BNL-103669-2014-TECH
AGS/RHIC/SN 043;BNL-103669-2014-IR

Spin Tracking Study of the AGS

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November 1996

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U.S. Department of Energy
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Spin Note

AGS/RHIC/SN No. 043

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Abstract

In the recent polarized proton runs in the AGS, a 5% partial snake is used successfully to overcome the imperfection depolarizing resonances. Although some depolarization at intrinsic resonances are expected, the level of the depolarization does not agree with a simple model calculation. A spin tracking program is then used to simulate the real polarized proton beam in the AGS. The results show that, due to the linear coupling introduced by a solenoidal 5% partial snake, the polarized beam will be partially depolarized also at the so-called coupling resonance, which is related to the horizontal betatron tune. The synchrotron oscillation also affect the beam polarization to a smaller extent. Some possible schemes to overcome the coupling resonances are discussed and simulated.

1. Introduction

In the recent polarized proton run in the AGS, a 5% partial snake [1] is used to overcome the imperfection depolarizing resonances [2]. Fig. 1 shows the measured absolute value of the vertical polarization at $G\gamma = n + \frac{1}{2}$ up to $G\gamma = 22.5$ (solid points). For these measurements, the betatron tunes are set at $\nu_x = 8.80$, $\nu_y = 8.70$, and the acceleration rate α is about 1.1×10^{-5}. The pulsed tune-jump quadrupoles were not used in this experiment. As shown in Fig. 1, the depolarization resulted only from the three intrinsic resonances, located at $G\gamma = 0 + \nu_y, 24 - \nu_y$, and $12 + \nu_y$, which the 5 % partial snake can not overcome. The observed level of depolarization at the intrinsic resonances were -65%, 63% and -49%, respectively.

Although some depolarization at intrinsic resonances are expected, the level of the depolarization does not agree with a simple model calculation. The unexpected high depolarization at $G\gamma = 24 - \nu_y$ is believed to be caused by the coincidence with the AGS transition energy. In a later run with different ν_y and faster acceleration rate, this resonance is crossed without depolarization.

When a polarized beam is accelerated through an isolated resonance, the final polarization is given by the Froissart-Stora formula [3],

$$\frac{P_f}{P_i} = 2e^{\frac{-\pi|\epsilon|^2}{2\alpha}} - 1, \qquad \alpha = \frac{d}{d\theta}[G\gamma - K] = \frac{1}{\omega_0}\frac{d}{dt}[G\gamma - K], \tag{1}$$

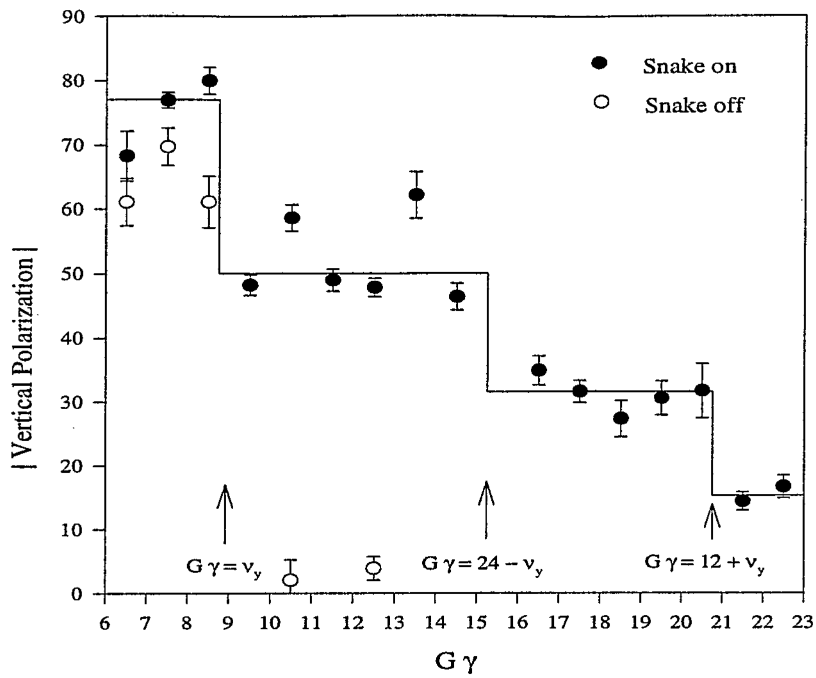

Fig. 1. The measured absolute value of the vertical polarization at $G\gamma = n + \frac{1}{2}$ up to $G\gamma = 22.5$ (solid points). The acceleration rate α is about 1.1×10^{-5}.

where P_f and P_i are the polarization before and after the resonance crossing, respectively, ϵ is the resonance strength, ω_0 is the revolution frequency, K is the resonance tune, and α is the resonance crossing rate. For intrinsic resonances, the resonance strength is proportional to $\sqrt{\epsilon_N}$, where ϵ_N is the normalized vertical emittance of the beam [4]. When calculating the intrinsic resonance depolarization, one has to take into account the beam distribution. Suppose the beam distribution is Gaussian with

$$\rho(\varepsilon) = \frac{1}{2\varepsilon_0} e^{-\frac{\varepsilon}{2\varepsilon_0}}, \tag{2}$$

where ε_0 is the rms emittance of the beam. Applying the Froissart-Stora formula to each particle, the effective polarization after passing through the resonance becomes,

$$\frac{P_f}{P_i} = \frac{[1 - \frac{\pi|\epsilon(\varepsilon_0)|^2}{\alpha}]}{[1 + \frac{\pi|\epsilon(\varepsilon_0)|^2}{\alpha}]}. \tag{3}$$

