai et al., Experimental test of coherent betatron resonance excitation, *Phys. Rev.* E **56**, 6002 (1997).

[6] T. Roser, private communication.