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Optimization of Spin Angles from a Helix Field Map

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Spin Note

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OPTIMIZATION OF SPIN ANGLES FROM A HELIX FIELD MAP *

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1. Orbit and spin angles in Siberian snakes and spin rotators

Four-helix Siberian snakes for RHIC should be able to rotate the proton spin by 180^0. The precession axis should be in the horizontal plane, forming an angle of $\pm 45^0$ with the longitudinal z direction. The particle trajectory should be compensated, i.e. a proton entering the snake on axis at zero angle should emerge from the snake still on axis and at zero angle. So, many conditions have to be fulfilled at all energies of the beam between, say 25 and 300 GeV.

In a snake, we have seven quantities to optimize. 4, for the orbit, the horizontal and vertical displacement and angle. 3, for the spin, the spin flipping angle μ, and two angles defining the direction of the precession axis, the latitude θ between the axis and the horizontal plane, and the azimuth ϕ, formed by the projection of the axis on the horizontal plane and z. If there are no trim coils in the helical magnets, we can only count on two parameters to adjust seven quantities, i.e. the field in the outer and inner helix pair, B_1 and B_2.

The orbit in a 4-helix Siberian snake is in good shape, since the field components, including fringe fields, are naturally compensated if the helices are pairwise identical [1]. Then, in principle, we are only left with three quantities, the spin angles, and two parameters to correct them. The symmetry properties of a 4-helix structure make $\theta = 0$. So, with two values of the field and a perfect structure, we can always find good values of the angles (μ, θ, ϕ) close to the values $(180^0, 0^0, \pm 45^0)$.

For a 4-helix spin rotator the situation is worse, since the rotator orbit is not naturally compensated. However, symmetry considerations, if the helices are all identical and perfect, show that we can do the trick with 4 parameters, two field values and two injection angles.

In previous work we integrated the motion and spin equations with the code *Snig* [1], using for the field the Blewett and Chasman (BC) analytical expression [2], with a plausible, quasi Maxwell-ian fringe field. We could find good values of the spin angles. Now, we repeated the calculation using the map of a realistic magnetic field [3], made with a numerical 3-D magnetic code [4].

2. The field map

The field map used in the present calculation is for the slotted type helical magnet prototype presently being constructed at Brookhaven [5]. Because of saturation in the iron, high field maps and low field maps have been prepared. They are not identical but, since the differences are small, for the present work we have used only the high field map, that we will call OH. Snakes and rotators are made of 4 helices, all identical

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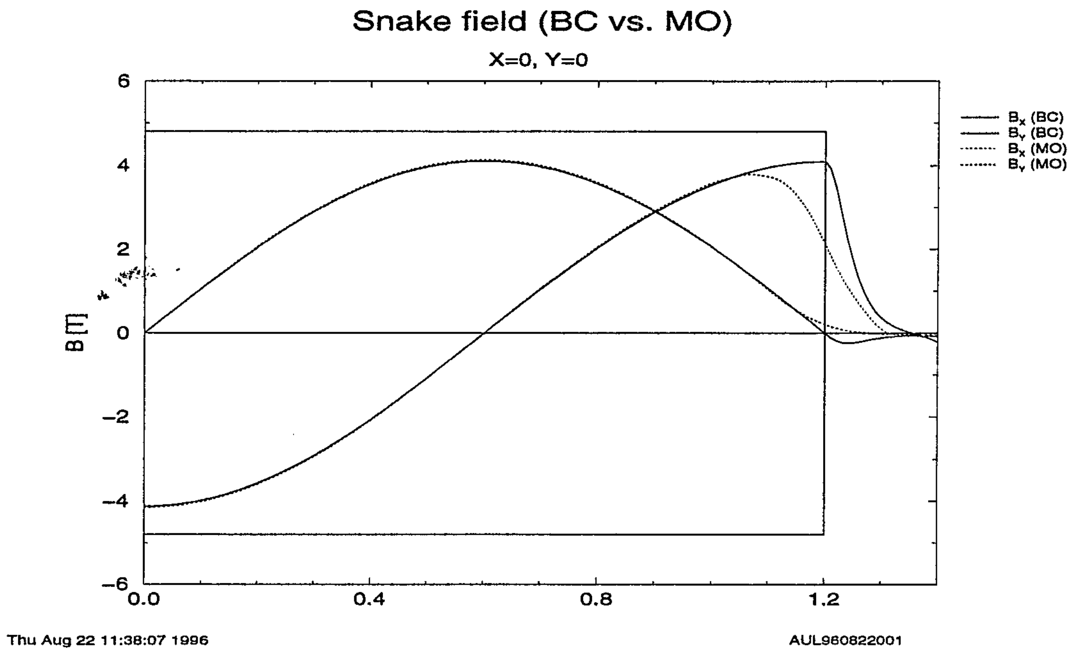

Figure 1: OH map field vs. BC field. On axis.

in length (2.4 m), and only differing for the right- or left- chirality. Map values were first normalized with respect to the maximum value on axis and then multiplied by the "nominal" field.