The measured normalized 95% emittance $\varepsilon_{N,95\%}$ in the AGS is about 25~30 π mm-mrad, the acceleration rate α is 1.1×10^{-5}, and the resonance strengths of $G\gamma = 0 + \nu_y$ and $G\gamma = 12 + \nu_y$ given by DEPOL [5] for a $\varepsilon_N = 10\pi$ mm-mrad beam are 0.0154 and 0.0054, respectively. With these numbers, the polarization after crossing each of the two resonances are:

$$\frac{P_f}{P_i} = -0.94 \quad \text{at} \quad G\gamma = 0 + \nu_y;$$

$$\frac{P_f}{P_i} = -0.61 \quad \text{at} \quad G\gamma = 12 + \nu_y.$$

they are different from the measured data, especially the one at $G\gamma = 0 + \nu_y$:

$$\frac{P_f}{P_i}|_{expt} = -0.65 \quad \text{at} \quad G\gamma = 0 + \nu_y;$$
$$\frac{P_f}{P_i}|_{expt} = -0.49 \quad \text{at} \quad G\gamma = 12 + \nu_y.$$

A spin tracking simulation is then needed to understand the spin dynamics when a partial snake is inserted in the ring.

DEPOL [5], a program written by E.D. Courant, calculates the depolarizing resonance strength by Fourier analysis. The inputs of DEPOL are the outputs of a machine code such as MAD or SYNCH. DEPOL is simple but it requires a smooth lattice condition which is not satisfied by a ring with a snake inserted. Moreover, there is no way to include the effect of synchrotron motion, and linear coupling effect of a solenoidal snake. So a spin tracking program is needed to analyze the data obtained from the AGS partial snake experiment.

2. SPINK Program

A tracking program SPINK [6] is used to track the polarized proton beam in the AGS. The idea is to track a group of protons, randomly generated with certain distribution in the phase space, through the machine lattice. Each proton is characterized by four transverse coordinates, two longitudinal coordinates and three spin components. Orbit matrices are built from a TWISS file, output of MAD. Spin matrices are built for each piecewise constant magnet, which includes bending magnets (both separate function and combine function), quadrupoles (both regular and skew), snakes, RF cavities, spin flippers.

The distribution of protons in the transverse phase is chosen as a Gaussian distribution:

$$\rho(\varepsilon_i) = \frac{1}{2\varepsilon_{i0}} e^{-\frac{\varepsilon_i}{2\varepsilon_{i0}}}, \tag{4}$$

where i can be x and y, respectively. The distribution of protons in the longitudinal phase is chosen as a parabolic distribution:

$$\rho(\delta) = \frac{1}{N_\delta}(1 - (\frac{\delta}{\hat{\delta}})^2), \tag{5}$$

$$\rho(\Delta\phi) = \frac{1}{N_{\Delta\phi}}(1 - (\frac{\Delta\phi}{\Delta\hat{\phi}})^2), \quad \Delta\phi = \phi - \phi_s, \tag{6}$$

Table 1

Resonances	Resonance strength DEPOL	SPINK
$0+\nu_y$	0.0151232	0.0149878
$24-\nu_y$	0.0005928	0.0006195
$12+\nu_y$	0.0052263	0.0051077
$36-\nu_y$	0.0137000	0.0135140
$24+\nu_y$	0.0012159	0.0012260
$48-\nu_y$	0.0015948	0.0016806
$36+\nu_y$	0.0265870	0.0269914

where δ is the momentum spread, ϕ is the synchrotron phase, and ϕ_s is the phase of the synchronous particle.

3. Comparison with DEPOL

SPINK can be used to check the strength calculated by DEPOL using the Froissart-Stora formula [3] if the resonances are well- separated and are not too strong. The first condition insures that the polarization, at the beginning of the tracking is stable around the value of one. The second condition should be met so that the Froissart-Stora would not saturate.

Suppose the initial polarization P_i is 1, then the resonance strength can be extracted from the final polarization $\langle P_f \rangle$ simulated by the tracking:

$$|\epsilon_k| = \sqrt{-\frac{2\alpha}{\pi}\ln\frac{(1+\langle P_f \rangle)}{2}}, \tag{7}$$

where $\langle P_f \rangle$ is the average of the polarization over a number of turns after crossing the resonance.

An example is given for the AGS lattice without partial snake. The betatron tunes are set to be $\nu_x = 8.717$ and $\nu_y = 8.766$, respectively. The same TWISS file is used as inputs for DEPOL and SPINK. For the imperfection resonances, the random error is chosen as Δy= 0.0003 m. The results for the imperfection resonances are plotted in Fig. 2. The results of SPINK agree well with those of DEPOL, except one at $G\gamma = 45$ where the resonance strength is so strong that the Froissart-Stora formula is saturated. For the intrinsic resonances, the particle is chosen on the boundary of the 10 π mm-mrad normalized 95% emittance. For a few very strong intrinsic resonance, the emittance is reduced to 0.1 π mm-mrad so that the Froissart-Stora formula would not saturate, then the resonance strength is projected to a 10 π mm-mrad emittance particle. The results for the intrinsic resonances are plotted in Fig. 3 and listed in Table 1. Again, it shows good agreement between DEPOL and SPINK results.

