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# Coupling effect on the proton optics from the electron lenses

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In this note we calculate the effect of the electron lens solenoids on the proton optics. Electron lenses (e-lenses) are to be used for head-on beam-beam compensation in the Relativistic Heavy Ion Collider (RHIC).

## 1 Introduction

Electron lenses are to be used for head-on beam-beam compensation in the polarized proton (pp) runs to compensate the large tune spread generated by the head-on proton-proton beam-beam interactions at IP6 and IP8 in the Relativistic Heavy Ion Collider (RHIC) [1]. The main part of an electron lens is a superconducting solenoid with a longitudinal magnetic field up to 6 T.

In the current design of RHIC head-on beam-beam compensation, there are two e-lenses, one for the Blue ring called BEL and one for the Yellow ring called YEL. The effective length of these e-lens solenoids is 2.0 m. They are located 1.5 m away from IP10. The actual operating solenoid field may range from 3 T to 6 T. Figure 1 shows the layout of RHIC head-on beam-beam compensation at 250 GeV. Figure 2 shows the locations of e-lenses around IP10.

As we know, solenoids will introduce betatron coupling into the proton linear optics. It will couple the horizontal orbit into the vertical plane. If there is horizontal dispersion in the solenoid, it will generate dispersion in the vertical plane. Beside the local coupling effect, the solenoid also will increase the eigen tune split. To cancel the effects of the two solenoids on the betatron coupling, in the current design of RHIC head-on beam-beam compensation, the two solenoids will be powered to have opposite magnetic fields.

In the following, we will estimate the e-lenses' effects on the  $\beta$  and dispersion functions with 100 GeV and 250 GeV pp run lattices. Table 1 lists some lattice and beam parameters to be used in the following study.

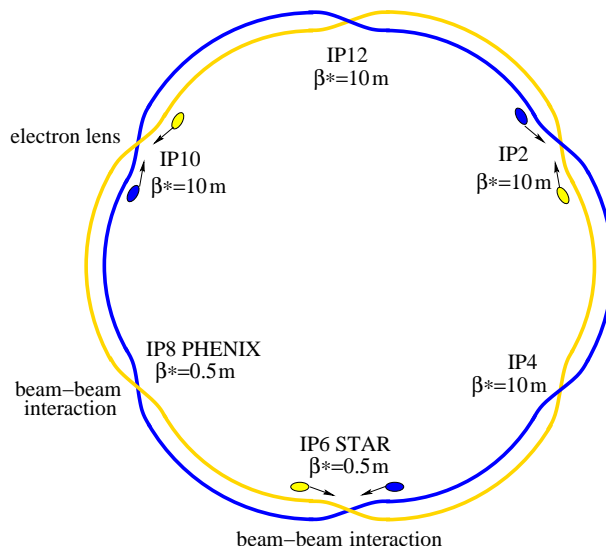


Figure 1: Layout of RHIC head-on beam-beam compensation.

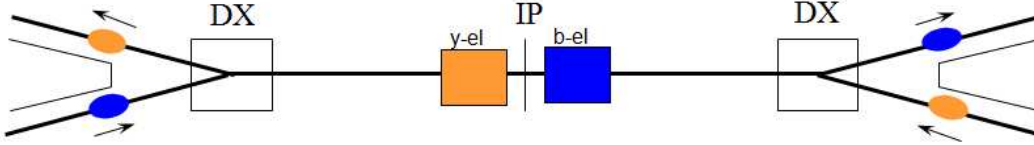


Figure 2: RHC e-lenses around IP10.

Table 1: Lattice and beam parameters used in this study

| parameter  | 100 GeV lattice | 250 GeV lattice |
|--|-----------------|-----------------|
| proton relativistic $\gamma$                         | 106             | 266             |
| $\beta_{x,y}^*$ at IP6 and IP8                       | 0.8 m           | 0.5 m           |
| $\beta_{x,y}^e$ at e-lens                            | 10 m            | 10 m            |
| normalized transverse rms emittance $\epsilon_{x,y}$ | 2.5 mm.mrad     |                 |
| non-colliding transverse tunes                       | (28.685,29.695) |                 |
| linear chromaticities                                | (1,1)           |                 |
| proton transverse rms beam size at IP10              | 0.48 mm         | 0.30 mm         |

## 2 Effects on $\beta$ function and coupling parameter $r$

First we calculate the effects of the two solenoids on the  $\beta$  function and coupling parameter  $r$  along the ring. As we know,  $r^2 + ||\mathbf{C}|| = 1$ , where  $\mathbf{C}$  is the coupling matrix. The eigen emittance exchange between the two eigen modes are proportional to  $1 - r^2/r^2$  [2, 3].