A comparison of the field in the map with the Blewett-Chasman field shows a significant difference near the end region of the helical magnet. As Fig. 1 shows, the field of the "real" OH magnet leaks out more than the BC. Consequently, the OH field integrals are smaller.

3. Integration and optimization in a field map

For the calculation of the orbit and of the spin motion we used *Snig*. The integrator in the code is almost symplectic, but since the field of the map is not perfectly Maxwell-ian anyway [6], the non perfect symplecticity is irrelevant. Moreover, in a single pass calculation the symplectic nature of the integration is not an issue.

3.1. Siberian snakes

The best angles were found by minimization of the function

$$f = \Delta_T^2 + \Delta_S^2$$

with the following orbit and spin terms

$$\Delta_T^2 = (x - x_0)^2 + (p_x - p_{x0})^2 + (y - y_0)^2 + (p_y - p_{y0})^2$$
$$\Delta_S^2 = (\mu - \pi)^2 + \theta^2 + (\phi \mp \pi/4)^2$$

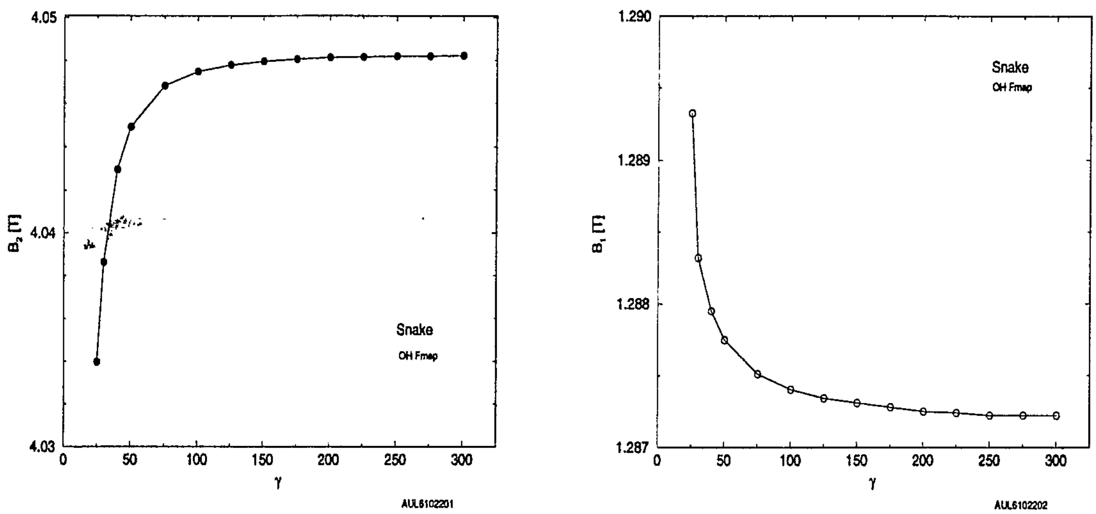

Figure 2: Snake field vs. proton energy for best angles. OH map.

The minimization routine is the "conjugate direction method", or Powell method of the Numerical Recipes [7]. The results are shown in Fig. 2. A similar optimization [8] using first order matrices based on the BC field [9] produced qualitatively similar results, shown in Fig. 3. The variation of the field vs. beam energy is due to the "1" in the numerator of

$$\omega = \frac{(1+G\gamma)B_0}{B\rho}$$

where B_0 is the nominal field. The field appears in the theoretical expressions for the spin matrix elements to first order only through the quantity ω. The field values that make $\omega = const$ are shown as a solid line in Fig. 3. Note that the optimized fields from field maps are $\approx 4\%$ higher than for BC, due to the faster fall-off of the field.

The optimized field values for a field map vs. energy are shown in Table I. Field values are in Tesla, angles in degrees. The table gives other parameters for the optimized snakes, for all energies, namely the maximum orbit excursion, and the orbit lengthening in the snake, in mm.

Fig. 4 shows the maximum orbit deformation, for a particle injected on axis, vs. γ. The figure shows also the orbit lengthening with respect to the device length.

Figs. 5, 6, and 7 show the magnetic field components, the orbit components and the spin components for the snake at $\gamma = 25$, respectively.

3.2. Spin rotators

Spin rotators compared to snakes have the additional problem that the orbit is not naturally compensated. A solution was given for a BC rotator by reducing the angle of rotation of the helix from the nominal value of 360^0 to 345^0 [1]. The structure represented by the OH map was designed to produce an "equivalent" 340^0 rotation as well as possible. We expected that injection angles different from zero were needed.