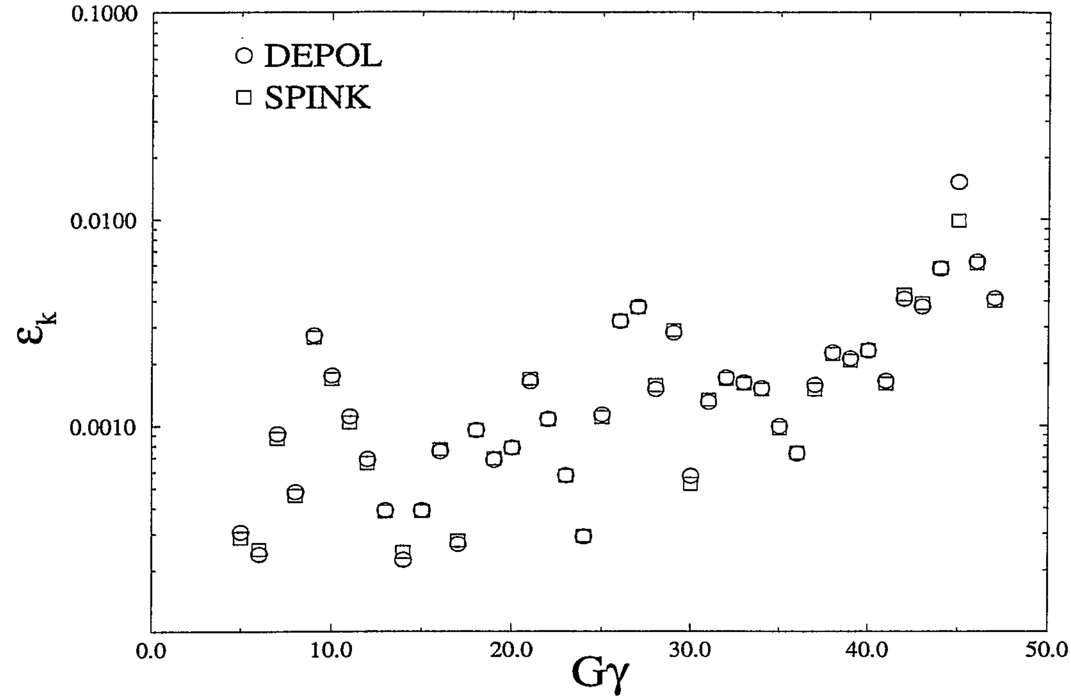

Fig. 2. The comparison of results of DEPOL and SPINK for imperfection resonances. The acceleration rate α used for SPINK is 4.5×10^{-5}.

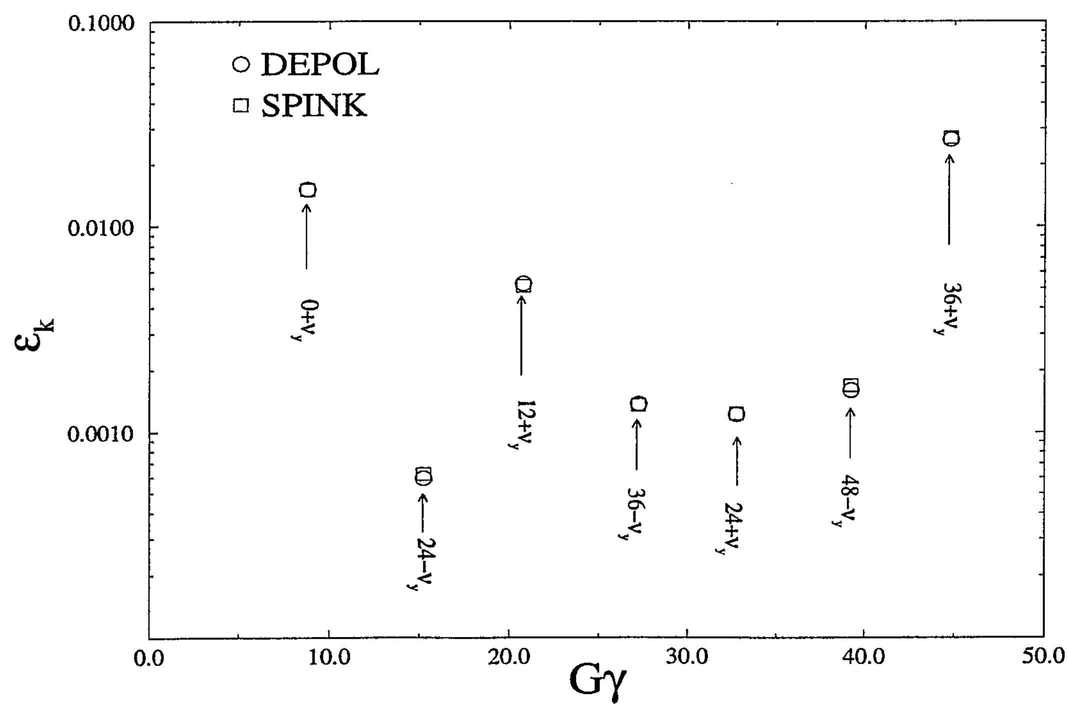

Fig. 3. The comparison of results of DEPOL and SPINK for intrinsic resonances. The acceleration rate α used for SPINK is 4.5×10^{-5}.

