

Measurements of Helical Magnetic Fields Using Flat Rotating Coils

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1 Introduction

Rotating coils are commonly used to measure the magnetic field coefficients (a_n, b_n) inside straight magnets [1]. They can also be employed to determine the multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$ (cf. Ref. [2]) in helical magnets.

We use a cylindrical coordinate system (r, θ, s) where s designates the coordinate along the longitudinal magnet axis. The area of a flat rotating coil ranges from r_1 to r_2 and from s_1 to s_2 . The magnetic flux through the coils is

$$\Phi(\theta) = N \int_{s_1}^{s_2} \int_{r_1}^{r_2} B_\theta(r, \theta) dr ds \quad (1)$$

where N is the number of coils windings. For rotating coils one has $\theta = \omega t$ and the induced voltage

$$U = -\frac{d\Phi}{dt} \quad (2)$$

is proportional to the angular velocity ω .

We will present formulae for the magnetic flux through a rotating coil for straight and helical magnets. Assuming the induced voltage is parameterized in terms of ordinary multipole coefficients (a_n, b_n) conversion formulas will be given to obtain the helical multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$.

2 Straight Magnetic Fields

The azimuthal field in straight magnets can be expressed in multipole coefficients (a_n, b_n) as [2]

$$B_\theta = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{r_0}\right)^n \left[b_n \cos((n+1)\theta) - a_n \sin((n+1)\theta) \right]. \quad (3)$$

B_0 is the main magnetic field and r_0 a reference radius. The magnetic flux (1) becomes

$$\Phi(\theta) = NB_0(s_2 - s_1) \sum_{n=0}^{\infty} K_n \left[b_n \cos((n+1)\theta) - a_n \sin((n+1)\theta) \right] \quad (4)$$

where the coefficients K_n are defined by

$$K_n = \frac{r_0}{n+1} \left[\left(\frac{r_2}{r_0} \right)^{(n+1)} - \left(\frac{r_1}{r_0} \right)^{(n+1)} \right] \quad (5)$$

and result from the integration over r in (1).

3 Helical Magnetic Fields

The azimuthal helical field can be written in terms of helical multipole coefficients (\tilde{a}_n, \tilde{b}_n) (cf. Ref. [2]) as

$$B_\theta = \frac{B_0}{kr} \sum_{n=0}^{\infty} f_n I_{n+1}((n+1)kr) \left[\tilde{b}_n \cos((n+1)(\theta - ks)) - \tilde{a}_n \sin((n+1)(\theta - ks)) \right], \quad (6)$$

with

$$f_n = \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \frac{1}{r_0^n k^n}. \quad (7)$$

Here B_0 denotes the transverse component of the main field close to the magnet axis. This field is vertical at the location $s = 0$. The magnetic flux (1) can be expressed as

$$\Phi(\theta) = NB_0 \sum_{n=0}^{\infty} R_n \left[\hat{b}_n \cos((n+1)\theta) - \hat{a}_n \sin((n+1)\theta) \right] \quad (8)$$

with new coefficients

$$R_n = \frac{f_n}{k} \int_{r_1}^{r_2} \frac{1}{r} I_{n+1}((n+1)kr) dr. \quad (9)$$

The integral in (9) can be computed numerically. In (8) new magnetic multipole coefficients

$$\begin{aligned} \hat{a}_n &= +\tilde{a}_n T_n + \tilde{b}_n S_n, \\ \hat{b}_n &= -\tilde{a}_n S_n + \tilde{b}_n T_n. \end{aligned} \quad (10)$$

are used for which

$$\begin{aligned} S_n &= \frac{1}{(n+1)k} \left[\cos((n+1)ks_2) - \cos((n+1)ks_1) \right] \\ &= -\frac{2}{(n+1)k} \sin \frac{(n+1)k(s_2 - s_1)}{2} \sin \frac{(n+1)k(s_2 + s_1)}{2} \end{aligned} \quad (11)$$

and

$$\begin{aligned}
T_n &= \frac{1}{(n+1)k} \left[\sin((n+1)ks_2) - \sin((n+1)ks_1) \right] \\
&= +\frac{2}{(n+1)k} \sin \frac{(n+1)k(s_2 - s_1)}{2} \cos \frac{(n+1)k(s_2 + s_1)}{2}
\end{aligned} \tag{12}$$

have been defined.

4 Conversion

We assume now a device that parameterizes the voltage (2) in terms of multipole coefficients (a_n, b_n) for straight magnets. If the measured magnetic field has helical symmetry, the coefficients $(\tilde{a}_n, \tilde{b}_n)$ in Eq. (8) can be derived as

$$\begin{aligned}
\tilde{a}_n &= \frac{K_n}{R_n}(s_2 - s_1) \cdot \frac{a_n T_n - b_n S_n}{S_n^2 + T_n^2}, \\
\tilde{b}_n &= \frac{K_n}{R_n}(s_2 - s_1) \cdot \frac{a_n S_n + b_n T_n}{S_n^2 + T_n^2}.
\end{aligned} \tag{13}$$

We consider three special cases.

(a) Measuring coil of one helical wavelength with $s_1 = s$, $s_2 = s + \lambda$.
From equations (11) and (12) we obtain

$$S_n = T_n = 0 \tag{14}$$

and with (10)

$$\hat{a} = \hat{b} = 0. \tag{15}$$

The magnetic flux (8) is therefore zero and the coefficients can not be obtained.

(b) Measuring coil of half helical wave length with $s_1 = 0$, $s_2 = \lambda/2$.
In this case one has

$$S_n = \begin{cases} -\frac{2}{(n+1)k} & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd} \end{cases} \quad \text{and} \quad T_n = 0.$$

Only coefficients with n even (i.e. helical dipole, sextupole etc. coefficients) can be measured. For those we have

$$\begin{aligned}
\tilde{a}_n &= +\frac{K_n(n+1)\pi}{R_n 2} b_n, \\
\tilde{b}_n &= -\frac{K_n(n+1)\pi}{R_n 2} a_n.
\end{aligned} \tag{16}$$

(c) Infinitely short measuring coil with $s_1 = s$, $s_2 = s + ds$.
Expanding (11) and (12) to first order in ds we obtain

$$\begin{aligned} S_n &= -\sin\left((n+1)ks\right)ds, \\ T_n &= +\cos\left((n+1)ks\right)ds \end{aligned} \tag{17}$$

and

$$\begin{aligned} \tilde{a} &= \frac{K_n}{R_n} \left[+a_n \cos\left((n+1)ks\right) + b_n \sin\left((n+1)ks\right) \right], \\ \tilde{b} &= \frac{K_n}{R_n} \left[-a_n \sin\left((n+1)ks\right) + b_n \cos\left((n+1)ks\right) \right]. \end{aligned} \tag{18}$$

If in addition $s = 0$, the $(\tilde{a}_n, \tilde{b}_n)$ can be obtained from the (a_n, b_n) by multiplication with K_n/R_n .

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References

- [1] P. Schmüser, “Magnetic measurements of the superconducting HERA magnets and analysis of systematic errors”, DESY HERA-p 92-1 (1992).
- [2] W. Fischer, “Magnetic field error coefficients for helical dipoles”, RHIC/AP/83 and AGS/RHIC/SN/17 (1996).