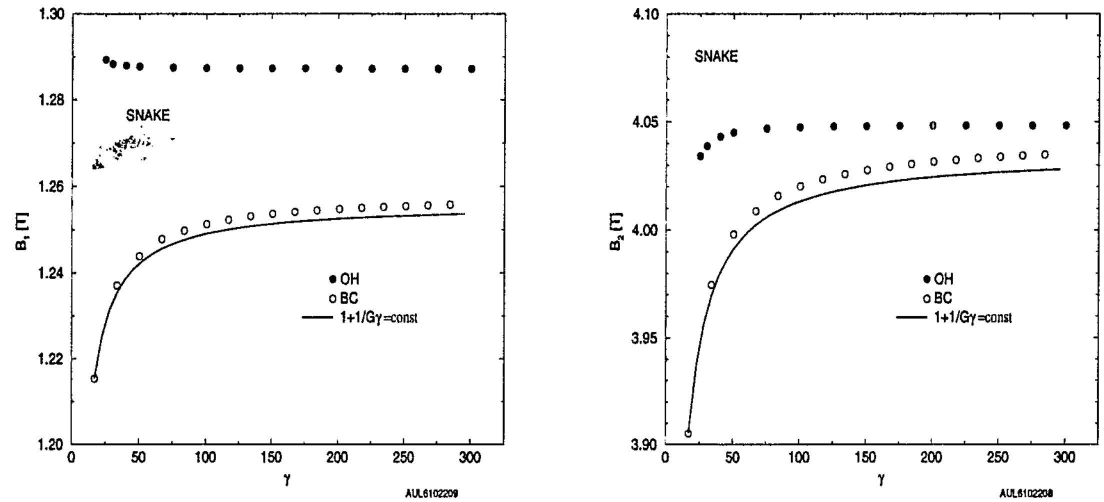

Figure 3: Snake field for best angles. OH fieldmap vs. Blewett/Chasman field.

Table I: Optimized parameters for snake based on a OH map.

γ	B_1	B_2	x_{max}	y_{max}	δL
25	1.28932	4.03397	15.430	32.495	2.065
30	1.28832	4.03864	12.749	27.078	1.435
40	1.28795	4.04294	9.489	20.299	0.808
50	1.28775	4.04491	7.562	16.237	0.517
75	1.28751	4.04683	5.034	10.824	0.230
100	1.28740	4.04749	3.782	8.117	0.129
125	1.28734	4.04780	3.028	6.493	0.083
150	1.28731	4.04796	2.524	5.411	0.058
175	1.28728	4.04806	2.164	4.637	0.042
200	1.28725	4.04814	1.894	4.058	0.032
225	1.28724	4.04817	1.684	3.607	0.026
250	1.28722	4.04820	1.515	3.246	0.021
275	1.28722	4.04822	1.378	2.951	0.017
300	1.28722	4.04824	1.263	2.705	0.014

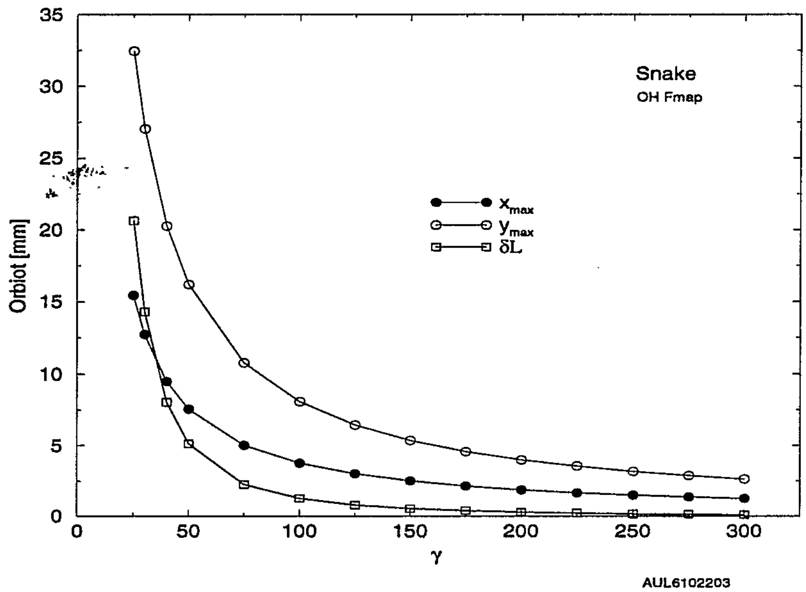

Figure 4: Snake orbit maxima and orbit lengthening ×10. OH fieldmap.