4. Tracking for the AGS Beam

The SPINK program is then used to simulate the real polarized proton beam in the AGS. During the experiment, the two betatron tunes were set at $\nu_x = 8.80$ and $\nu_y = 8.70$, respectively. The longitudinal beam emittance is measured as about 0.8 eVs, and the measured normalized 95% transverse emittances are $\varepsilon_x \approx \varepsilon_y \approx 30\pi$ mm-mrad. The acceleration rate $\alpha \approx 1.1 \times 10^{-5}$. In the simulation, the betatron tunes and acceleration rate are chosen as the experiment values; the normalized longitudinal emittance is set at 0.8 eVs; and for the simplicity, the two transverse emittances are set at the same value $\varepsilon = 30\pi$ mm-mrad. A group of 200 particles randomly chosen with a Gaussian distribution in transverse phase space and a parabolic distribution in longitudinal phase space are used in the tracking. It takes about 10 hours machine time of a fast workstation to track 200 particles crossing one resonance. The tracking results for $G\gamma = 0 + \nu_y$ and $G\gamma = 12 + \nu_y$ are plotted in Figs. 4 and 5.

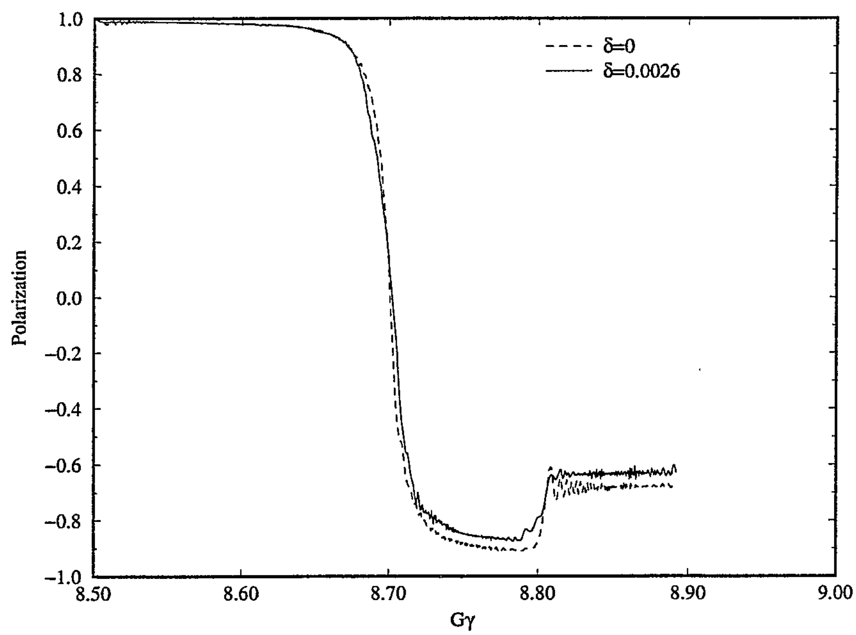

Fig. 4. The simulation of crossing $G\gamma = 0 + \nu_y$, where $\nu_x = 8.80$ and $\nu_y = 8.70$. The dashed line is without momentum spread, and the solid line is with momentum spread $\delta = 0.0026$ which corresponds to emittance 0.8 eVs. The transverse emittances $\epsilon_x = \epsilon_y = 30\pi$ mm-mrad. The final polarization $P = -0.63$, which agrees with the experimental data $P_{expt} = -0.65$ within the error bar ± 0.05. The acceleration rate α is 1.1×10^{-5}.

The Figs. 4 and 5 clearly show that there is an extra resonance adjacent to the intrinsic resonance, which causes spin to flip the other way and reduces the polarization. This additional resonance can be easily understood as a linear coupling effect. The solenoidal partial snake introduces considerable linear coupling between the two transverse betatron motions. Due to the coupling, the vertical betatron motion also has a component with the horizontal betatron frequency. As a consequence, the beam will see an additional resonance, the so-called coupling resonance. Besides the linear coupling effect, the synchrotron motion also affect the beam polarization but to a smaller extent. The results show good agreement with the experimental data.

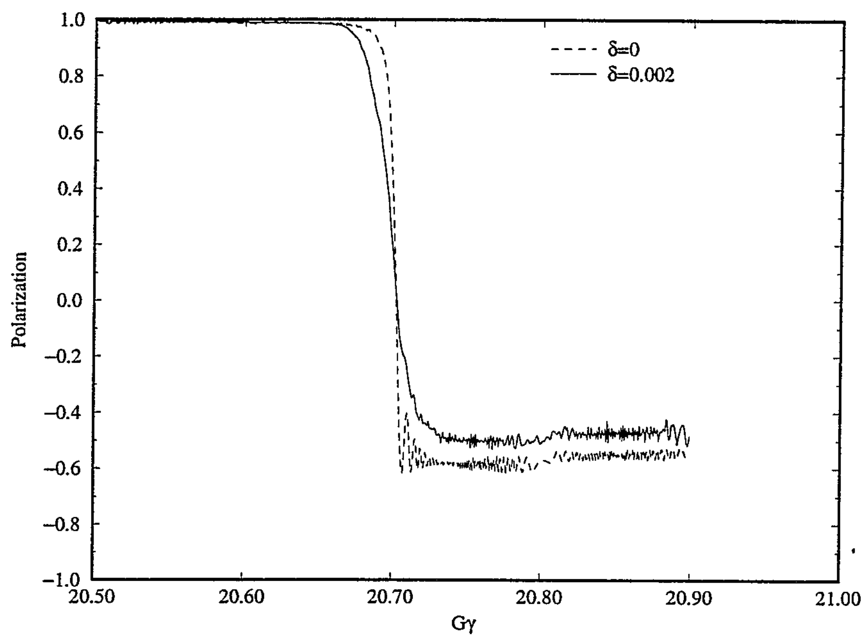

Fig. 5. The simulation of crossing $G\gamma = 12 + \nu_y$, where $\nu_x = 8.80$ and $\nu_y = 8.70$. The dashed line is without momentum spread, and the solid line is with momentum spread $\delta = 0.002$ which corresponds to emittance 0.8 eVs. The transverse emittances $\epsilon_x = \epsilon_y = 30\pi$ mm-mrad. The final polarization $P = -0.47$, which agrees with the experimental data $P_{expt} = -0.49$ within the error bar ± 0.05. The acceleration rate α is 1.1×10^{-5}.