In this calculation, we found that there is no significant changes in the eigen  $\beta$  functions and the horizontal dispersion along the ring.

Figure 3 and Figure 4 shows the coupling parameter  $r$  along the ring with opposite and same polarities of the two e-elens solenoids. Here 100 GeV pp run lattice is used. With the opposite polarities of the two solenoids, the effects on the local coupling parameter is localized in the ring and is neglectable. With same polarities of the two solenoids, the coupling can be seen in the whole ring and its amplitude is about 0.97.

In the Appendix, we show the  $\beta$  functions and coupling parameter  $r$  in the scan of solenoid field amplitude for the 100 GeV and 2500 GeV pp run lattices. The solenoids are powered with same or different polarities, and even with one solenoid off.

## 3 Effect on vertical dispersion

Figure 5 and Figure 6 show the vertical dispersion along the ring due to the e-lens solenoids with 250 GeV and 100 GeV pp run lattices. The solenoid magnetic field is set to 6 T. We calculate different polarity combinations of the two e-lens solenoids.

From Figure 5 and Figure 6, with opposite polarities of the two solenoids, the vertical dispersion is localized between the two solenoids. With same polarities of the two solenoids, the vertical dispersion leaks out to the rest of the ring.

Also from Figure 5 and Figure 6, with opposite solenoid polarities, the maximum vertical dispersion is less than 1 mm for the 100 GeV pp lattice. However, with same solenoid polarity, the maximum vertical dispersion is about 7 mm and 3.5 mm in the triplets of IR6 and IR8 for the 100 GeV and 250 GeV lattices.

## 4 Effect on dQmin

With betatron coupling, there is a minimum eigen tune split, which we call dQmin. As we know, the fractional tune split is given by [4]

$$|Q_1 - Q_2 - p| = \sqrt{|Q_{x,0} - Q_{y,0} - p| + |C^-|}, \quad (1)$$

where  $|Q_1 - Q_2 - p|$  and  $|Q_{x,0} - Q_{y,0} - p|$  are fractional eigen tune split and set tune split,  $|C^-|$  is global coupling coefficient.

