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Further Studies of the Fast Injection Bump and Comparison with Analytic Expressions and with Results from the BEAM Program

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Observations and Conclusion

The purpose of this study was to examine in some detail the dependence of both the sine and cosine components of the equilibrium orbit (E.O.) harmonics on the relative magnitudes of the kicks at A13 and B7. Data were obtained in the same way as in the previous studies (AGS Studies Reports 171, 172) using the ring PUE system and the ORBED/NORB programs. The NORB program analyzes the E.O. and arbitrarily defines the angular zero for its fourier analysis as straight section (S.S.) L20. With the magnitude of the A13 kick fixed at 1.5 V the B7 kick was swept through a series of values ranging from 0.6 V to 3.0 V. The sine and cosine components of the 9th harmonic of the E.O. obtained for each case are listed in Table I and are plotted in Figure 1. These data were obtained with a horizontal tune of 8.75. Note that the sine and cosine components appear to vary linearly with the magnitude of the kick at B7. Fourier analysis of the analytic expression for the deformed E.O. shows that this is indeed the case, which we now show.

The general expression for the deformed E.O. is[1]

$$\eta(\phi) = \frac{\nu}{2 \sin \pi\nu} \int_{\phi}^{\phi+2\pi} F(\psi) \cos \nu(\pi+\phi-\psi)\, d\psi \qquad (1)$$

where

$$\phi = \phi(s) = \int_{o}^{s} \frac{d\bar{s}}{\nu\beta(\bar{s})}, \qquad (2)$$

$$f(\psi) = \beta^{3/2}(s)\, F(s), \qquad (3)$$

[1]E.D. Courant and H.S. Snyder, Annals of Physics:3, 1-48 (1958).

s is the distance along the undeformed orbit of circumference c, F(s) is the perturbation term due to perturbing dipoles in the ring, and the other symbols have their usual meanings. For the case of two delta function kicks at s_1 and s_2 respectively ($0<s_1<s_2<c$), F(s) is a periodic function of the form

$$F(s) = -\theta_1 \delta(s-s_1) - \theta_2 \delta(s-s_2), \quad 0<s<c \tag{4}$$

where θ_1 (θ_2) is the magnitude of the kick at s_1(s_2). Inserting (4) into (3) and (1), the radial displacement

$$y(s) = \beta^{1/2}(s)\ \eta(\phi) \tag{5}$$

of the E.O. due to the perturbing kicks is

$$y(s) = \frac{\beta^{1/2}(s)}{2 \sin \pi\nu} \left[\Delta_1 \cos \nu(\phi-\psi_1+\pi) + \Delta_2 \cos \nu(\phi-\psi_2+\pi)\right], \quad 0<s<s_1$$

$$y(s) = \frac{\beta^{1/2}(s)}{2 \sin \pi\nu} \left[\Delta_1 \cos \nu(\phi-\psi_1-\pi) + \Delta_2 \cos \nu(\phi-\psi_2+\pi)\right], \quad s_1<s<s_2 \tag{6}$$

$$y(s) = \frac{\beta^{1/2}(s)}{2 \sin \pi\nu} \left[\Delta_1 \cos \nu(\phi-\psi_1-\pi) + \Delta_2 \cos \nu(\phi-\psi_2-\pi)\right], \quad s_2<s<c$$

where

$$\Delta_1 \equiv -\theta_1 \beta^{1/2}(s_1), \quad \Delta_2 \equiv -\theta_2 \beta^{1/2}(s_2) \tag{7}$$

$$\psi_1 \equiv \psi(s_1) = \int_0^{s_1} \frac{d\bar{s}}{\nu\beta(\bar{s})}, \quad \psi_2 \equiv \psi(s_2) = \int_0^{s_2} \frac{d\bar{s}}{\nu\beta(\bar{s})} \tag{8}$$

Now we consider the specific case at hand in which s_1 is the location (along the unperturbed E.O.) of the A13 kick and s_2 is the location of the B7 kick. θ_1 and θ_2 are respectively the magnitudes of these kicks and $\beta(s_1)$ and $\beta(s_2)$ are the values of the betatron function at A13 and B7. The locations of these kicks have been chosen so that nominally they produce a half-lambda bump. Thus, we have

$$\nu\psi_2 = \nu\psi_1 + \pi. \tag{9}$$

The location of the A13 kick (W.R.T. S.S. L20) is also such that one has nominally

$$\nu\psi_1 = \pi. \tag{10}$$

(This is because we happen to inject at A20, one superperiod away from our angular zero reference (S.S. L20) or 1/12 $(2\pi\nu) \approx 3\pi/2$ away, and the kick wants to be $\lambda/4 = \pi/2$ away from injection--hence placing the A13 kick at π.)