Fig. 6 shows the beam polarization dependence on the momentum spread when crossing $G\gamma = 0 + \nu_y$ while the other parameters fixed. Due to the synchrotron oscillation, some particles may cross the resonance line several times, which effectively reduces the spin flip efficiency. The larger the synchrotron oscillation, the less the spin flip. This effect will be more severe when the acceleration is very slow and the chromaticity match is needed.

The strength of the coupling resonance depends on the separation of the two betatron tunes, partial snake strength and the adjacent intrinsic resonance strength. It is then expected that for given betatron tunes and partial snake strength, the ratio of the coupling resonance strength to the intrinsic resonance strength is about a constant for different pair of resonances. The resonance strength is extracted from the polarization levels before and after crossing each resonance by using Eq. (3). A series simulations are performed for several strong intrinsic resonances such as $0+\nu_y, 36-\nu_y$ and $36+\nu_y$ with tune separation 0.1 unit and a 5% snake. The ratio for all of them

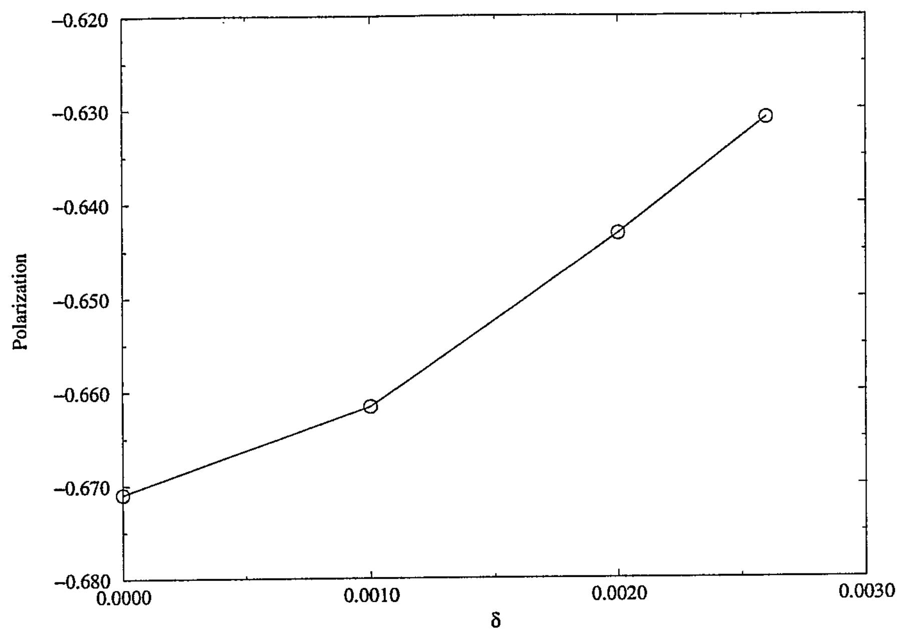

Fig. 6. The simulation of crossing $G\gamma = 0 + \nu_y$ with different momentum spread δ. The transverse emittances $\epsilon_x = \epsilon_y = 30\pi$ mm-mrad and $\nu_x = 8.80$ and $\nu_y = 8.70$. The acceleration rate α is 1.1×10^{-5}.

is about 0.06.

In the polarized proton run, the polarization was measured at $G\gamma = 10.5$, after passing through the $G\gamma = 0+\nu_y$ resonance, as a function of the vertical betatron tune ν_y for fixed horizontal tune ν_x. The adiabatic 80% spin flip for the $G\gamma = 0+\nu_y$ intrinsic resonance was diminished by the linear coupling. A series simulations are performed to reproduce the data. A simple analytical model for two isolated resonances was also used to understand the data. The results are summarized in Fig. 7. All data show that the spin flip will be diminished when $\nu_x \approx \nu_y$. When the two tunes are separated further, the spin flip efficiency increased. But when ν_y approaches 8.5, both analytical calculation and simulation give more spin flip efficiency while experiment shows less spin flip. There is no known depolarization mechanism that could account for this phenomenon. Further spin tracking study is needed to understand in detail the depolarization mechanism. Fig. 7 indicates less spin flip when ν_y approaches 8.5. Although the depolarization mechanism is unknown, one possible way to avoid it is to swamp the two betatron tunes: choose ν_x close to 8.5, and ν_y close to 9.0. A series simulations for a fixed $\nu_x = 8.60$ and different ν_y were performed and plotted in Fig. 8. It shows that the further apart the two betatron tunes, the smaller the coupling resonance strength.