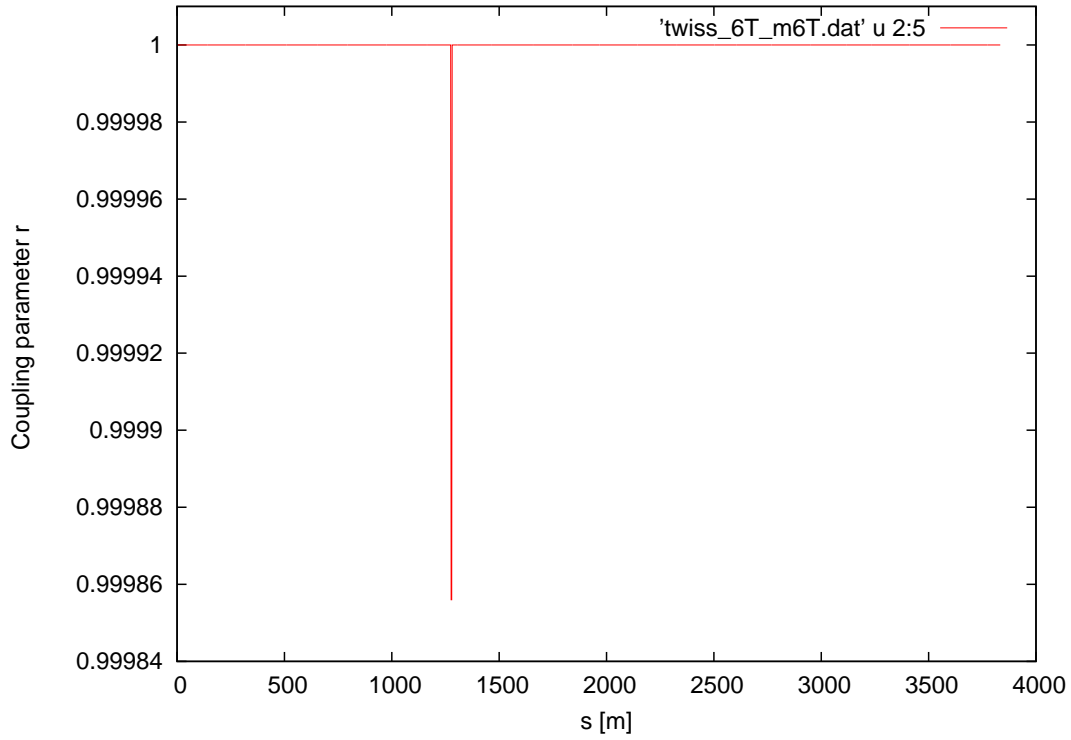


Figure 3: 100 GeV: coupling parameter  $r$  along the ring with opposite soleoid polarities.

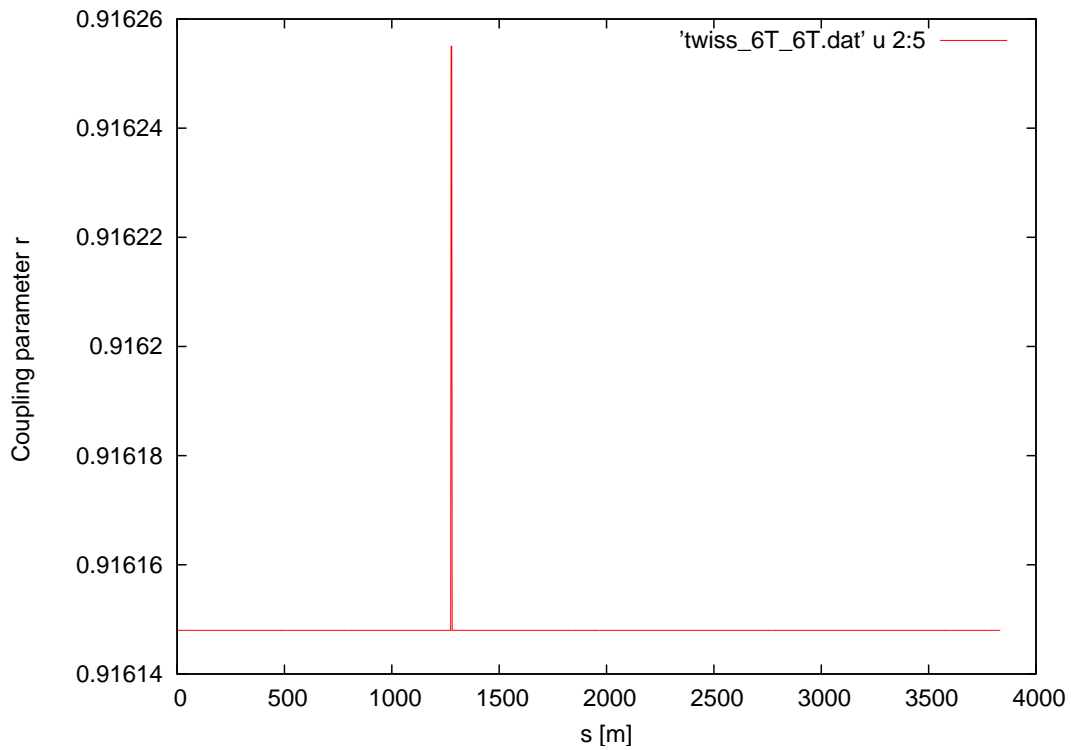


Figure 4: 100 GeV: coupling parameter  $r$  along the ring with same soleoid polarities.