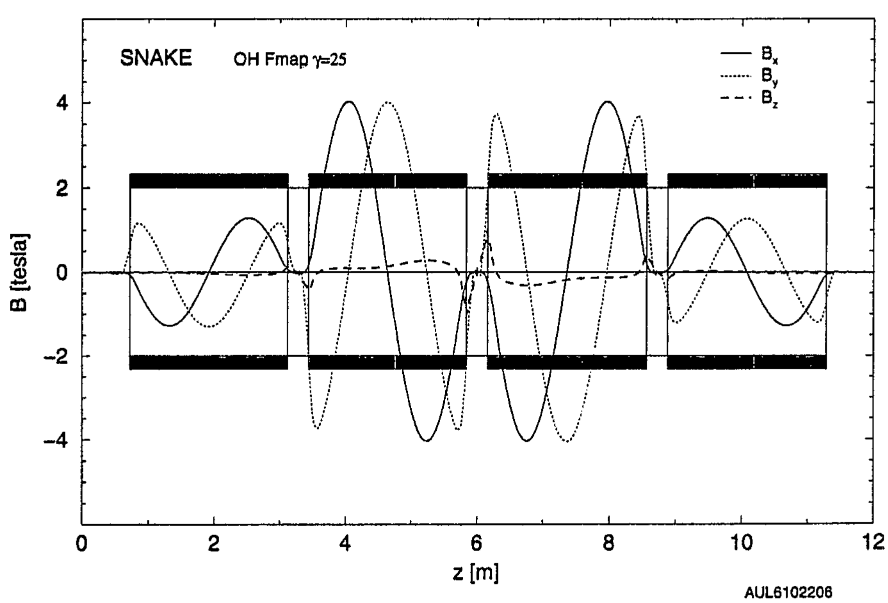

Figure 5: Snake magnetic field components. $\gamma = 25$. OH fieldmap.

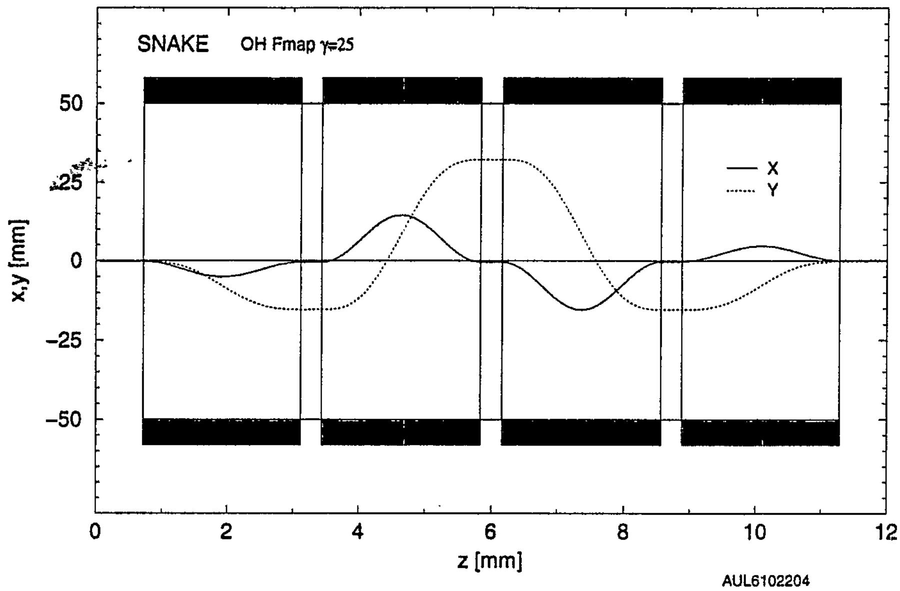

Figure 6: Snake orbit components. $\gamma = 25$. OH fieldmap.

