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Magnetic Field Analysis of Helical Magnets

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Spin Note

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Three dimensional (3D) magnetic field calculated by the computer code TOSCA was analyzed including the fringing field region. The magnetic field in the median plane was well simulated by a simple function. Off median plane, contributions from the coils should be taken into account.

1 Introduction

Beam trajectory and spin motion are numerically calculated, if the magnetic field are given along the beam path. However it is not practical to store whole thefield strengths along the helical snakes. Measured data have usually some errors and they may not satisfy Maxwell equations. It is necessary to get analytic expressions of the magnetic field not only to reduce the storage area of calculated/measured data but also to keep the accuracy of transfer matrix of spin and orbit.

2 Multipole expansion of the magnetic field

2.1 Cartesian coordinate system

In a current free region in vacuum where the electric field $\vec{E}$ is constant, the magnetic field $\vec{B}$ can be derived from a scalar potential Ψ as

$$\vec{B} = -\nabla\Psi \tag{1}$$

In a Cartesian coordinate system, the scalar potential is expanded in power series of x and y coordinates.

$$\Psi = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{m,\ n}(z)\frac{x^n}{n!}\frac{y^m}{m!} \tag{2}$$

where z is a coordinate of axis. If the magnetic field has a median plane symmetry, Ψ is an odd function of y; i.e., $m = odd$ [1]. The Laplace equation, $\Delta\Psi = 0$, gives the following recurrent relation between coefficients:

$$A_{m,\ n+2}(z) + A_{m+2,\ n}(z) + A^{(2)}_{m,\ n}(z) = 0 \tag{3}$$

Coefficients, $A_{0,\ n}(z)$ and $A_{1,\ n}(z)$ are obtained as functions of z, from the expansion of $B_x(x,0,z)$ and $B_y(x,0,z)$ in the median plane in series of x at each z-position.

Therefor, all the coefficients are determined from the analysis of measured/calculated magnetic field only in the median plane.

$$A_{0,\ n} = -(\frac{\partial^n B_x}{\partial x^n})_{x=y=0} \tag{4}$$

$$A_{1,\ n} = -(\frac{\partial^n B_y}{\partial x^n})_{x=y=0} \tag{5}$$

$$A^{(k)}_{m,\ n} = \frac{d^k A_{m,n}}{dz^k} \tag{6}$$

With these coefficients, $B_x(x,y,z)$, $B_y(x,y,z)$ and $B_z(x,y,z)$ are calculated off median plane as below.

$$B_x = -\frac{\partial \Psi}{\partial x} = -\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{m,\ n+1}(z)\frac{x^n}{n!}\frac{y^m}{m!} \tag{7}$$

$$B_y = -\frac{\partial \Psi}{\partial y} = -\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{m+1,\ n}(z)\frac{x^n}{n!}\frac{y^m}{m!} \tag{8}$$

$$B_z = -\frac{\partial \Psi}{\partial z} = -\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A'_{m,\ n}(z)\frac{x^n}{n!}\frac{y^m}{m!} \tag{9}$$

2.2 Cylindrical coordinate system

In the central region of a helical dipole magnet or for a helical magnet of infinit length, the magnetic field has a cylindrical symmetry and a solution of the Laplace equation $\Delta\Psi = 0$ is given by [2],

$$\Psi = -B_0\{\sum_{n=0}^{\infty} \frac{2^{n+1}(n+1)!}{(n+1)^{n+2}} \frac{1}{r_0^n k^{n+1}} I_{n+1}((n+1)kr)$$

$$\times [\tilde{a}_n \cos((n+1)\tilde{\theta}) + \tilde{b}_n \sin((n+1)\tilde{\theta})]\}, \tag{10}$$

where I_n are modified Bessel functions and $\tilde{\theta}$ is defined as

$$\tilde{\theta} = \theta - kz. \tag{11}$$

where $k = 2\pi/\lambda$ and λ is the wave length of the helix. Modified Bessel functions are expanded in the form of the ascending series of r,

$$I_{n+1}((n+1)kr) = \sum_{j=0}^{\infty} \frac{1}{j!(n+j+1)!}(\frac{(n+1)kr}{2})^{2j+n+1}. \tag{12}$$