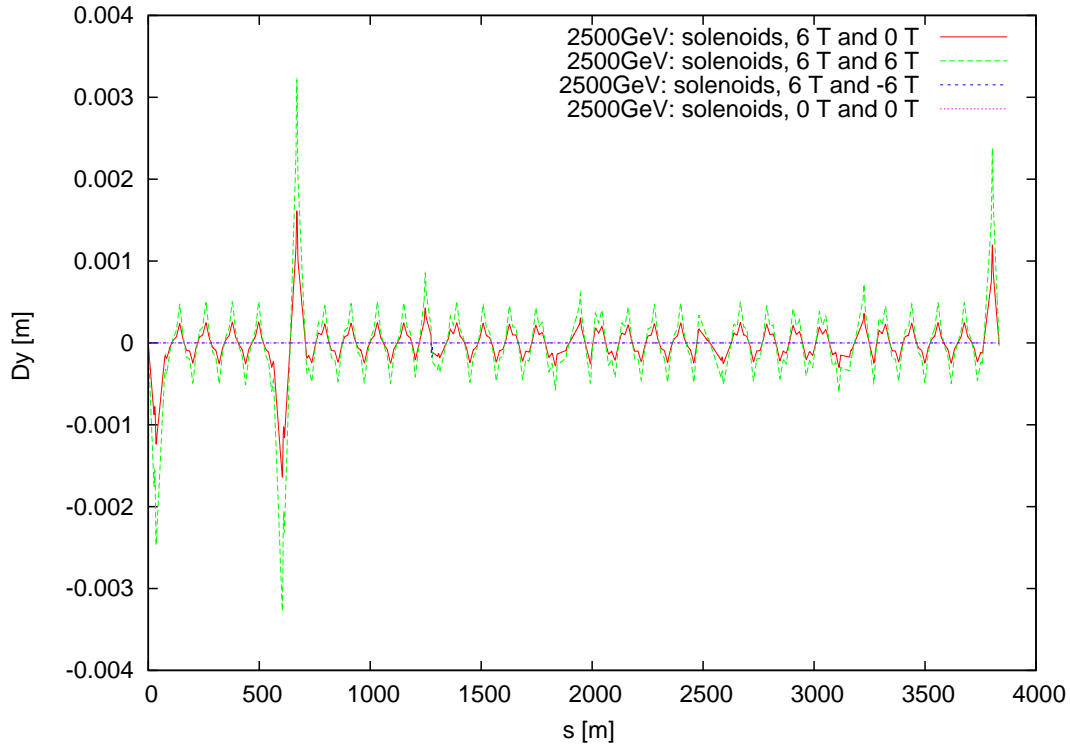


Figure 5: 250 GeV: Vertical dispersion along the ring with different solenoid powering cases

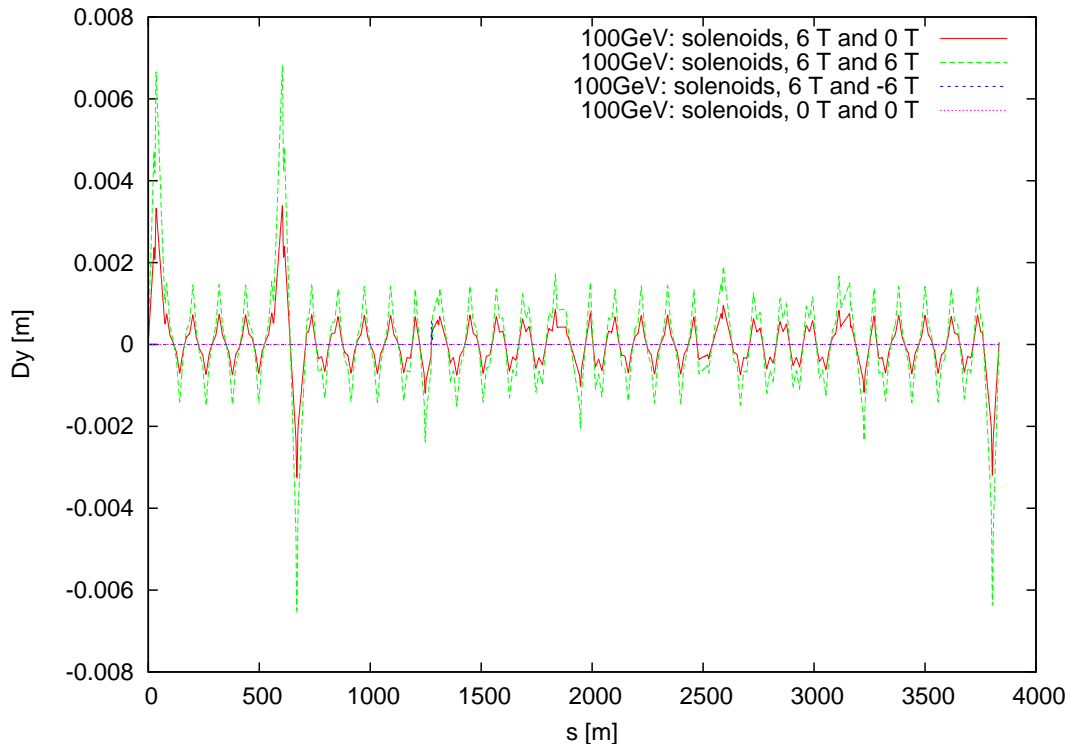


Figure 6: 100 GeV: Vertical dispersion along the ring with different solenoid powering cases