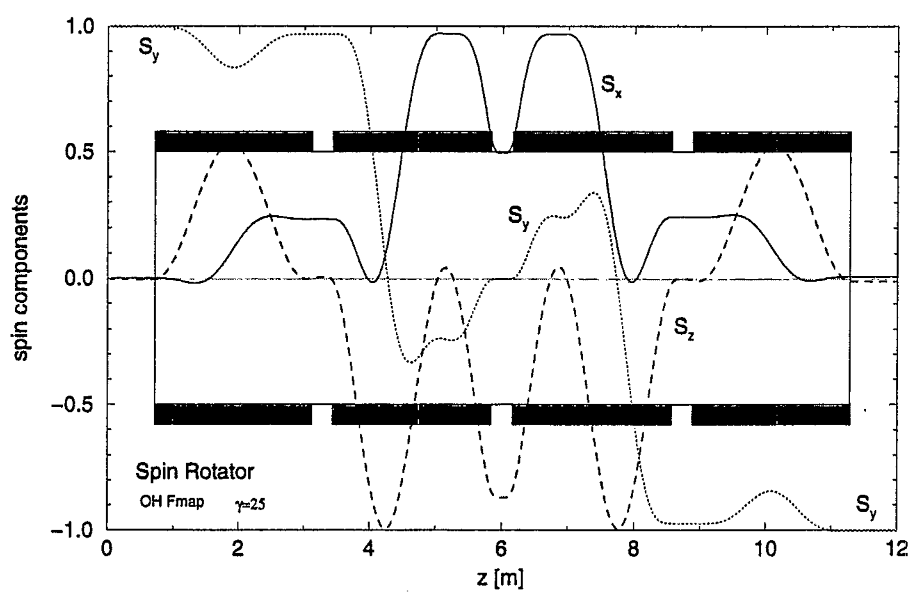

Figure 7: Snake spin components. $\gamma = 25$. OH fieldmap.

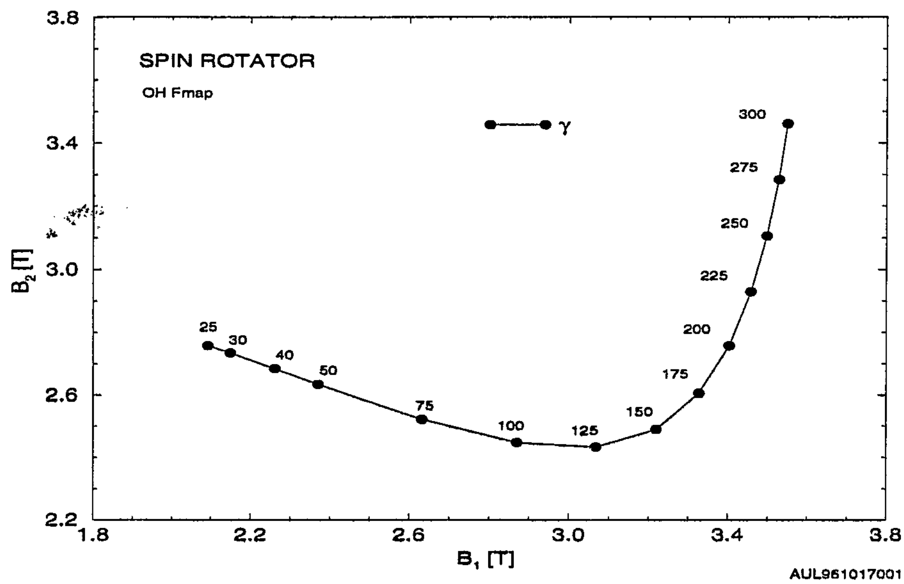

Figure 8: Rotator field vs. proton energy for best angles. OH map.

The spin flip angle for a rotator is $\mu = 90^0$. The final spin orientation should be in the horizontal plane at an angle with z strongly depending on energy, because the section of RHIC where the rotators are inserted is at some angle $\phi^* = 3.674\ mrad$ with the direction of the adjacent interaction straight [10]. Accordingly, the angle for the spin direction at rotator's exit should be $G\gamma\phi^*$.

Figs. 8 and 9 show the field needed to flip the spin from the vertical to the horizontal, with the appropriate angle $G\gamma\phi^*$. The field is on the average 5% higher than for the corresponding spin rotator calculated from the BC model [1]. The optimization was done as for the snakes, by minimization of some quadratic expression. Table II shows the values of the field in the rotator helices, the value of $G\gamma\phi^*$, in degrees for each beam energy, and the required injection angles to produce a balanced orbit. The table shows also the maximum orbit excursion and the orbit lengthening, in mm.

Fig. 10 shows the maximum orbit distortion vs. γ for a spin rotator. The same figure shows also the orbit lengthening. Fig. 11 shows the injection angle needed to produce a balanced orbit, due to the non perfect compensation of the field integrals. The small non-zero horizontal injection angle is due to map errors. Fig. 12 shows the (absolute) field integral along the orbit needed to obtain the appropriate values of the angles μ and $G\gamma\phi^*$ at all energies. Because of the variation of the latter angle, the transverse field integrals increase with beam energy. The integral of the longitudinal component of the field decreases with the energy, since at higher energies the beam feels less and less a longitudinal field (this field is zero on axis).