It seems that the linear coupling can be diminished by well-separated betatron tunes. The betatron tunes of the AGS can be separated by more than one unit, so a simulation with $\nu_x = 7.70, \nu_y = 8.80$ is done (see Fig. 9). Although there is no depolarization at $G\gamma = 0+\nu_x = 7.70$, beam depolarization happens at $G\gamma = 1+\nu_x =$

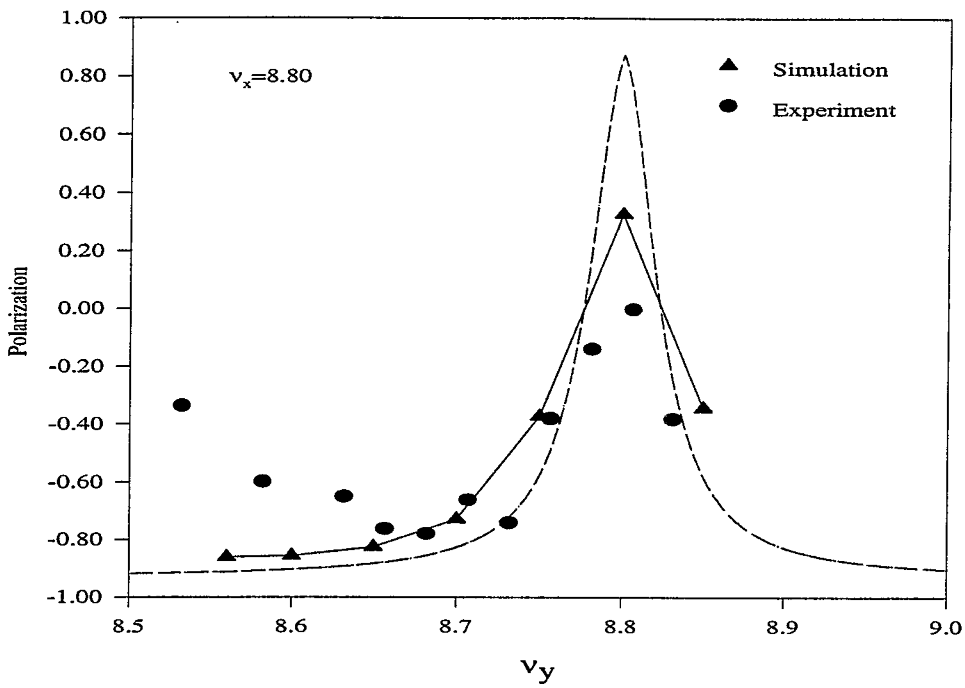

Fig. 7. Comparison of the simulation, analytical calculation and the experiment data after crossing $G\gamma = 0 + \nu_y$ and $G\gamma = 0 + \nu_x$. The dashed line is the result of the analytical calculation. The vertical axis is the ratio of P_f/P_i.

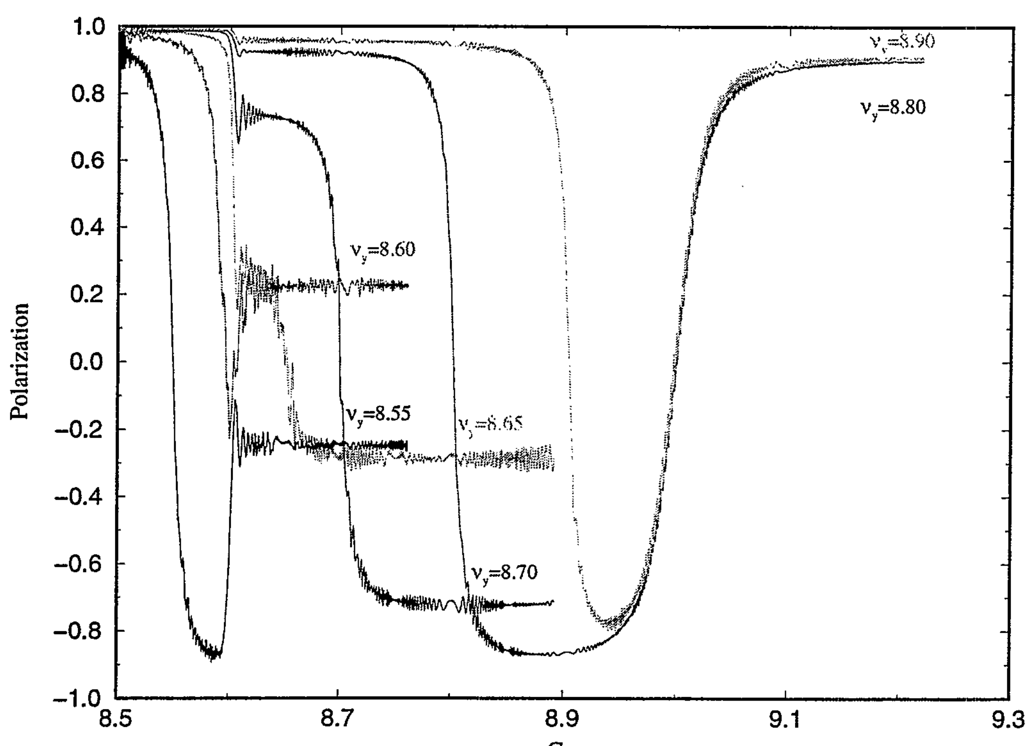

Fig. 8. The simulation results for $\nu_x = 8.60$. $\varepsilon_x = \varepsilon_y = 25\pi$, $\delta = 0$, $\alpha = 1.1 \times 10^{-5}$.