To numerically calculate the minimum tune split  $dQ_{min}$ , we first match the set tunes to (28.69, 29.69), then,

$$dQ_{min} = |C^-| = |Q_1 - Q_2 - p|. \quad (2)$$

Following list shows the calculated  $dQ_{min}$  for 100 GeV and 250 GeV pp lattices.

<<  $dQ_{min}$  due to e-lenses at 100GeV and 250 GeV >>

|         | BEL | YEL | $dQ_{min}$ |
|---------|-----|-----|------------|
| 100GeV: |     |     |            |
|         | 6T  | 6T  | 0.0108     |
|         | 6T  | -6T | 0.0006     |
|         | 6T  | 0T  | 0.0054     |
| 250GeV: |     |     |            |
|         | 6T  | 6T  | 0.0046     |
|         | 6T  | -6T | 0.0001     |
|         | 6T  | 0T  | 0.0023     |

With opposite polarities of the two solenoids, the  $dQ_{min}$  are about  $1 \times 10^{-4}$  and  $6 \times 10^{-4}$  at 100 GeV and 250 GeV, which are neglectible for real operation. However, with same polarities of the two solenoids, the  $dQ_{min}$  is about 0.005 and 0.01 at 100 GeV and 250 GeV, which must be corrected with skew quadrupoles. However, the skew quadrupole almost reach their maximum current limits at 100 GeV and 250 GeV pp run to correct the residual betatron coupling in the ring. Therefore, in the operation of e-lenses, the two solenoids must be powered with opposite polarities.

## 5 Summary

In this note we calculated the effects of the e-lens solenoids on the proton optics. We found that the effect of these two solenoids on the  $\beta$  functions and horizontal dispersion are neglectible. However, regarding to the vertical dispersion and global coupling, we should have the two e-lens solenoids powered with opposite polarities.

## References

- [1] Y. Luo and W. Fischer, *Outline of using an electron lens for the RHIC head-on beam-beam compensation ( v 0.1 )*, BNL C-AD AP Note 286, Dec., 2007.
- [2] Y. Luo, Phys. Rev. ST Accel. Beam **7**, 124001 (2004).
- [3] Y. Luo, NIM A 562 (2006) 57-64.
- [4] G. Guignard, Phys. Rev., **E 51** (1995) 6104.



## 6 Appendix:

<< 250 GeV : e-lenses' effect at IP6 and IP8 >>

1) BEL and YEL have different polarity

| BEL | YEL | Q1      | Q2      | -----IP6----- |          |   | -----IP8----- |          |   |
|-----|-----|---------|---------|---------------|----------|---|---------------|----------|---|
|     |     |         |         | beta1         | beta2    | r | beta1         | beta2    | r |
| -6  | 6   | 28.685  | 29.6949 | 0.518719      | 0.51975  | 1 | 0.518796      | 0.519645 | 1 |
| -5  | 5   | 28.6849 | 29.6949 | 0.518745      | 0.519723 | 1 | 0.518799      | 0.519651 | 1 |
| -4  | 4   | 28.6849 | 29.6949 | 0.518767      | 0.519702 | 1 | 0.518801      | 0.519656 | 1 |
| -3  | 3   | 28.6849 | 29.6949 | 0.518784      | 0.519685 | 1 | 0.518803      | 0.519659 | 1 |
| -2  | 2   | 28.6849 | 29.6949 | 0.518796      | 0.519674 | 1 | 0.518805      | 0.519662 | 1 |
| -1  | 1   | 28.6849 | 29.6949 | 0.518803      | 0.519666 | 1 | 0.518805      | 0.519664 | 1 |
| 0   | 0   | 28.6849 | 29.6949 | 0.518806      | 0.519664 | 1 | 0.518806      | 0.519664 | 1 |
| 1   | -1  | 28.6849 | 29.6949 | 0.518803      | 0.519666 | 1 | 0.518805      | 0.519664 | 1 |
| 2   | -2  | 28.6849 | 29.6949 | 0.518796      | 0.519674 | 1 | 0.518805      | 0.519662 | 1 |
| 3   | -3  | 28.6849 | 29.6949 | 0.518784      | 0.519685 | 1 | 0.518803      | 0.519659 | 1 |
| 4   | -4  | 28.6849 | 29.6949 | 0.518767      | 0.519702 | 1 | 0.518801      | 0.519656 | 1 |
| 5   | -5  | 28.6849 | 29.6949 | 0.518745      | 0.519723 | 1 | 0.518799      | 0.519651 | 1 |
| 6   | -6  | 28.685  | 29.6949 | 0.518719      | 0.51975  | 1 | 0.518796      | 0.519645 | 1 |