The spin rotator field components at $\gamma = 250$ and the spin components at two energies are shown in Figs. 13, 14, and 15, respectively. Note the final values of the x vs. z components of the spin at $\gamma = 25$ vs. $\gamma = 250$ needed to produce the required

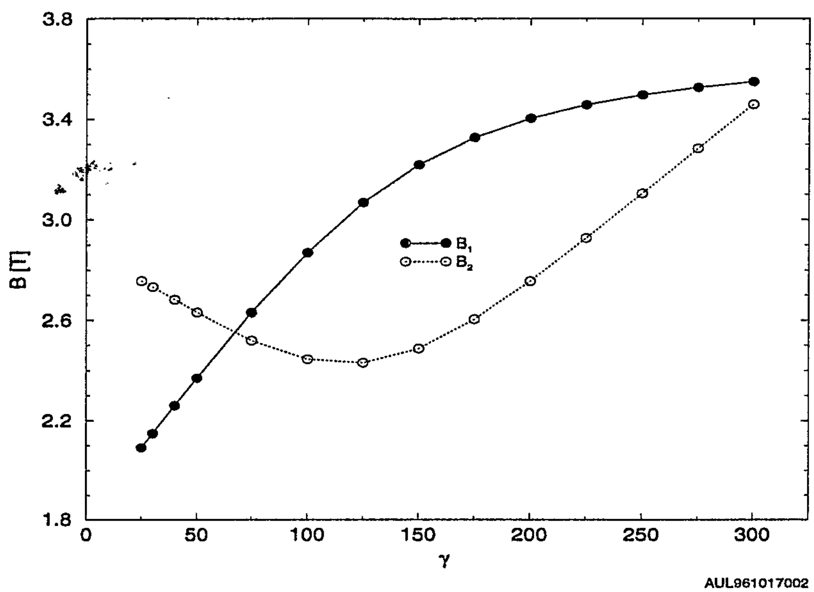

Figure 9: Rotator field vs. proton energy for best angles. Same data as in figure 8. OH map.

Table II: Optimized parameters for spin rotator based on a OH map.

γ	$G\gamma\phi^*$	B_1	B_2	x_{max}	y_{max}	δL
25	9.435	2.09136	2.75730	24.598	9.677	1.376
30	11.322	2.14865	2.73350	21.059	8.018	0.964
40	15.096	2.26009	2.68289	16.613	5.912	0.552
50	18.870	2.36950	2.63263	13.932	4.643	0.360
75	28.304	2.63169	2.52116	10.316	3.185	0.169
100	37.739	2.86892	2.44692	8.434	2.611	0.102
125	47.174	3.06702	2.43263	7.213	2.236	0.070
150	56.609	3.21818	2.48920	6.307	1.956	0.053
175	66.044	3.32668	2.60530	5.589	1.733	0.042
200	75.479	3.40319	2.75782	5.003	1.551	0.034
225	84.913	3.45782	2.92831	4.518	1.400	0.029
250	94.348	3.49755	3.10555	4.109	1.479	0.025
275	103.783	3.52719	3.28370	3.767	1.360	0.022
300	113.218	3.54935	3.46052	3.475	1.258	0.020

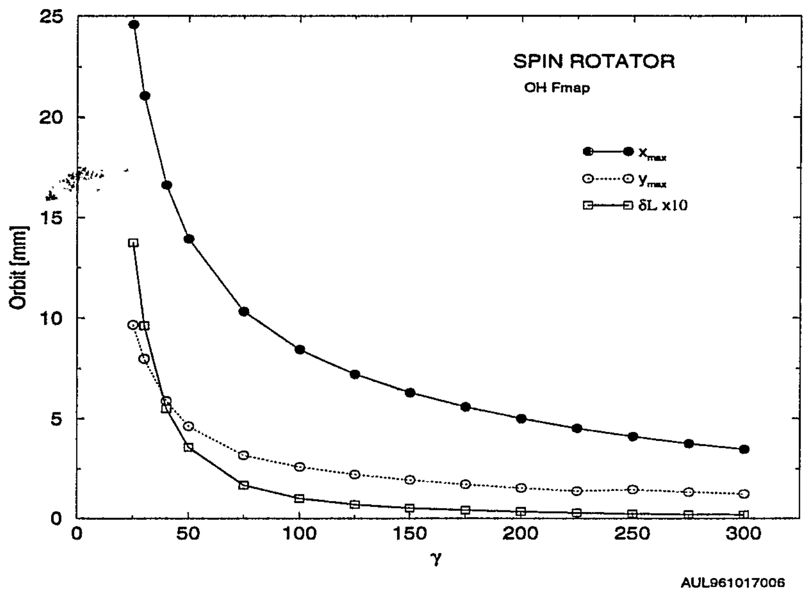

Figure 10: Rotator maximum orbit and lengthening for best angles. OH map.

