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Dependence of the Frequency on the Radius at Extraction

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AGS STUDIES REPORT
BNL AGS Dept.

SUBJECT: Dependence of the Frequency on the Radius at Extraction

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REPORTED BY: E. Bleser

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I. INTRODUCTION

A given orbit in the AGS has a total path length which we call the circumference, C, and, since it is very nearly circular, by convention we define a radius, R, which is just:

R = C/2pi

The frequency of revolution is then:

f(rev) = beta*c/2pi*R

The RF frequency is:

f(rf) = 12*f(rev)

where "beta" is the particle velocity divided by the speed of light, c. If we fix the particle velocity, we have:

df/f = -dR/R

However in the AGS when we change the radius we change the momentum, and the correct formula to use is:

df/f = -[dR/R][1-{gamma(tr)/gamma}**2] 1

where gamma is the usual:

gamma = 1/sqrt(1-beta**2)

and gamma(tr) is gamma at transition.

In a note of 11/26/85, Ahrens, Allard, and Brown, discuss this formula and detail the technique for determining the beam radius by RF frequency measurements. In this note we examine a small set of data taken during an experiment on a different topic and find that the measurements of the frequency and the radii are remarkably precise but that there may be a calibration constant incorrectly set in the PUE system.

II. DATA

The data are shown in Table I. The frequency meter and the orbit acquisition were triggered simultaneously just before the the time for turning on the H5 extraction bumps, however a full record of the data taking conditions was not made and thus can not be given here. RS7A gives the setting of the radius shifter. The Avg. Pos. gives the average position of the radius as given by the ORBED program averaging over all the operating PUE's. The (L4+L18)/2 column is calculated from the PUE listing.

The radius shifter uses the signal from (L4+L18)/2 to set the radius and therefore there should be a close correlation between columns 3 and 5. This is plotted in Figure 1. The line is the result of a least squares fit, which in this and all subsequent cases assumes errors occur only in the ordinate. The formula for the line is:

$$Y = (0.150 \pm 0.005) + (0.364 \pm 0.004) * 10^{-2} * X$$

The data are recorded to 0.01 centimeters and if we assume this is our error, the fit has a Chi Square of 2.6 which is reasonable for three degrees of freedom.

Figure 2 shows (L4+L18)/2 versus the Average Position. The formula for the line is:

$$Y = (0.376 \pm 0.012) + (1.013 \pm 0.021) * X$$

The slope should be one as it is within the errors. If we assume an error of 0.01 on the ordinate we get a Chi Square of 9, suggesting an error on the abscissa of 0.014 cm which is not unreasonable, but which is larger than we would get from simply averaging the data.

Figure 3 shows the Average Position vs the radius shifter. The formula for the line is:

$$Y = (-0.224 \pm 0.006) + (0.359 \pm 0.005) * 10^{-2} * X$$

Assuming an error of 0.01 cm on the ordinate gives a Chi Square of 3.6, quite reasonable.

Figure 4 shows the frequency versus the radius shifter with a fitted line given by:

$$Y = (4,454,915 \pm 3) + (-1.379 \pm 0.021) * X$$

Assuming an error of 5 cycles in the ordinate gives a Chi Square of 3, which is reasonable since we only recorded data to an accuracy of 10 cycles.

Figure 5 shows the results we are really interested in, the frequency versus the average position. The fitted line is:

$$Y = (4,454,830 \pm 5) + (-384 \pm 9) * X$$

Assuming an error of 5 cycles in the ordinate gives a Chi Square of 7.5, but Figure 2 has already suggested that we must take account of an error in average position, which would presumably give us a very reasonable Chi Square.