2) BEL and YEL have same polarity

| BEL | YEL | Q1      | Q2      | -----IP6----- |          |          | -----IP8----- |          |          |
|-----|-----|---------|---------|---------------|----------|----------|---------------|----------|----------|
|     |     |         |         | beta1         | beta2    | r        | beta1         | beta2    | r        |
| -6  | -6  | 28.6845 | 29.6955 | 0.518748      | 0.519718 | 0.976794 | 0.518825      | 0.519632 | 0.976794 |
| -5  | -5  | 28.6846 | 29.6953 | 0.518766      | 0.519701 | 0.983255 | 0.518819      | 0.519641 | 0.983255 |
| -4  | -4  | 28.6847 | 29.6952 | 0.51878       | 0.519688 | 0.988926 | 0.518815      | 0.51965  | 0.988926 |
| -3  | -3  | 28.6848 | 29.695  | 0.518791      | 0.519677 | 0.993604 | 0.518811      | 0.519656 | 0.993604 |
| -2  | -2  | 28.6849 | 29.695  | 0.518799      | 0.51967  | 0.997101 | 0.518808      | 0.51966  | 0.997101 |
| -1  | -1  | 28.6849 | 29.6949 | 0.518804      | 0.519666 | 0.999267 | 0.518806      | 0.519663 | 0.999267 |
| 0   | 0   | 28.6849 | 29.6949 | 0.518806      | 0.519664 | 1        | 0.518806      | 0.519664 | 1        |
| 1   | 1   | 28.6849 | 29.6949 | 0.518804      | 0.519666 | 0.999267 | 0.518806      | 0.519663 | 0.999267 |
| 2   | 2   | 28.6849 | 29.695  | 0.518799      | 0.51967  | 0.997101 | 0.518808      | 0.51966  | 0.997101 |
| 3   | 3   | 28.6848 | 29.695  | 0.518791      | 0.519677 | 0.993604 | 0.518811      | 0.519656 | 0.993604 |
| 4   | 4   | 28.6847 | 29.6952 | 0.51878       | 0.519688 | 0.988926 | 0.518815      | 0.51965  | 0.988926 |
| 5   | 5   | 28.6846 | 29.6953 | 0.518766      | 0.519701 | 0.983255 | 0.518819      | 0.519641 | 0.983255 |
| 6   | 6   | 28.6845 | 29.6955 | 0.518748      | 0.519718 | 0.976794 | 0.518825      | 0.519632 | 0.976794 |

3) BEL in on and YEL is off

| BEL | YEL | Q1      | Q2      | -----IP6----- |          |          | -----IP8----- |          |          |
|-----|-----|---------|---------|---------------|----------|----------|---------------|----------|----------|
|     |     |         |         | beta1         | beta2    | r        | beta1         | beta2    | r        |
| -6  | 0   | 28.6848 | 29.6951 | 0.518798      | 0.519672 | 0.993602 | 0.518771      | 0.519687 | 0.993602 |
| -5  | 0   | 28.6848 | 29.695  | 0.518801      | 0.519669 | 0.99551  | 0.518782      | 0.51968  | 0.99551  |
| -4  | 0   | 28.6849 | 29.695  | 0.518802      | 0.519667 | 0.997101 | 0.51879       | 0.519674 | 0.997101 |
| -3  | 0   | 28.6849 | 29.6949 | 0.518804      | 0.519666 | 0.998358 | 0.518797      | 0.51967  | 0.998358 |
| -2  | 0   | 28.6849 | 29.6949 | 0.518805      | 0.519665 | 0.999267 | 0.518802      | 0.519667 | 0.999267 |
| -1  | 0   | 28.6849 | 29.6949 | 0.518805      | 0.519664 | 0.999816 | 0.518805      | 0.519665 | 0.999816 |
| 0   | 0   | 28.6849 | 29.6949 | 0.518806      | 0.519664 | 1        | 0.518806      | 0.519664 | 1        |
| 1   | 0   | 28.6849 | 29.6949 | 0.518805      | 0.519664 | 0.999816 | 0.518805      | 0.519665 | 0.999816 |