Putting (9) and (10) into (6) and letting the tune $\nu = 8.75$ we find

$$y(s) = \frac{\beta^{1/2}(s)}{2\sin\pi\nu}(\Delta_1-\Delta_2)(\sqrt{2}/2)\left[\cos\nu\phi + \sin\nu\phi\right],\ 0<s<s_1 \tag{11}$$

$$y(s) = \frac{\beta^{1/2}(s)}{2\sin\pi\nu}(\sqrt{2}/2)\left[(\Delta_1-\Delta_2)\cos\nu\phi - (\Delta_1+\Delta_2)\sin\nu\phi\right],\ s_1<s<s_2$$

$$y(s) = \frac{\beta^{1/2}(s)}{2\sin\pi\nu}(\Delta_1-\Delta_2)(\sqrt{2}/2)\left[\cos\nu\phi - \cos\nu\phi\right],\ s_2<s<c.$$

The fourier sine and cosine components of the perturbed orbit y(s) are respectively

$$A_N = \frac{1}{\pi}\int_0^{2\pi} d\phi\, y(s)\sin N\phi \tag{12}$$

$$B_N = \frac{1}{\pi}\int_0^{2\pi} d\phi\, y(s)\cos N\phi$$

Taking $N = 9$ and putting (11) into (12) with $\beta^{1/2}(s)$ replaced by the average, $\beta_{AV}^{1/2}$, we find the following values for the ninth harmonic of the deformed E.O.:

$$A_9 = (\beta_{AV}^{1/2}/2\sin\pi\nu)\ \frac{1}{\pi}(\sqrt{2}/2)\ \left[-0.34\ \Delta_1\right]\ \text{(sine)} \tag{13}$$

$$B_9 = (\beta_{AV}^{1/2}/2\sin\pi\nu)\ \frac{1}{\pi}(\sqrt{2}/2)\ \left[3.97(\Delta_1-\Delta_2) + 0.02(\Delta_1+\Delta_2)\right]\ \text{(cosine)}$$

(We note here that Equations (13) are exact only when $\nu = 8.75$ and (9) and (10) are satisfied. In practice (9) and (10) are only approximately true, so in general there will be small corrections to (13)). Thus we see that with the magnitude (θ_1) of the kick at A13 fixed, the cosine component varies linearly with the magnitude (θ_2) of the kick at B7, while the sine component remains constant. Our observations (see Figure 1) are consistent with these results.

We note from Equations (11) that the residuals will have zero amplitude when $\Delta_1 = \Delta_2$. Thus, using (7), the condition for nulled residuals is

$$(\theta_2/\theta_1)^2 = \beta(s)_1/\beta(s_2). \tag{14}$$

Comparing (13) and Figure 1, we see that the value of the B7 kick for which the cosine component is zero is very nearly the value for which $\Delta_1 = \Delta_2$. Thus, our measured results show that the residuals are nulled when $(\theta_2/\theta_1)^2 = (1.95/1.5)^2 = 1.7$, which agrees with the result obtained in AGS Studies Report No. 172, and as mentioned there is slightly less than the value of 2.1 one expects if $\beta_H(A13) = \beta_{max} \simeq 22$ m, $\beta_H(B7) = \beta_{min} \simeq 10.5$m.