Now, the magnetic field can be computed as

$$B_r = B_0\{\sum_{n=0}^{\infty} \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \frac{1}{r_0^n k^n} I'_{n+1}((n+1)kr)$$

$$\times [\tilde{a}_n \cos((n+1)\tilde{\theta}) + \tilde{b}_n \sin((n+1)\tilde{\theta})]\}, \tag{13}$$

$$B_\theta = B_0\{\sum_{n=0}^{\infty} \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \frac{1}{r_0^n k^n} \frac{1}{kr} I_{n+1}((n+1)kr)$$

$$\times [\tilde{b}_n \cos((n+1)\tilde{\theta}) - \tilde{a}_n \sin((n+1)\tilde{\theta})]\}, \tag{14}$$

$$B_z = -krB_\theta, \tag{15}$$

$$B_x = B_r \cos(\theta) - B_\theta \sin(\theta), \tag{16}$$

$$B_y = B_r \sin(\theta) + B_\theta \cos(\theta). \tag{17}$$

Multipole coefficients, $\tilde{a}_n$ and $\tilde{b}_n$ are related with a_n and b_n in a two-dimensional approximation (a straight magnet) [3],

$$a_n = \tilde{a}_n \cos((n+1)kz) - \tilde{b}_n \sin((n+1)kz), \tag{18}$$

$$b_n = \tilde{a}_n \sin((n+1)kz) + \tilde{b}_n \cos((n+1)kz). \tag{19}$$

If we expand B_y in series of r at $z = 0$, normal components are

$$B_y/B_0 = [1 + \frac{1}{4}(kr)^2 + (\text{higher order of } r^n)] \cdot b_0$$

$$+\{[b_2 - \frac{1}{8}(kr_0)^2 \cdot b_0](\frac{r}{r_0})^2 + (\text{higher order of } r^n)\} \cos(2\theta) + \{\text{higher multipoles}\} \tag{20}$$

Helical dipoles have sextupole components originating from the structure [4]. When the magnet length, $\lambda = 240\ cm$ and the reference radius, $r_0 = 3.5\ cm$, then the sextupole contribution from the dipole field becomes

$$\frac{1}{8}(kr_0)^2 \simeq 1 \times 10^{-3}. \tag{21}$$

3 Numerical analysis

3.1 Fitting procedure

In order to analyze the magnetic field calculated by Dr. Okamura [5], the scalar potential was assumed to be expanded as

$$\Psi = -B_0 \sum_{n=0}^{\infty} (\frac{1}{r_0})^n \{[\sum_{j=0}^{\infty} \frac{r^{n+1}}{n+2j+1} (\frac{r}{r_0})^{2j} \cdot a_{n,\ j}(z)] \cos((n+1)\theta)$$

$$+ [\sum_{j=0}^{\infty} \frac{r^{n+1}}{n+2j+1} (\frac{r}{r_0})^{2j} \cdot b_{n,\ j}(z)] \sin((n+1)\theta)\}. \tag{22}$$

Here, $a_{n,\ j}(z)$ and $b_{n,\ j}(z)$ are functions of z, and they are $\cos((n+1)kz)$ - or $\sin((n+1)kz)$ - like functions for an infinitely long magnet. The order of magnitude of their derivatives are estimated as

$$a_{n,\ j}^{(2m)}(z) = \frac{d^{2m}}{dz^{2m}} a_{n,\ j}(z) = o(((n+1)k)^{2m}) \cdot a_{n,\ j}$$

$$\sim o(10^{-3m}(n+1)^{2m}) \cdot a_{n,\ j}. \tag{23}$$

From the Laplace equation, $\Delta\Psi = 0$, $a_{n,\ j}(z)$ and $a_{n,\ j}(z)$ satisfy following relations,

$$a_{n,\ j+1} = -\frac{n+2j+3}{4(j+1)(n+j+2)} \cdot \frac{1}{n+2j+1} \cdot a^{(2)}_{n,\ j}, \tag{24}$$

$$b_{n,\ j+1} = -\frac{n+2j+3}{4(j+1)(n+j+2)} \cdot \frac{1}{n+2j+1} \cdot b^{(2)}_{n,\ j}. \tag{25}$$