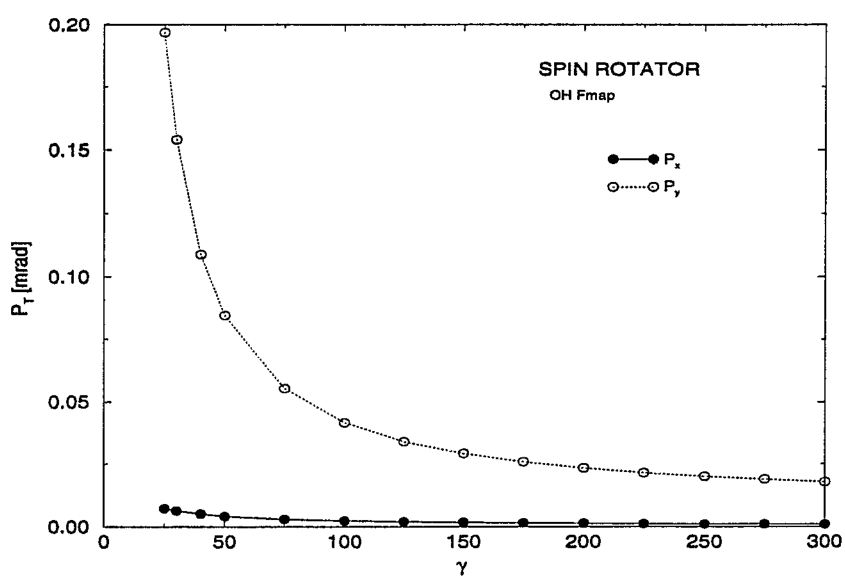

Figure 11: Rotator injection orbit angles. OH map.

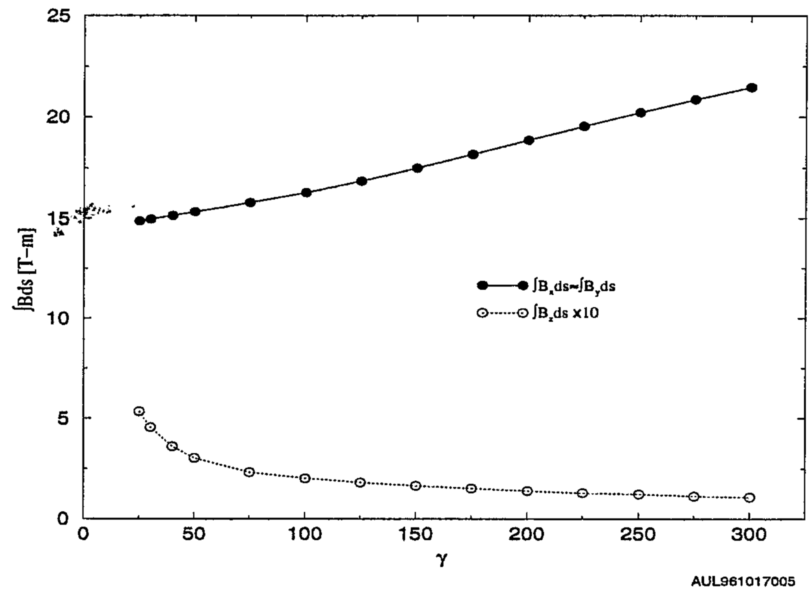

Figure 12: Rotator field integrals. OH map.

$G\gamma\phi^*$ values.

4. Conclusion

The field integrals for a 4-helix snake or spin rotator calculated from a field map (slotted type construction) are smaller than for a Blewett-Chasman model with the same nominal field. Consequently, to obtain the appropriate spin flipping angles and axis angles, the field should be slightly raised (up to 4-5 %). These values are well within the tolerance margin of the magnet prototypes currently being fabricated at Brookhaven.

For a rotator, the field integrals of the map calculated along the orbit are not exactly compensated. This may require some trim coil capability in order to inject particles parallel to the rotator axis.