Figure 2 shows the E.O.'s obtained from the ring PUE system for three cases. In case (b) the magnitude of the A13 and B7 kicks have been adjusted to null the residuals. In case (a) the B7 kick is smaller than the value required to null the residuals while in case (c) it is larger than the value required for a null. Equilibrium orbits obtained by the BEAM program for the three corresponding cases are shown in Figure 3. Comparison of the corresponding cases in the two figures shows that the general features of the orbits agree quite well. (Note especially the behavior near the A13 and B7 kicks).

mvh

Table I

Kick Magnitudes (volts)		Ninth Harmonic of Deformed E.O. (mm)	
A13	B7	Sin	Cos
1.5	0.6	-0.06	+0.45
1.5	1.0	-0.07	+0.33
1.5	1.3	-0.08	+0.22
1.5	1.5	-0.07	+0.14
1.5	1.8	-0.08	+0.02
1.5	2.0	-0.09	-0.02
1.5	2.1	-0.09	-0.07
1.5	2.2	-0.09	-0.10
1.5	2.3	-0.10	-0.15
1.5	2.5	-0.10	-0.23
1.5	3.0	-0.10	-0.36

FIGURE 1

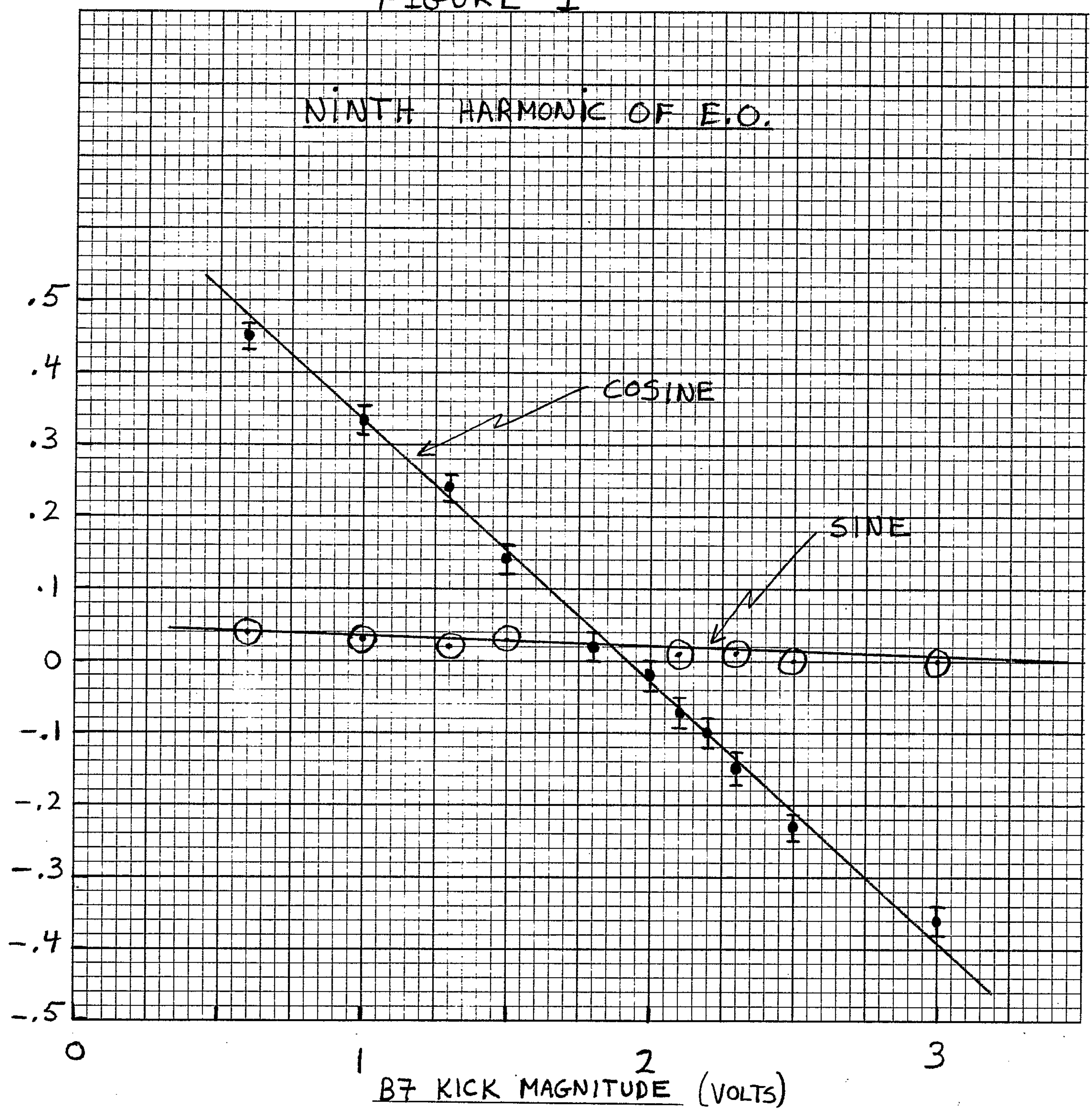

NINTH HARMONIC OF E.O.
COSINE
SINE
.5
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
-.5
0
1
2
3
B7 KICK MAGNITUDE (VOLTS)