III. ANALYSIS

From the slope calculated in Figure 5 we have the measured value:

$$DF/DR = -384 \pm 9 \text{ Hz per cm} \qquad 2$$

We can also calculate this value since from Equation 1 we can write:

$$dF/dR = -[F/R] * [1 - \{gamma(tr)/gamma\}**2] \qquad 3$$

We must evaluate F, R, and gamma. From TN 217 we know that the optimun central orbit (OCO) corresponds to a displacement of -0.4 cm in the PUE's. From Figure 5 this orbit has a frequency of:

$$F(OCO) = 4,454,984 \pm 6 \text{ Hz}$$

Table A7 of Tn 217 gives the chord length of the OCO, adding the saggita gives:

$$R = 12,845.36 \text{ cm}$$

This is a theoretical number and can be taken to be exact as stated. The recent radial survey of the ring (R. Thern, to be distributed) suggests the radius may be low by 0.25 cm, but this is well within the survey errors, so we shall take this number to be the uncertainty on R. Then:

$$F/R = 346.8 \pm 0.01 \text{ Hz per cm}$$

For this R and F we find beta and thus P and gamma:

$$P = 28.84 \text{ and } gamma = 30.75$$

We take gamma(tr) as 8.75, to get from Eq. 3:

$$dF/dR = -[346.8 \pm 0.01]*[1-0.081]$$

$$= -318.7 \pm 0.01 \text{ Hz per cm}$$

which is 7 standard deviations from the measured result in Eq. 2.

IV. DISCUSSION

The discrepancy between the measured and calculated values of dF/dR is unacceptably large, 20 percent. Neither F nor R can be in error by this much, and the gamma terms contribute only a small correction. The analysis presented here could be wrong, and if so will hopefully be corrected by wiser heads. If not, the only remaining source of error is in the calibration of the PUE's. Thus we would suggest that:

r(actual displacement from center of PUE)= 1.2*r(reported by PUE)

The people familiar with the PUE's will readily concede that the calibration could be in error, however a 20 percent error seems very large. Since the PUE reponse may be momentum (freqency) dependent, additional measurements at other momenta will be interesting.

One possible source of confusion might be the location of the PUE's relative to the maxima and minima of the dispersion function. However, they are very cleverly placed at positions where the dispersion function has its average value, and therefore the radius reported by the PUE's is a fair estimate of the average radius as we vary the the beam position. Figure 7 of TN 217 shows the dispersion function.

TABLE I. DATA

Run No.	Frequency Hz	RS7A	Avg. Pos. cm	(L4+L18)/2 cm
1	4,455,230	-230	-1.058	-0.680
2	4,455,100	-130	-0.685	-0.335
3	4,454,960	-30	-0.319	0.045
4	4,454,910	0	-0.223	0.145
5	4,454,820	70	0.017	0.410