Coefficients $a_{n,\ j}(z)$ and $b_{n,\ j}(z)$ were determined by fitting the radial field B_r and B_θ in the median plane ($\theta = 0$ or π), respectively. In the fitting procedure, the maximum value of n was taken 6 and odd n values were also included. The j's were taken to be zero except for $n = 0$ (dipole component). Only the second derivatives of $a_{0,\ 0}$ and $b_{0,\ 0}$ were taken into account and higher order derivatives were negrected. This assumption is reasonable from the previous estimation of the order of magnitudes for coefficient $a_{n,\ j}$ and $b_{n,\ j}$ in (23). Following expansion was used in the fitting procedure,

$$B_r = -\frac{\partial\Psi}{\partial r} = B_0\{[a_{00} + a_{01}(\frac{r}{r_0})^2]\cos(\theta) + [b_{00} + b_{01}(\frac{r}{r_0})^2]\sin(\theta)$$

$$+ \sum_{n=1}^{6}(\frac{r}{r_0})^n[a_{n0}\cos((n+1)\theta) + b_{n0}\sin((n+1)\theta)]\}, \tag{26}$$

$$B_\theta = -\frac{\partial\Psi}{\partial\theta} = B_0\{[b_{00} + \frac{1}{3}b_{01}(\frac{r}{r_0})^2]\cos(\theta) - [a_{00} + \frac{1}{3}a_{01}(\frac{r}{r_0})^2]\sin(\theta)$$

$$+ \sum_{n=1}^{6}(\frac{r}{r_0})^n[b_{n0}\cos((n+1)\theta) - a_{n0}\sin((n+1)\theta)]\}. \tag{27}$$

The longitudinal field B_z was computed with coefficients obtanted above,

$$B_z = -\frac{\partial\Psi}{\partial z} = B_0 \cdot r\{[a'_{00} + \frac{1}{3}a'_{01}(\frac{r}{r_0})^2]\cos(\theta) + [b'_{00} + \frac{1}{3}b'_{01}(\frac{r}{r_0})^2]\sin(\theta)$$

$$+ \sum_{n=1}^{6}\frac{1}{n+1}(\frac{r}{r_0})^n[a'_{n0}\cos((n+1)\theta) + b'_{n0}\sin((n+1)\theta)]\}. \tag{28}$$

3.2 Results

At each z-position, coefficients $a_{n0}(z)$ and $b_{n0}(z)$ were determined by fitting the 3D-calculated field B_r and B_θ in the median plane ($y = 0$) with equations (26) and (27). The 3D-field was calculated from x, y = -35 mm to 35 mm in 5 mm step including the iron core [5]. So, 14 coefficients were determined by fitting the calculated fields at 30 points with the Simplex minimizing procedure. Figure 1 shows a comparison of the 3D-field and that fitted with equations (26) and (27). In the median plane, the 3D-field B_x and B_y are well approximated including the fringing field region by the function assuming a cylindrical symmetry. The longitudinal field B_z was "predicted" with the equation (28) and it agrees with the 3D-field very well.