FIGURE 2

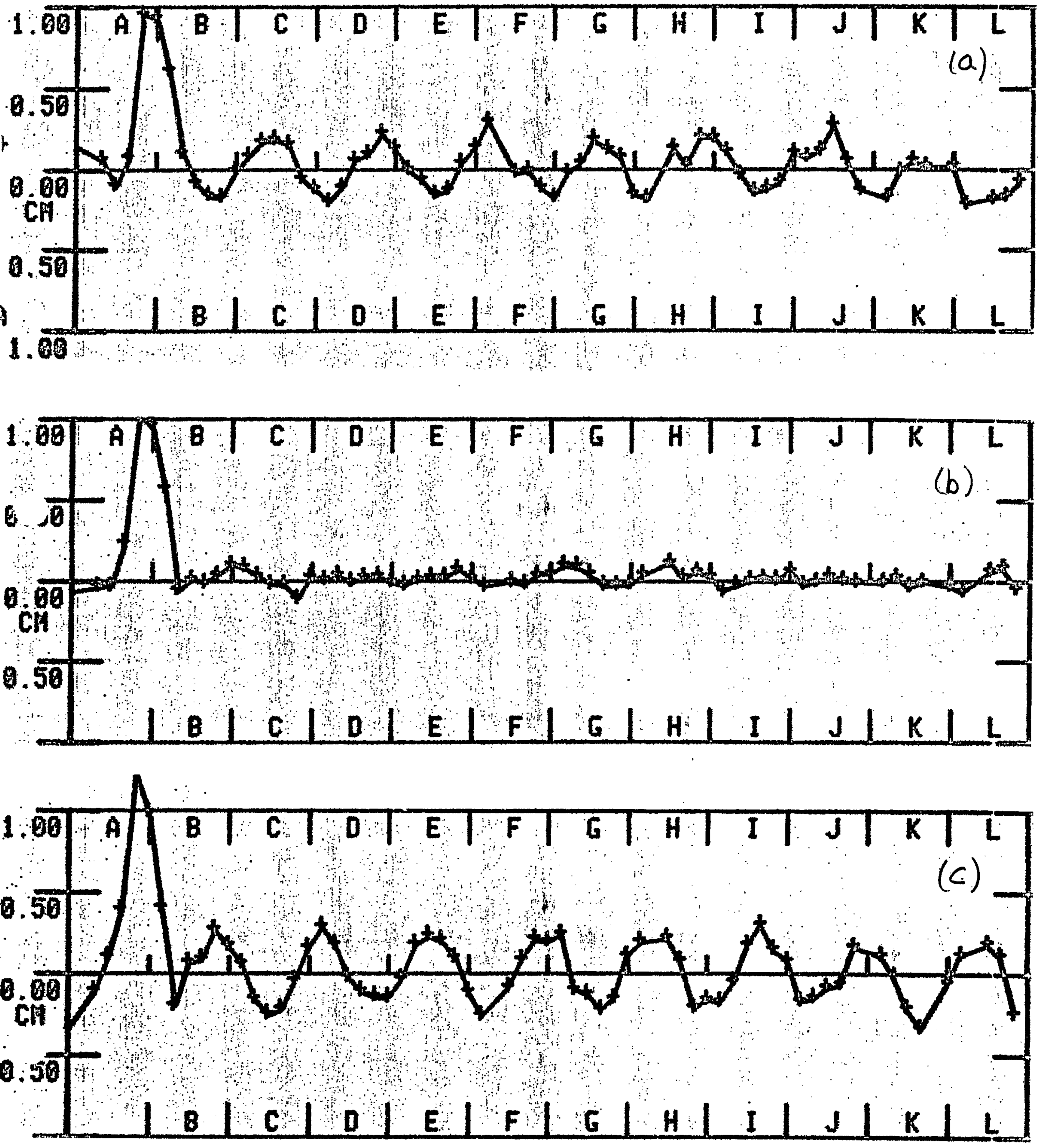
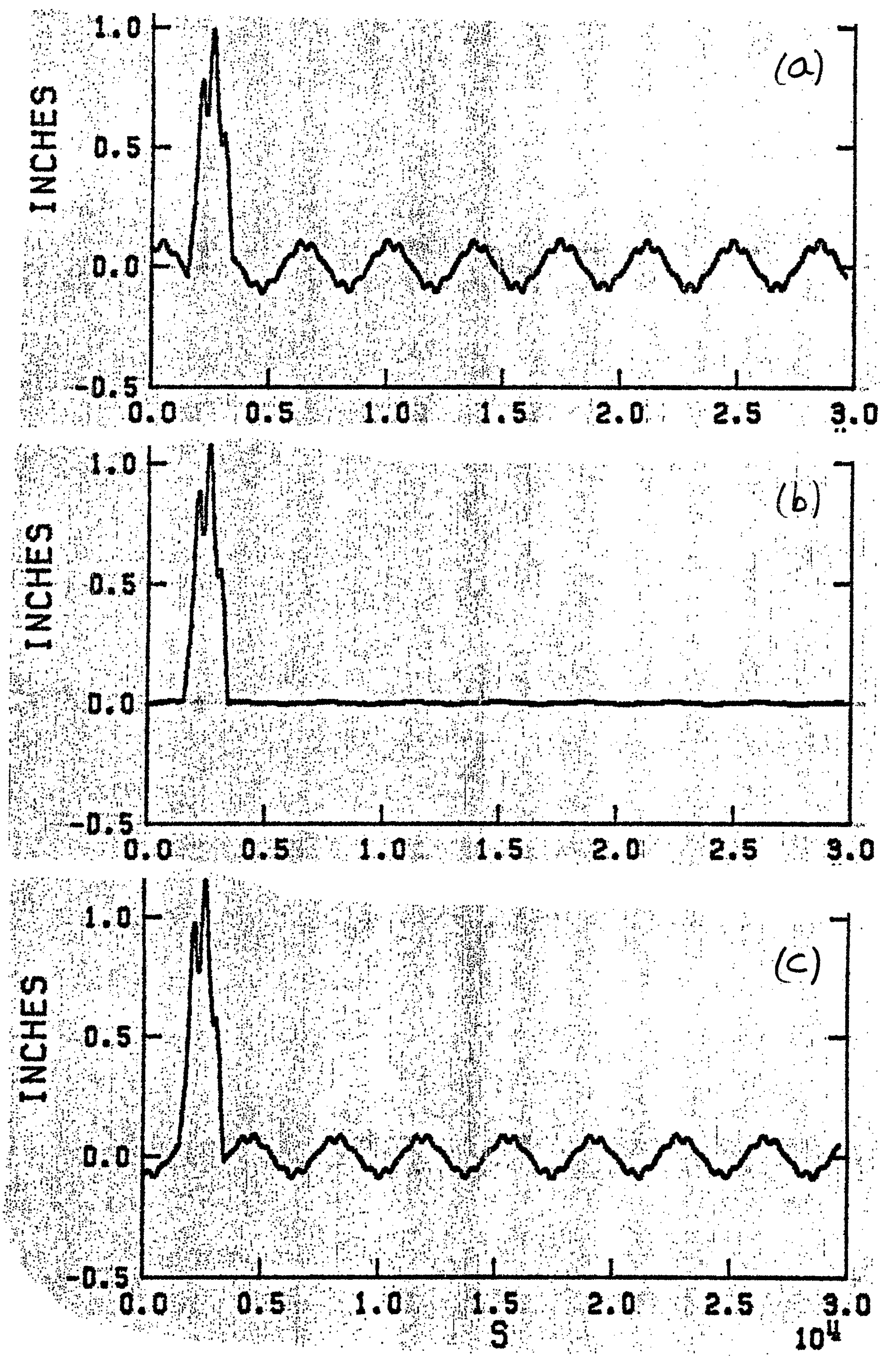


FIGURE 3


(a)
INCHES
1.0
0.5
0.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
(b)
INCHES
1.0
0.5
0.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
(c)
INCHES
1.0
0.5
0.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
S
10⁴