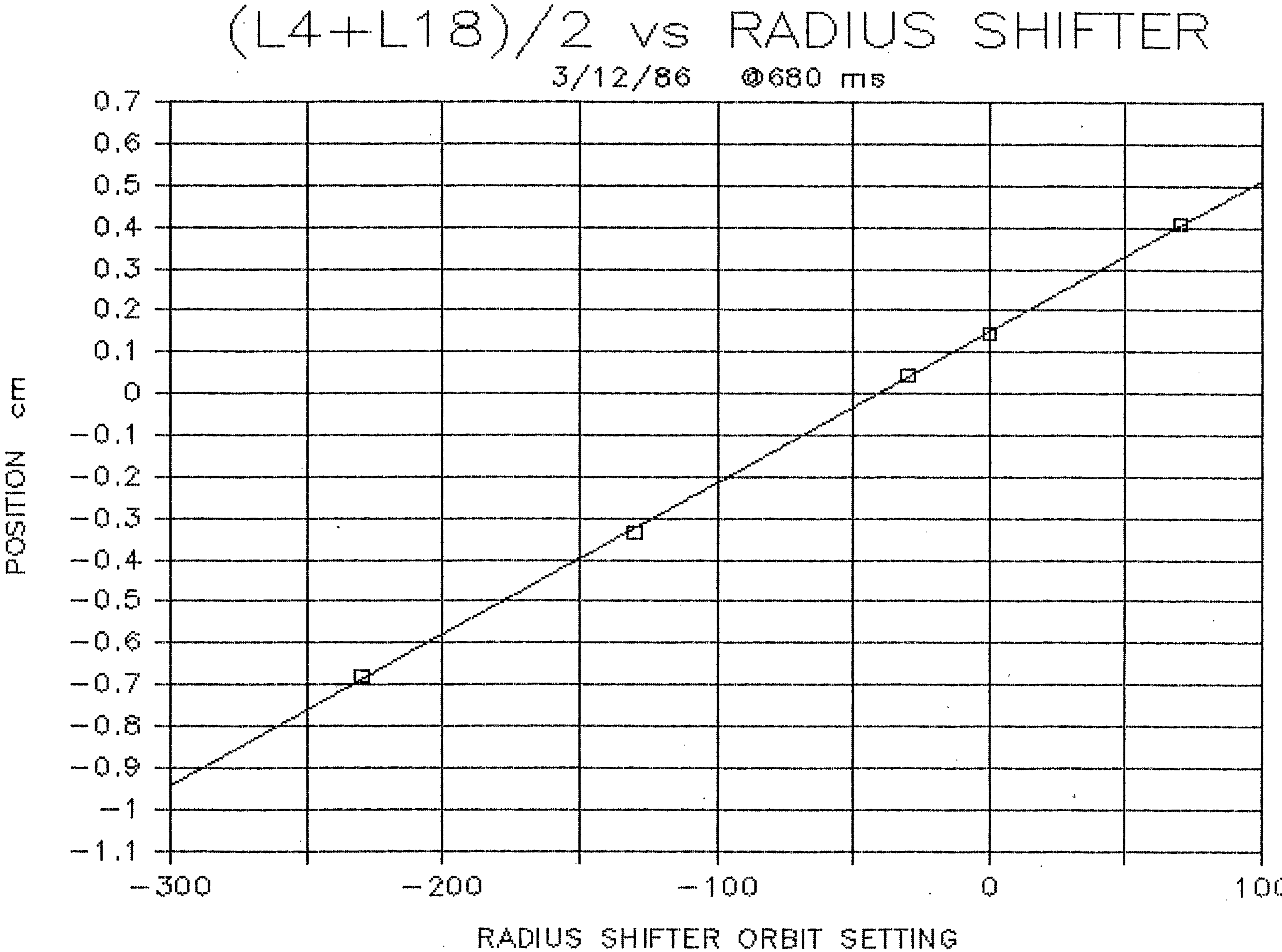