Off the median plane, for example along the line $x = 0$ and $y = 3\ cm$, relatively large differences are observed between 3D-results and our approximation, especially at the magnet edge. The vector potential $\vec{A}$ by a current $d\vec{I}$ parallel to the x-axis at $y = y_0$ and $z = z_0$ is given by

$$\vec{A} = \mu_0 \frac{d\vec{I}}{4\pi r} = \mu_0 \frac{d\vec{I}}{4\pi\sqrt{(y-y_0)^2 + (z-z_0)^2}} \tag{29}$$

The magnetic field from this vector potential is calculated as

$$\vec{B} = rot\vec{A} \propto (0, \frac{z-z_0}{[(y-y_0)^2 + (z-z_0)^2]^{\frac{3}{2}}}, \frac{y_0-y}{[(y-y_0)^2 + (z-z_0)^2]^{\frac{3}{2}}}) \tag{30}$$

As shown in Fig. 3, the fitting errors observed for the B_y field at the magnet edge are well described by the sum of three terms with the functional form (30) corresponding to a magnetic field generated by a line current parallel to the x-axis. Fitting errors for the B_z field at the edge is also described by the functional form derived from three line currents at the same positions, if the these currents are reduced by a factor of four. At the edge of a helical magnet, there are currents not only parallel to the x-axis but also those parallel to the z-axis. The latter cause additional B_y field but do not have B_z component. Altough the field (30) overestimates the contribution for B_z due to such a simplification, it shows that we have to take account of the effects of the coil at the end of a magnet. Inside the magnet, on the other hand, magnetic field is well described by a simple function of cylindrical symmetry including the fringing field region.

4 Summary

Calculated 3D-field was analyzed to obtain the analytic expressins satisfying Maxwell equations. In the median plane, the field B_x and B_y were well expanded with simple functons of a cylindrical symmetry including the fringing field region. The longitudinal field B_z was also well "predicted" with parameters obtaind by fitting B_x and B_y. Off median plane, on the other hand, effects of coils parallel to the x-axis were required to explain the difference between 3D-field and the "predicted" field from the analysis. Present procedure can be applied to analyze measured magnetic fields for helical magnets.