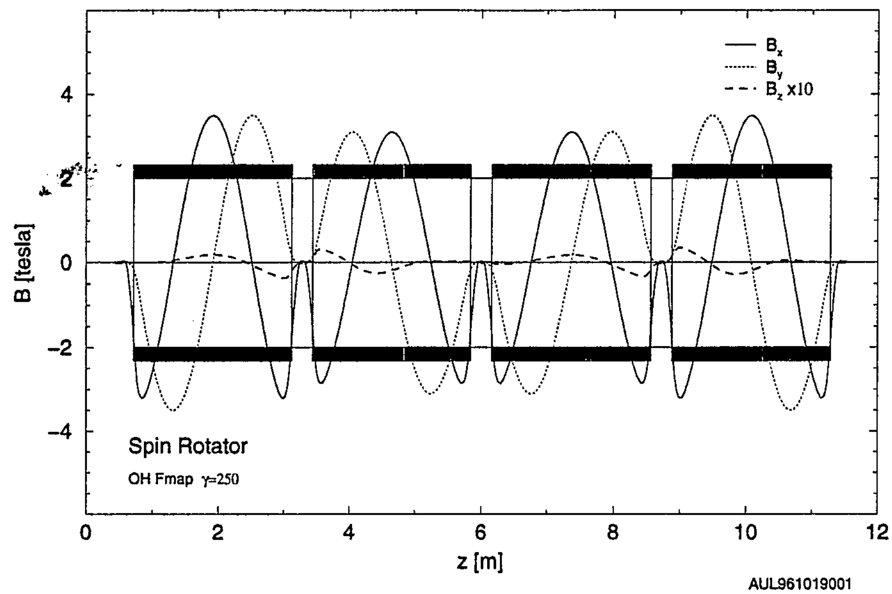

Figure 13: Rotator field at $\gamma = 250$. OH map.

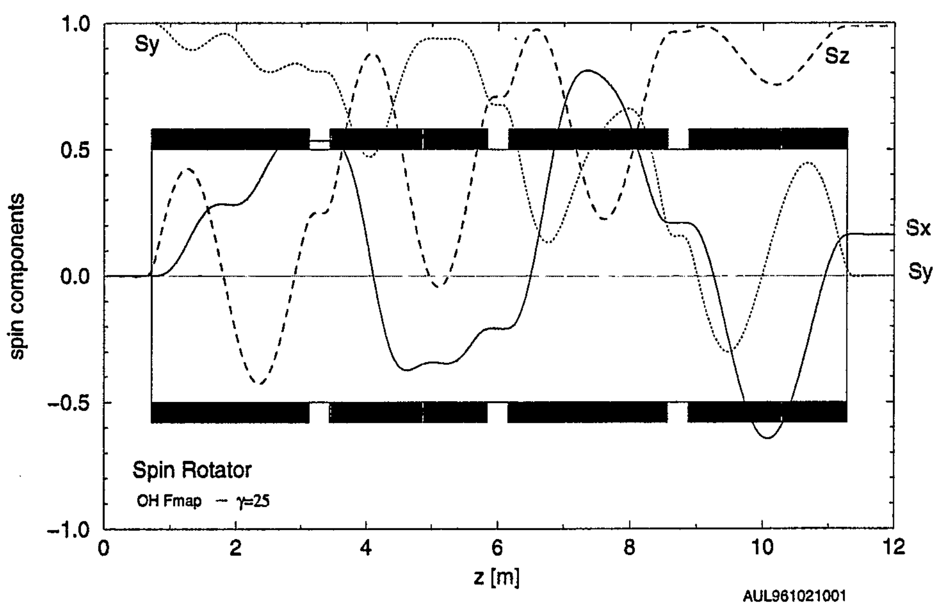

Figure 14: Rotator spin components at $\gamma = 25$. OH map.

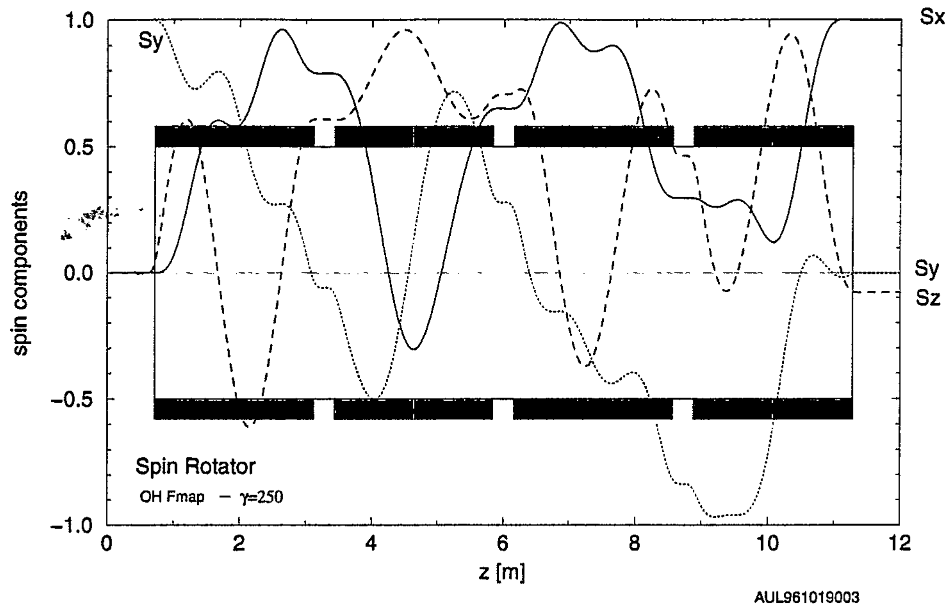

Figure 15: Rotator spin components at $\gamma = 250$. OH map.

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