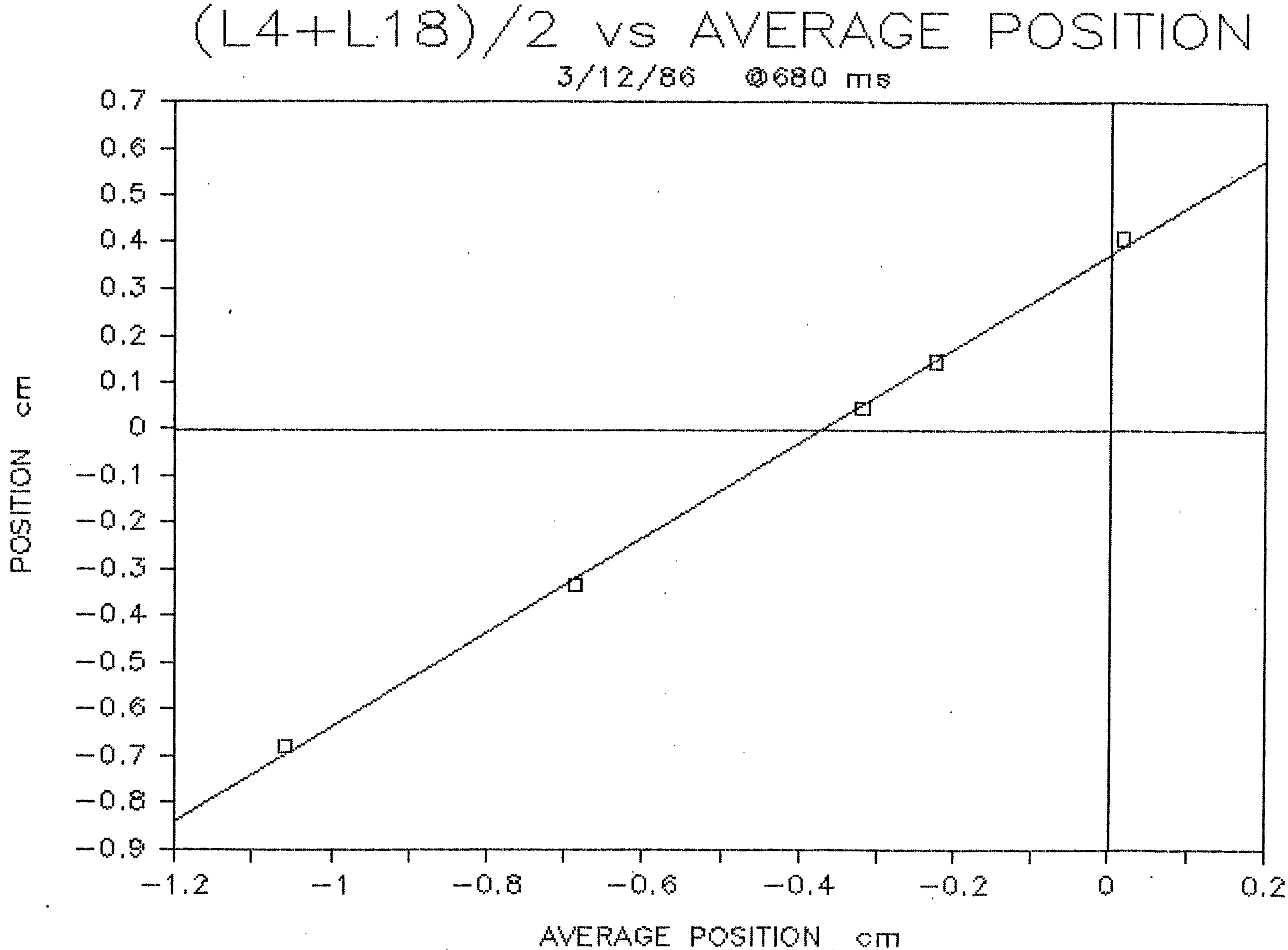