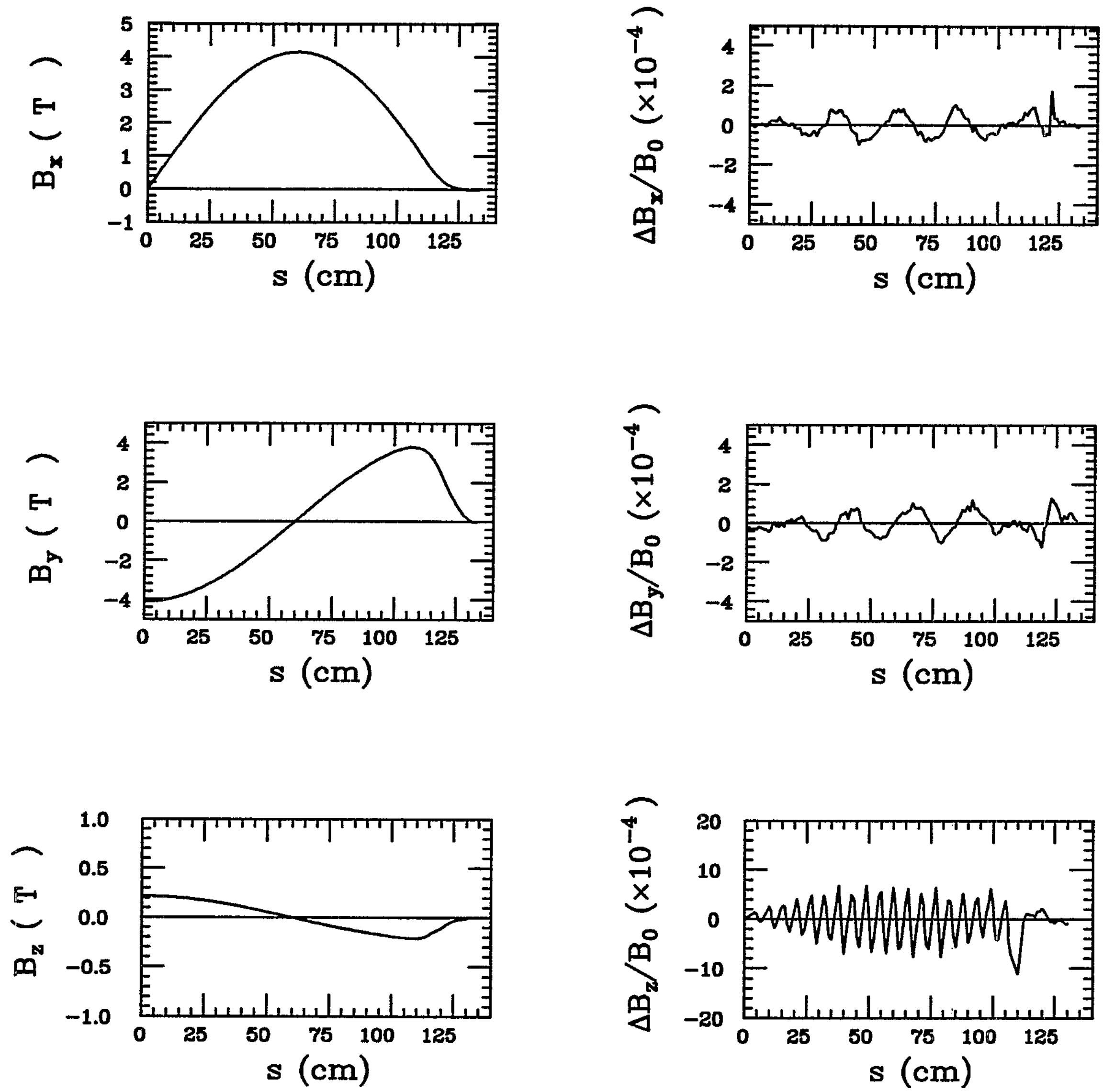

Fig. 1. Left figures show the calculated 3D-field (solid curves) and the "predicted" field (dashed curves) by equations (26)-(28) in the median plane, $x = 3$ *cm* and $y = 0$ *cm*. Right figures shows their difference divided by B_0. The magnet center is at $z = 0$ *cm* and the end is at $z = 120$ *cm*.

