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Measurements of Chromaticity and Momentum Spread at Injection in the AGS Using PIP

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Observations and Conclusion

The purpose of this study was to use PIP[1] to measure the AGS chromaticity at injection as a function of the current in the ring sextupole magnets and to determine the momentum spread of the injected beam. Among the seven parameters determined by PIP are the tune (ν), the tune shift per turn ($\Delta\nu$), and the tune spread ($\delta\nu$) of the beam in the AGS. In terms of these parameters, the horizontal (H) and vertical (V) chromaticities and the momentum spread are respectively

$$\xi_H = - \frac{\Delta\nu_H/\nu_H}{\Delta B/B} \tag{1}$$

$$\xi_V = - \frac{\Delta\nu_V/\nu_V}{\Delta B/B} \tag{2}$$

$$\frac{\Delta P}{P} = (\delta\nu_H/\nu_H)/\xi_H = (\delta\nu_V/\nu_V)/\xi_V \tag{3}$$

where B is the field and ΔB is the change in field per turn at injection. (Nominally B = 250 G and ΔB = 0.0207 G at injection.)

We note here that the validity of the values of ξ and $\Delta P/P$ determined from (1-3) depend on the validity of the fitting function used by PIP to obtain the parameters ν, $\Delta\nu$, $\delta\nu$. The fitting function assumes, for example, that the horizontal and vertical betatron oscillations are not coupled and that the decay in the amplitude of the coherent betatron oscillations is due only to the tune spread of the beam. If these assumptions are not true, because of coupling or because the decay in the coherent oscillations is due to something other than the tune spread, then the values of $\Delta\nu$ and $\delta\nu$ yielded by PIP will be affected and the resulting values of ξ and $\Delta P/P$ will not be correct. For the data obtained during this study we believe that the assumptions inherent in the fitting function are valid.

A total of 16 sextupoles were used to vary the chromaticity. These are divided into two groups of eight. The sextupoles of the first group are located at horizontal beta-maximums in straight sections B13, C13, E13, F13, H13, I13, K13, L13 and are called horizontal sextupoles, while those of the second group are located at vertical beta-maximums in straight sections B7, C7, E7, F7, H7, I7, K7, L7 and are called vertical sextupoles. The fields produced by each sextupole are[2]

$$B_X = + B''XZ \tag{4}$$

$$B_Z = \frac{B''}{2!}(X^2 - Z^2) \tag{5}$$

where the sextupole strength is[3]

$$\frac{B''\ell}{2!} = 17.151\ I\ (\text{Gauss/Inch}), \tag{6}$$

ℓ is the effective length of the sextupole, I is the current (in amps) flowing through the sextupole, and X and Z refer to the horizontal (radial) and vertical directions respectively.

These fields contribute[4]

$$\xi_H = \frac{1}{4\pi\nu_H} \int \frac{B''(s)\beta_H(s)\alpha_P(s)}{B\rho} \qquad (7)$$

and

$$\xi_V = \frac{-1}{4\pi\nu_V} \int \frac{B''(s)\beta_V(s)\alpha_P(s)}{B\rho} \qquad (8)$$

respectively to the horizontal and vertical chromaticities, where the magnetic rigidity at injection is

$$B\rho = 8.464 \times 10^5 \text{ (Gauss-Inches)} \qquad (9)$$

and the momentum compaction function is[5]

$$\alpha_P(s) \approx \beta_{AV}^{1/2} \, \beta_H^{1/2} / \nu_H \qquad (10)$$

The contributions of each of the horizontal sextupoles to the horizontal and vertical chromaticities are then approximately

$$\xi_H^{13} = \frac{+1}{4\pi\nu_H} \frac{B''\ell}{B\rho} (B_{AV}^{1/2}/\nu_H) \, \beta_{H13}^{3/2} \qquad (11)$$

$$\xi_V^{13} = \frac{-1}{4\pi\nu_V} \frac{B''\ell}{B\rho} (\beta_{AV}^{1/2}/\nu_H) \, \beta_{H13}^{1/2} \quad \beta_{V13} \qquad (12)$$

and the contributions of each of the vertical sextupoles to the horizontal and vertical chromaticities are approximately

$$\xi_H^7 = \frac{+1}{4\pi\nu_H} \frac{B''\ell}{B\rho} (B_{AV}^{1/2}/\nu_H)\ \beta_{H7}^{3/2} \tag{13}$$

$$\xi_V^7 = \frac{-1}{4\pi\nu_V} \frac{B''\ell}{B\rho} (\beta_{AV}^{1/2}/\nu_H)\ \beta_{H7}^{1/2}\ \beta_{V7} \tag{14}$$

where

$$\beta_{AV} = R/\nu_H \tag{15}$$

$$R = 5057.5'' \tag{16}$$

$$\beta_{H13} \simeq \beta_{V7} \simeq 866'' \tag{17}$$

$$\beta_{H7} \simeq \beta_{V13} \simeq 413'' \tag{18}$$

Using (6), (9) and (15-18) in (11-14), we find that when the current increases by one amp in each of the eight horizontal sextupoles, the horizontal and vertical chromaticities change by the amounts