(L4+L18)/2 vs RADIUS SHIFTER
3/12/86 @680 ms
POSITION cm
RADIUS SHIFTER ORBIT SETTING
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1
−1.1
−300
−200
−100
0
100

FIGURE 1



(L4+L18)/2 vs AVERAGE POSITION
3/12/86 @680 ms
POSITION cm
AVERAGE POSITION cm
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1.2
−1
−0.8
−0.6
−0.4
−0.2
0
0.2

FIGURE 2

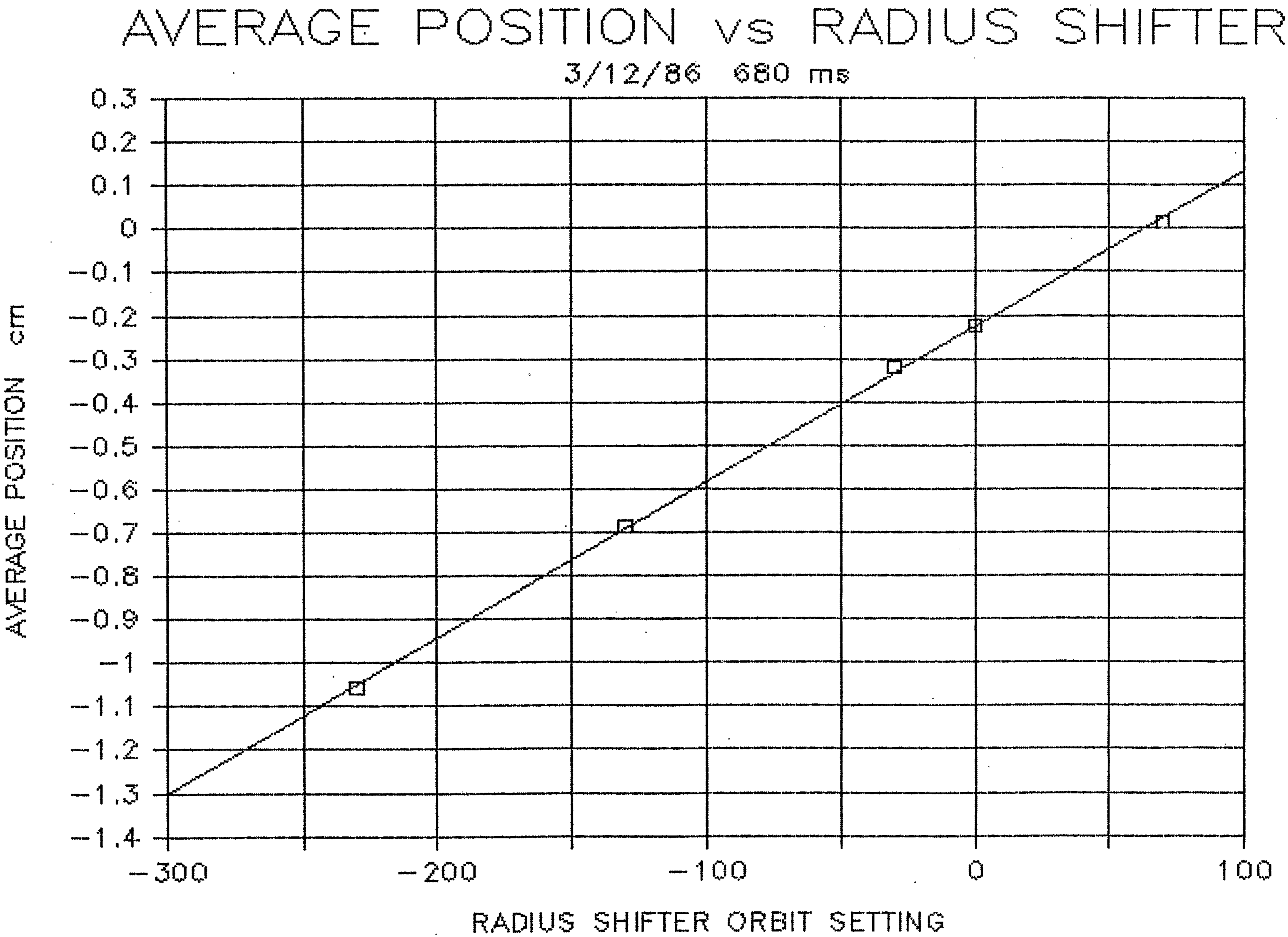
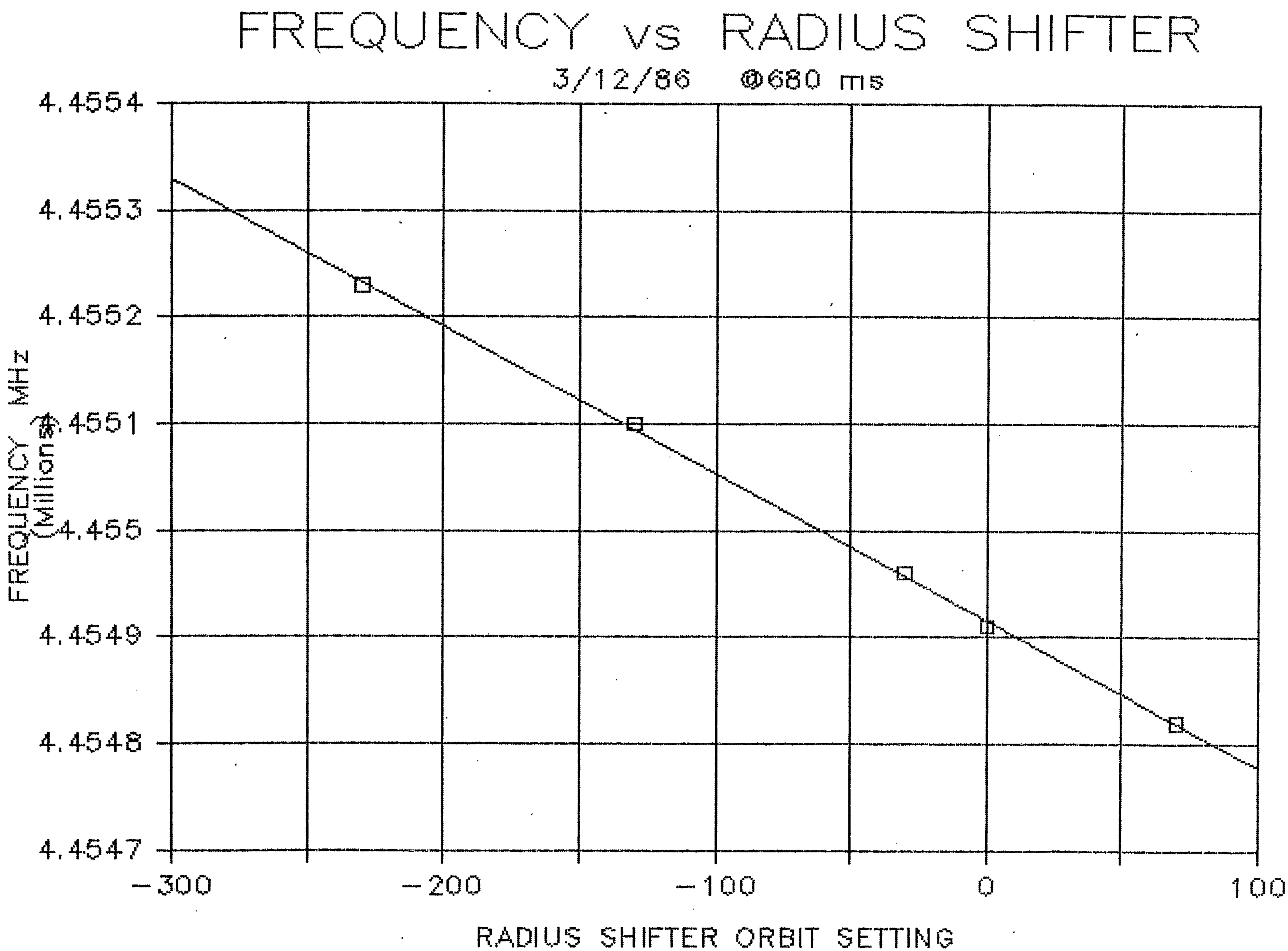