|   |   |         |         |          |          |          |          |          |          |
|---|---|---------|---------|----------|----------|----------|----------|----------|----------|
| 2 | 0 | 28.6849 | 29.6949 | 0.518805 | 0.519665 | 0.999267 | 0.518802 | 0.519667 | 0.999267 |
| 3 | 0 | 28.6849 | 29.6949 | 0.518804 | 0.519666 | 0.998358 | 0.518797 | 0.51967  | 0.998358 |
| 4 | 0 | 28.6849 | 29.695  | 0.518802 | 0.519667 | 0.997101 | 0.51879  | 0.519674 | 0.997101 |
| 5 | 0 | 28.6848 | 29.695  | 0.518801 | 0.519669 | 0.99551  | 0.518782 | 0.51968  | 0.99551  |
| 6 | 0 | 28.6848 | 29.6951 | 0.518798 | 0.519672 | 0.993602 | 0.518771 | 0.519687 | 0.993602 |

<<100 GeV : e-lenses' effect at IP6 and IP8>>

1) BEL and YEL have different polarity

| BEL | YEL | Q1      | Q2      | -----IP6----- |          |   | -----IP8----- |          |   |
|-----|-----|---------|---------|---------------|----------|---|---------------|----------|---|
|     |     |         |         | beta1         | beta2    | r | beta1         | beta2    | r |
| -6  | 6   | 28.6852 | 29.6952 | 0.758689      | 0.77116  | 1 | 0.7594        | 0.769891 | 1 |
| -5  | 5   | 28.6851 | 29.6951 | 0.759034      | 0.770917 | 1 | 0.759528      | 0.770035 | 1 |
| -4  | 4   | 28.685  | 29.695  | 0.759317      | 0.770717 | 1 | 0.759634      | 0.770153 | 1 |
| -3  | 3   | 28.685  | 29.695  | 0.759538      | 0.770562 | 1 | 0.759716      | 0.770245 | 1 |
| -2  | 2   | 28.6849 | 29.6949 | 0.759695      | 0.770451 | 1 | 0.759774      | 0.77031  | 1 |
| -1  | 1   | 28.6849 | 29.6949 | 0.75979       | 0.770385 | 1 | 0.75981       | 0.77035  | 1 |
| 0   | -0  | 28.6849 | 29.6949 | 0.759821      | 0.770363 | 1 | 0.759821      | 0.770363 | 1 |
| 1   | -1  | 28.6849 | 29.6949 | 0.75979       | 0.770385 | 1 | 0.75981       | 0.77035  | 1 |
| 2   | -2  | 28.6849 | 29.6949 | 0.759695      | 0.770451 | 1 | 0.759774      | 0.77031  | 1 |
| 3   | -3  | 28.685  | 29.695  | 0.759538      | 0.770562 | 1 | 0.759716      | 0.770245 | 1 |
| 4   | -4  | 28.685  | 29.695  | 0.759317      | 0.770717 | 1 | 0.759634      | 0.770153 | 1 |
| 5   | -5  | 28.6851 | 29.6951 | 0.759034      | 0.770917 | 1 | 0.759528      | 0.770035 | 1 |
| 6   | -6  | 28.6852 | 29.6952 | 0.758689      | 0.77116  | 1 | 0.7594        | 0.769891 | 1 |