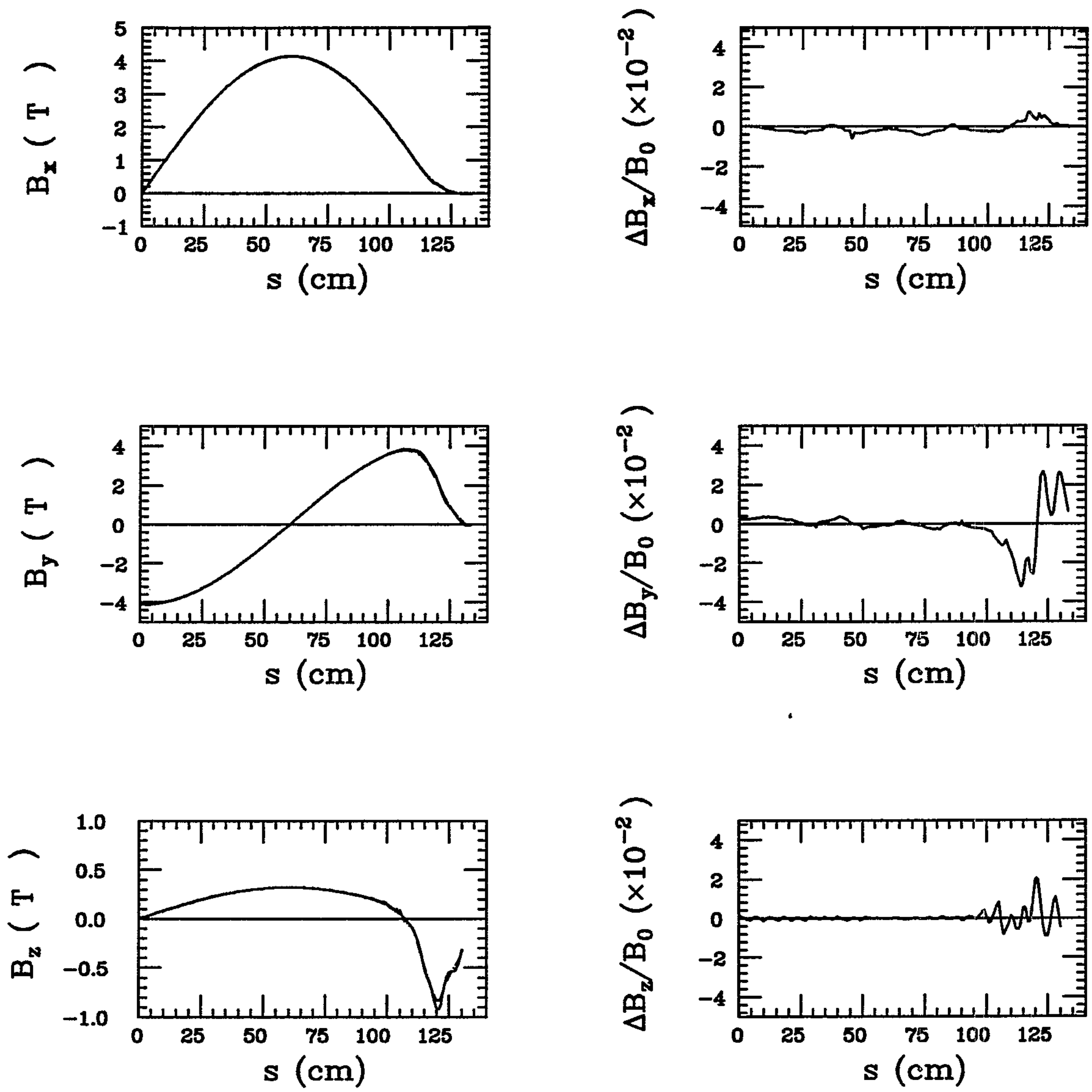

Fig. 2. Same as Fig. 1, but off the median plane, $x = 0\ cm$ and $y = 3\ cm$.

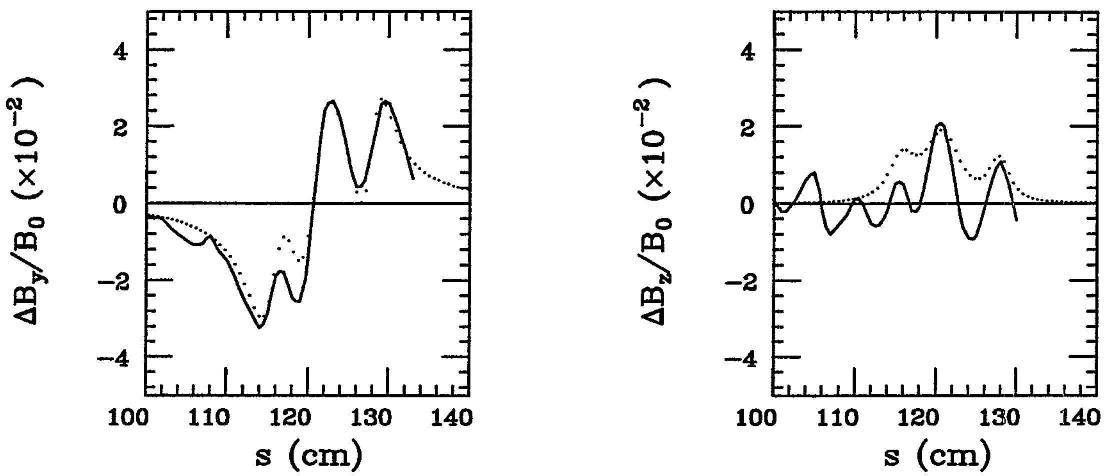

Fig. 3. Dotted curves show the simulation of the magnetic field generated by three line currents parallel to the x-axis. Their positions and strengths are optimized to fit the B_y field. The corresponding B_z field is reduced by a factor of four, although the current positions are same.

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