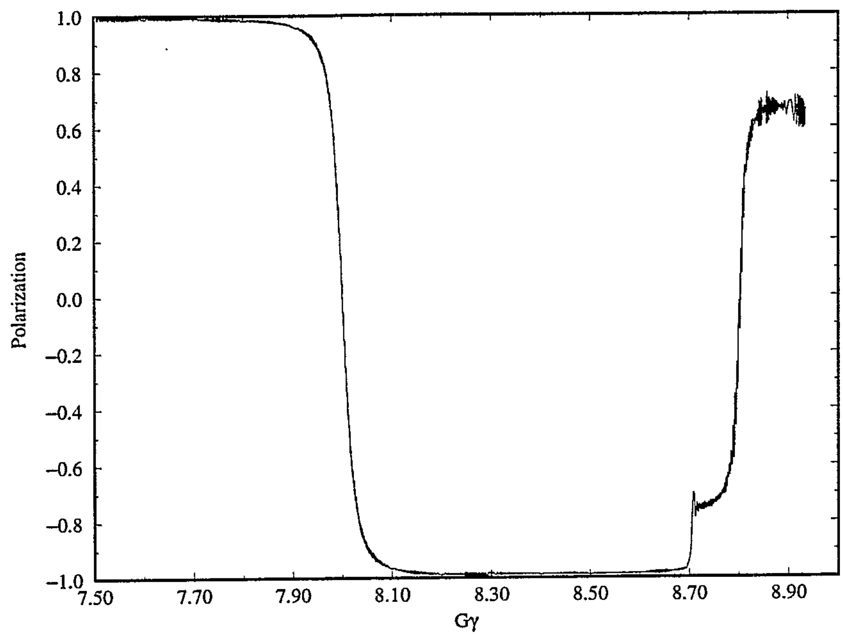

Fig. 9. The simulation results for $\nu_y = 8.80$ and $\nu_x = 7.70$. $\varepsilon_x = \varepsilon_y = 30\pi$, $\delta = 0$, $\alpha = 1.1 \times 10^{-5}$.

8.70 and the depolarization level is about the same as the one with $\nu_x = 8.70, \nu_y = 8.80$ (see Fig. 4). Because there is only one coupling element, the solenoidal partial snake, in the AGS, the coupling kick has all Fourier components $G\gamma = N \pm \nu_x$ and with the same amplitude. Whichever Fourier component is closest to the vertical betatron tune, it will pick up the strength of the intrinsic resonance and become the strongest coupling resonance. For example, when $\nu_y = 8.80$, if $\nu_x = 7.60$, the strongest coupling resonance is $G\gamma = 1 + \nu_x = 8.60$; if $\nu_x = 8.30$, the strongest coupling resonance is $G\gamma = 17 - \nu_x = 8.70$. So only the tune difference in the same half unit will affect the coupling resonance strength. Considering the stopbands at half integers and integers, the available tune separation is less than 0.5 unit.

5. Overcome the Coupling Resonances

As shown in the simulations, it is clear that the unexpected depolarization level after crossing the strong intrinsic resonances is due to the combined effect of the coupling resonance and synchrotron oscillation. The coupling resonance strength is inversely proportional to the betatron tune separation but separating the two tunes for more than 0.5 unit does not help. This coupling resonance makes preserving the polarization very challenging. The ratio of the coupling resonance strength to the intrinsic resonance strength with a 5% partial snake and 0.1 unit tune separation is about 0.06. With larger tune separation, say 0.3, the ratio can go down to 0.03 but not zero.

This ratio makes it impossible to cross them at one speed without losing po-

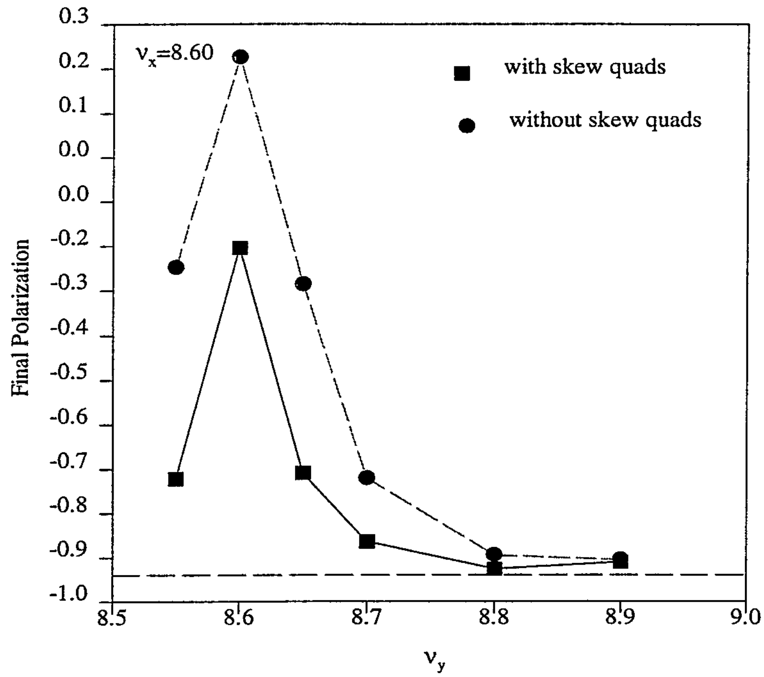

Fig. 10. The simulation with different horizontal tunes and fixed vertical tune $\nu_x = 8.60$. The solid round points are for skew quads off and the square points are for skew quads with 1.5A current. The dashed line is the polarization level without coupling resonances (at -0.94).