AVERAGE POSITION vs RADIUS SHIFTER
3/12/86 680 ms
AVERAGE POSITION cm
RADIUS SHIFTER ORBIT SETTING
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1
−1.1
−1.2
−1.3
−1.4
−300
−200
−100
0
100

FIGURE 3



FREQUENCY vs RADIUS SHIFTER
3/12/86 @680 ms
FREQUENCY MHz
(Millions)
4.4554
4.4553
4.4552
4.4551
4.455
4.4549
4.4548
4.4547
-300
-200
-100
0
100
RADIUS SHIFTER ORBIT SETTING

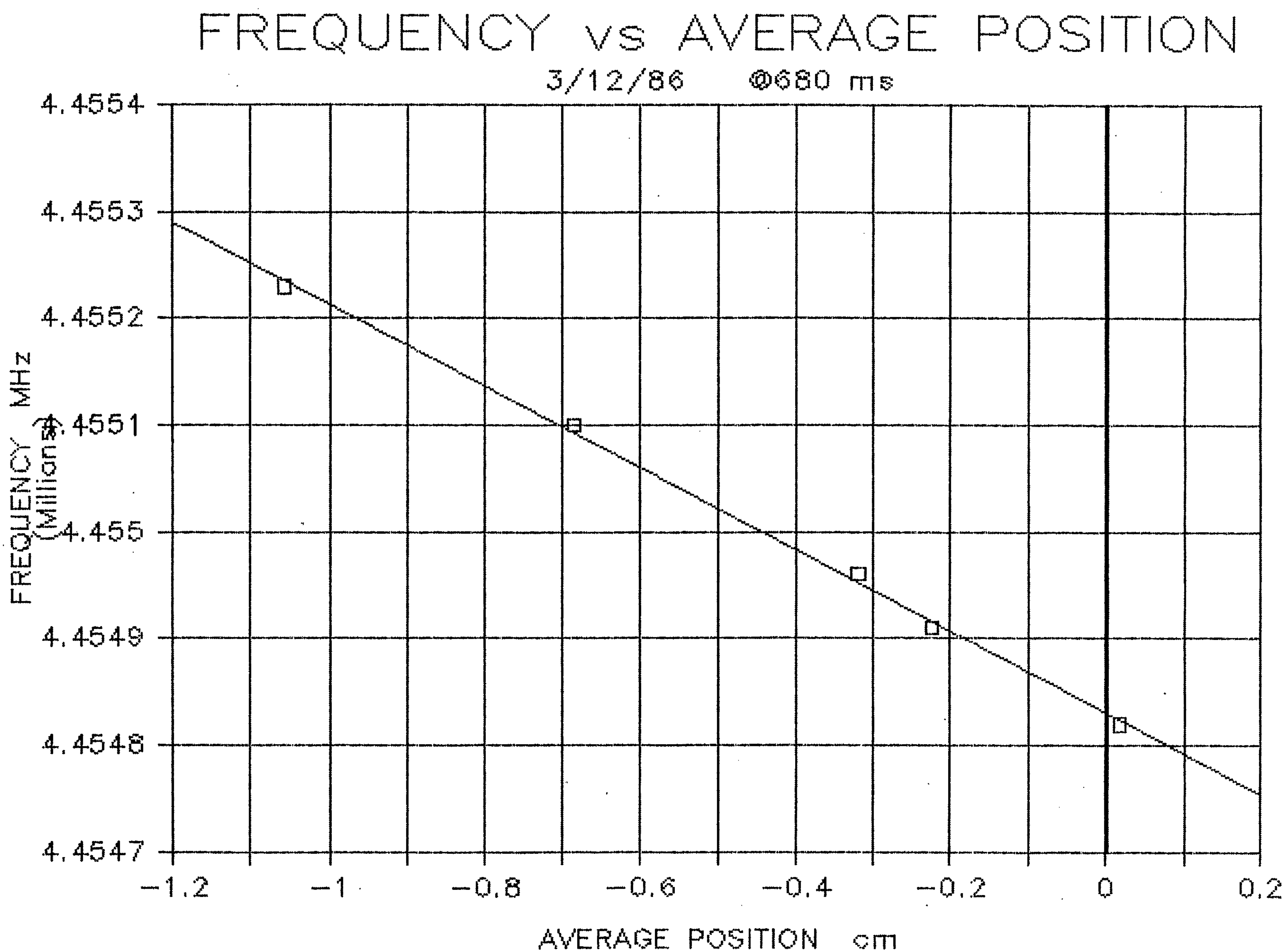

FREQUENCY vs AVERAGE POSITION
3/12/86 @680 ms
FREQUENCY MHz
(Millions)
4.4554
4.4553
4.4552
4.4551
4.455
4.4549
4.4548
4.4547
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
AVERAGE POSITION cm

FIGURE 5