2) BEL and YEL have same polarity

| BEL | YEL | Q1      | Q2      | -----IP6----- |          |          | -----IP8----- |          |          |
|-----|-----|---------|---------|---------------|----------|----------|---------------|----------|----------|
|     |     |         |         | beta1         | beta2    | r        | beta1         | beta2    | r        |
| -6  | -6  | 28.6828 | 29.6975 | 0.75882       | 0.771272 | 0.916148 | 0.759659      | 0.76971  | 0.916148 |
| -5  | -5  | 28.6834 | 29.6968 | 0.75913       | 0.770998 | 0.933456 | 0.759718      | 0.769902 | 0.933456 |
| -4  | -4  | 28.6839 | 29.6962 | 0.759382      | 0.770772 | 0.951596 | 0.759761      | 0.770064 | 0.951596 |
| -3  | -3  | 28.6843 | 29.6957 | 0.759576      | 0.770594 | 0.969432 | 0.759791      | 0.770192 | 0.969432 |
| -2  | -2  | 28.6846 | 29.6953 | 0.759713      | 0.770466 | 0.985071 | 0.759809      | 0.770286 | 0.985071 |
| -1  | -1  | 28.6848 | 29.695  | 0.759794      | 0.770389 | 0.996027 | 0.759818      | 0.770343 | 0.996027 |
| 0   | 0   | 28.6849 | 29.6949 | 0.759821      | 0.770363 | 1        | 0.759821      | 0.770363 | 1        |
| 1   | 1   | 28.6848 | 29.695  | 0.759794      | 0.770389 | 0.996027 | 0.759818      | 0.770343 | 0.996027 |
| 2   | 2   | 28.6846 | 29.6953 | 0.759713      | 0.770466 | 0.985071 | 0.759809      | 0.770286 | 0.985071 |
| 3   | 3   | 28.6843 | 29.6957 | 0.759576      | 0.770594 | 0.969432 | 0.759791      | 0.770192 | 0.969432 |
| 4   | 4   | 28.6839 | 29.6962 | 0.759382      | 0.770772 | 0.951596 | 0.759761      | 0.770064 | 0.951596 |
| 5   | 5   | 28.6834 | 29.6968 | 0.75913       | 0.770998 | 0.933456 | 0.759718      | 0.769902 | 0.933456 |
| 6   | 6   | 28.6828 | 29.6975 | 0.75882       | 0.771272 | 0.916148 | 0.759659      | 0.76971  | 0.916148 |

3) BEL in on and YEL is off

| BEL | YEL | Q1      | Q2      | -----IP6----- |          |   | -----IP8----- |          |   |
|-----|-----|---------|---------|---------------|----------|---|---------------|----------|---|
|     |     |         |         | beta1         | beta2    | r | beta1         | beta2    | r |
| -6  | 6   | 28.6852 | 29.6952 | 0.758689      | 0.77116  | 1 | 0.7594        | 0.769891 | 1 |
| -5  | 5   | 28.6851 | 29.6951 | 0.759034      | 0.770917 | 1 | 0.759528      | 0.770035 | 1 |

|    |    |         |         |          |          |   |          |          |   |
|----|----|---------|---------|----------|----------|---|----------|----------|---|
| -4 | 4  | 28.685  | 29.695  | 0.759317 | 0.770717 | 1 | 0.759634 | 0.770153 | 1 |
| -3 | 3  | 28.685  | 29.695  | 0.759538 | 0.770562 | 1 | 0.759716 | 0.770245 | 1 |
| -2 | 2  | 28.6849 | 29.6949 | 0.759695 | 0.770451 | 1 | 0.759774 | 0.77031  | 1 |
| -1 | 1  | 28.6849 | 29.6949 | 0.75979  | 0.770385 | 1 | 0.75981  | 0.77035  | 1 |
| 0  | -0 | 28.6849 | 29.6949 | 0.759821 | 0.770363 | 1 | 0.759821 | 0.770363 | 1 |
| 1  | -1 | 28.6849 | 29.6949 | 0.75979  | 0.770385 | 1 | 0.75981  | 0.77035  | 1 |
| 2  | -2 | 28.6849 | 29.6949 | 0.759695 | 0.770451 | 1 | 0.759774 | 0.77031  | 1 |
| 3  | -3 | 28.685  | 29.695  | 0.759538 | 0.770562 | 1 | 0.759716 | 0.770245 | 1 |
| 4  | -4 | 28.685  | 29.695  | 0.759317 | 0.770717 | 1 | 0.759634 | 0.770153 | 1 |
| 5  | -5 | 28.6851 | 29.6951 | 0.759034 | 0.770917 | 1 | 0.759528 | 0.770035 | 1 |
| 6  | -6 | 28.6852 | 29.6952 | 0.758689 | 0.77116  | 1 | 0.7594   | 0.769891 | 1 |

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