$$\Delta\xi_H^{13} = +0.21 \text{ per amp} \tag{19}$$

$$\Delta\xi_V^{13} = -0.10 \text{ per amp} \tag{20}$$

and when the current increases by one amp in each of the eight vertical sextupoles, the chromaticities change by the amounts

$$\Delta\xi_H^7 = +0.07 \text{ per amp} \tag{21}$$

$$\Delta\xi_V^7 = -0.14 \text{ per amp} \tag{22}$$

Table I and Figure 1 show the values of ξ_H and ξ_V which were obtained using Equations 1-3) from the parameters measured by PIP for various values of current in the 16 sextupoles. Here we see that the measured chromaticities vary linearly with the sextupole current, and from the slopes of the lines fitted to the data points we find that when the current increases by 1 amp in all 16 sextupoles, the measured chromaticities change by amounts

$$\begin{aligned} \Delta\xi_H^{meas} &= +0.31(5) \\ \Delta\xi_V^{meas} &= -0.29(5). \end{aligned} \tag{23}$$

These are to be compared with the calculated changes from Equations (19)-(22):

$$\begin{aligned} \Delta\xi_H^{calc} &= \Delta\xi_H^{13} + \Delta\xi_H^{7} = +0.28 \\ \Delta\xi_V^{calc} &= \Delta\xi_V^{13} + \Delta\xi_V^{7} = -0.24 \end{aligned} \tag{24}$$

Thus the measured and calculated values are in good agreement.

In addition to the chromaticity measurements, the PIP data allow for an estimate of the beam momentum spread $\Delta P/P$ at injection. Plugging into Equations (3) the values of the parameters $\delta\nu$, ν, ξ determined by PIP, we obtain the momentum spreads summarized in Table II. Within the quoted errors these values of $\Delta P/P$ are all consistent with a momentum spread of about 0.1% and show, as expected, no dependence on the sextupole current.

References

1. A description of PIP may be found in the paper "A Method for Determining the Position, Angle, and other Injection Parameters of a Short Pulsed Beam in the Brookhaven AGS" submitted to the 1985 Particle Accelerator Conference, Vancouver, Canada by Gardner and Ahrens (BNL 36490).

2. CERN report #77-13 ("Theoretical Aspects of the Behavior of Beams in Accelerators and Storage Rings"), P. 116.

3. Taken from one of Dave Barge's notebooks.

4. Reference #2, P. 120.

5. M. Month, AGSCD-38, Jan. 22, 1969.

mvh/ld

TABLE I

Current in Each of 16 Sextupoles (Amps)	Tune		Tune Shift/Turn		Chromaticity	
	ν_H	ν_V	$\Delta\nu_H \times 10^3$	$\Delta\nu_V \times 10^3$	ξ_H	ξ_V
-1.5	8.731(1)	8.926(1)	1.53(5)	0.40(5)	-2.1(1)	-0.5(1)
0	8.742(1)	8.917(1)	1.12(4)	0.70(6)	-1.5(1)	-0.9(1)
+1.5	8.752(1)	8.907(1)	0.85(3)	1.01(7)	-1.2(1)	-1.4(1)

TABLE II

Current in Each of 16 Sextupoles (Amps)	Tune Spread		Momentum Spread*	
	$\delta\nu_H$	$\delta\nu_V$	$(\Delta P/P)_H \times 10^3$	$(\Delta P/P)_V \times 10^3$
-1.5	0.0172(5)	0.0063(10)	0.93(7)	1.33(30)
0	0.0139(5)	0.0079(10)	1.03(7)	0.94(20)
+1.5	0.0101(5)	0.0090(10)	0.99(7)	0.73(20)

* The subscript H(V) on $\Delta P/P$ indicates that the number was obtained from equation (3) using horizontal (vertical) data.

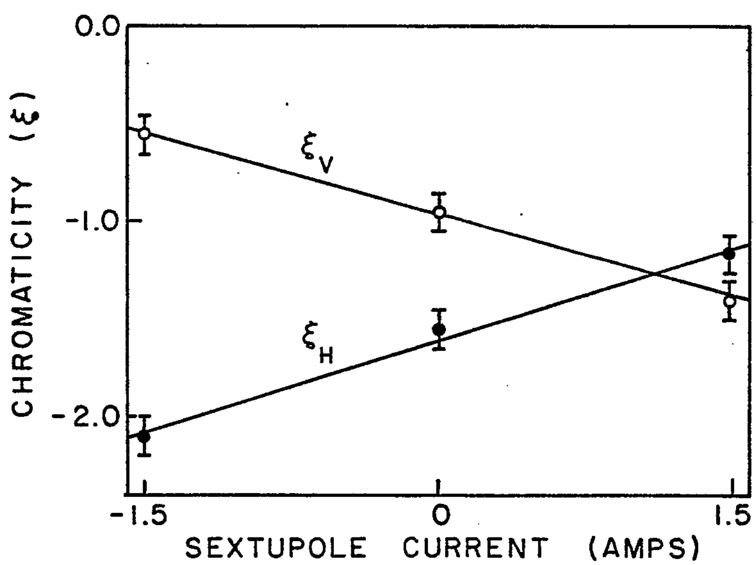
0.0
-1.0
-2.0
CHROMATICITY (ξ)
ξ_V
ξ_H
-1.5
0
1.5
SEXTUPOLE CURRENT (AMPS)

Figure 1