larization. With a 20π mm-mrad typical polarized beam in the AGS, the coupling resonances adjacent to those weak intrinsic resonances such as $24 - \nu_y, 24 + \nu_y$ and $48 - \nu_y$ will have little effect with the regular acceleration rate $\alpha = 4.5 \times 10^{-5}$. But the coupling resonances adjacent to the strong intrinsic resonances such as $0 + \nu_y, 12 + \nu_y, 36 - \nu_y$ and $36 + \nu_y$ will cause certain depolarization. Using a super slow crossing speed could full flip spin after the intrinsic resonance, but on the other hand, it also increase the spin flip of the coupling resonances which flip the spin the other way. Even the traditional tune-jump quadrupole method is challenged. The tune jump method changes the betatron tunes in less than one orbit turn to effectively make the resonance crossing speed very fast. If the two tunes are separated, the coupling resonance may be crossed by the beam at the normal crossing speed and causes polarization loss. Moreover, the tune jump may increase the beam emittance and increase the coupling resonance strength if the coupling resonance is after the intrinsic resonance. In short, the benefits of all these methods will be reduced or diminished without extra efforts to overcome the coupling resonances. Another method to overcome intrinsic resonances is to excite the coherent betatron motion to increase the spin flip efficiency. But in the presence of coupling resonance, the spin flip efficiency of the coupling resonance may also be increased if it is after the intrinsic resonance. To avoid that, ν_x can be chosen smaller than ν_y and coherent betatron motion is excited after crossing the coupling resonance. Also, the acceleration rate has to be maximized to minimize the depolarization at the coupling resonance.

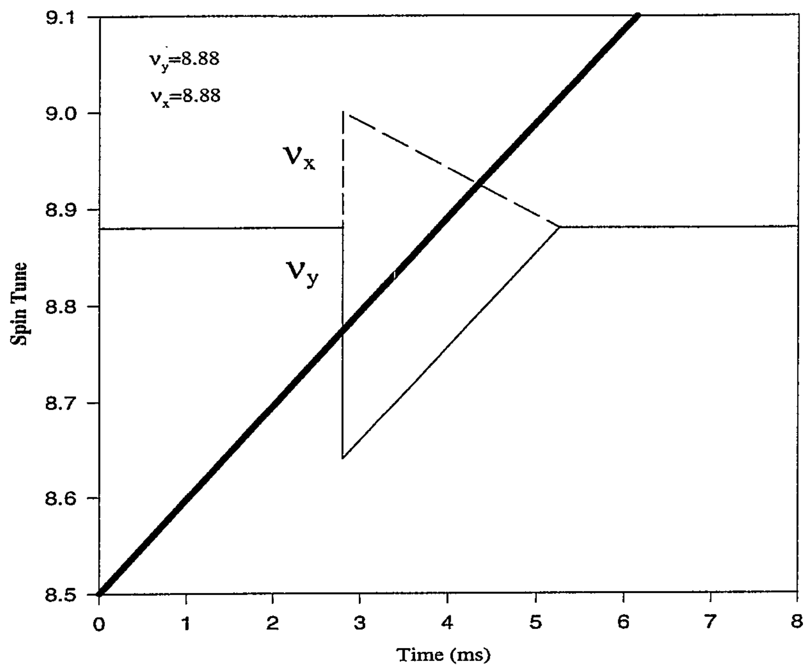

Fig. 11. The schematic plot of tune-jump method. The solid line is the resonance line of $0 + \nu_y$ and the dashed line is the resonance line of $0 + \nu_x$. The thick line is the spin tune which crosses $0 + \nu_y$ during the tune-jump (very fast) but crosses $0 + \nu_x$ with regular crossing speed.

The coupling resonances can be eliminated if the linear coupling can be compensated. Two sets of strong skew quadrupoles could compensate the linear coupling of the solenoidal partial snake globally and make the above intrinsic resonance correction methods work. Currently, the skew quadrupoles in the AGS are not strong enough to compensate the coupling of the solenoidal snake. Fig. 10 plots the simulations with and without skew quadrupoles. On the other hand, with a global decoupling, the orbit is flat globally but not locally, which means that at certain points the coupling will be non-zero and may still cause some problems. The another way is to eliminate only the coupling resonance closest to the intrinsic resonance instead of eliminating everyone. Some further simulation is needed to test the idea.

With tune-jump method, it is worthwhile to try to set the two betatron tunes equal and jump through them together. But since when ν_y jumps down, ν_x will jump the opposite way, the beam still cross the coupling resonance with normal crossing speed (see Fig. 11). Moreover, the coupling resonance strength with two tunes equal is much stronger than the one with separate tunes. Some experiments are needed to test if this is true.

Another prospective method is to cross the intrinsic resonance and the coupling resonances with different acceleration rates. In other words, the intrinsic resonances are crossed with tune-jump, super slow acceleration rate or coherent betatron motion excitation; while the coupling resonances are crossed using a fast radial jump at the resonance to generate an energy jump. The radial jump is accomplished by rapidly changing the beam circumference using the powerful AGS RF system. Since the beam has to be accelerated during the radial jump, this method is limited by the maximum RF voltage. For the AGS, the maximum acceleration rate is $\alpha \approx 8 \times 10^{-5}$, which is fast enough to cross most of the coupling resonances without polarization loss except the one associated with $36 + \nu_y$. While the coupling resonances are crossed by radial jump, all the three methods mentioned above can be used to overcome the intrinsic resonances. For the super slow acceleration rate, the effect of spin chromaticity (i.e., multiple crossing of depolarizing resonance due to the synchrotron oscillation) has to be taken care of.

6. Conclusion

As shown in the simulations, it is clear that the unexpected depolarization level after crossing the strong intrinsic resonances is due to the combined effect of the coupling resonance and synchrotron oscillation. The coupling resonance strength is inversely proportional to the betatron tune separation but separating the two tunes for more than 0.5 unit does not help. Several possible schemes to overcome the coupling resonances are discussed and simulated. Further simulation is needed for the tune-jump and radial jump methods.

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