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E. Gianfelice

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RCS closed orbit correction for preserving the beam polarization

E. Gianfelice

Brookhaven National Laboratory, Upton, NY, USA

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Abstract

In this note we discuss some non empirical closed orbit correction methods aiming at improving the equilibrium beam spin polarization in electron storage rings. The corresponding minimization of the spin diffusion phenomenon is of importance also for preserving polarization in ramping machines as the EIC Rapid Cycling Synchrotron.

I. INTRODUCTION

The RCS is supposed to accelerate single electron bunches with $\approx 90\%$ initial polarization from 0.75 GeV to 5 GeV, 9 GeV and 18 GeV, with a polarization preservation of at least 91%. During the ramp the beam crosses a number of spin-orbit resonances occurring at

$$\nu_0 = m \pm m_x Q_x \pm m_y Q_y \pm m_s Q_s \,, \tag{1}$$

where ν_0 is the spin tune which in a perfectly planar ring without rotators is related to the energy by $\nu_0 = a\gamma$, with a gyromagnetic anomaly, which value for electrons is ≈ 0.00116.

In a perfectly planar ring in the absence of spin-rotators the periodic solution, $\hat{n}_0$, to the Thomas-BMT equation is vertical everywhere and the equilibrium electron beam vertical dimension is so small that no depolarizing resonance appears. The resonances appear in presence of vertical orbit distortions which deflect $\hat{n}_0$ from the vertical direction and cause the vertical beam dimension to be not negligible, in particular in presence of betatron coupling.

The RCS energy ramp rate is pushed to 0.1725 GeV/msec for minimizing the time the beam spends on resonances. Moreover the RCS lattices has been designed to avoid the most important resonances [1], namely the imperfection resonances [2]

$$\nu_0 = [Q_y] + kP_B$$

where P_B is the periodicity of the bending magnets, which is 96 for the RCS. However the remaining resonances may still negatively impact the attainable transmission especially on the way to 18 GeV, as spin diffusion is larger at high energy.

At HERAe harmonic bumps were optimized empirically based on the on-line polarization measurement. This allowed to increase the Sokolov-Ternov equilibrium polarization at 27 GeV from 5-10% to above 60%. Such an empirical optimization is not easily applied in an accelerating ring as the RCS. Therefore we are considering here algorithms solely based on the orbit measurement at the BPMs or on the knowledge of the magnet mis-alignments.

For the following simulations we have used the RCS lattice version V5S0 with nominal tunes Q_x=43.13 and Q_y=56.21. We are assuming that at least for the 18 GeV stage, each of the 352 quadrupoles is equipped with a dual plane corrector and a BPM. The assumed

quadrupole rms misalignments are $\delta_x^Q = \delta_y^Q$=100 μm and the rms quadrupole roll angles value is δ_ψ^Q=400 μrad. The same seed is used for all simulations.

II. BPMS BASED HARMONIC CORRECTION

In a planar designed ring without rotators and assuming an iso-magnetic guide, the average along the ring of the spin diffusion term $\gamma\partial\hat{n}/\partial\gamma$ is related to the vertical closed orbit harmonics by [3][4]

$$< \left(\gamma\frac{\partial\hat{n}}{\partial\gamma}\right)^2 >= \nu_0^2 \sum_q \frac{|w_q|^2}{(\nu_0 - k)^2}$$

with $|w_q|^2 = u_q^2 + v_q^2$ and

$$\left\{ \begin{array}{c} u_q \\ v_q \end{array} \right\} = \frac{1}{\pi} \sum_{i=1}^{M} y_i \, \Delta\alpha_i \left\{ \begin{array}{c} \cos q\alpha_i \\ \sin q\alpha_i \end{array} \right\}$$

where y_i is the vertical orbit at the quadrupoles, α_i is the cumulated bending angle at the i^{th} quadrupole and

$$\Delta\alpha_i = \frac{\alpha_{i+1} - \alpha_{i-1}}{2}$$

The beam position at the quadrupoles is known if there is a BPM close to each quadrupole. The harmonics introduced by powering vertical correctors can be computed and the harmonics closest to the spin tune can be compensated by inverting the corresponding system of equations. This "deterministic harmonic spin matching", as it was named at CERN, was applied with somewhat mixed success at LEP. As reported for the 1993 operation in [5], it allowed, with some extra empirical optimizations, to improve polarization from $\approx$ 16% to between 35% and 70%. The cause of this great uncertainty could be traced back to the LEP's BPM system, which was not very sophisticated.

For the RCS where the spin tune spans from 1.7 at 0.75 GeV to 40.8 at 18 GeV and the whole energy ramp takes only 100 msec it is maybe difficult to ramp up and down correctors for compensating few harmonics at a time. One may instead attempt to correct all the relevant harmonics. Figure 1 shows the expected polarization vs. energy for the particular error realization here considered, after orbit correction with MADX MICADO. The polarization along the ramp is computed using the Bmad long_term_tracking code [6]. The initial 100% polarization becomes about 72% at 18 GeV. The black and orange

vertical lines denote the integer and the vertical intrinsic resonances location respectively. Depolarization becomes evident starting with E=12.8 GeV where ν_0=29.

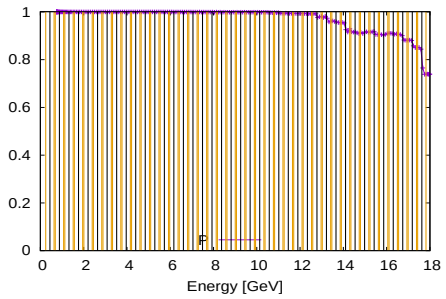


Figure 1: Expected polarization vs. energy during the RCS ramp for a particular error realization after closed orbit correction with MADX MICADO. The final polarization is 72%.

A. Harmonic correction through vertical closed orbit bumps.

We arrange the 320 correctors in the machine arcs in groups of 40 three-correctors bumps. We correct all the 20 harmonics from 21 to 40 at a time, while asking that the harmonics 20 and 41 are unchanged. It is worth to stress that in this approach the corrector kicks are constant, which means their current is monotonically ramped with energy. Between the different combinations of 40 bumps we shall choose that one corresponding to the largest determinant. A crude algorithm was used for sorting the bumps, it is likely that a better combination exists. The vertical closed orbit variation for compensating the relevant orbit harmonics is shown in Figure 2. Table I shows the

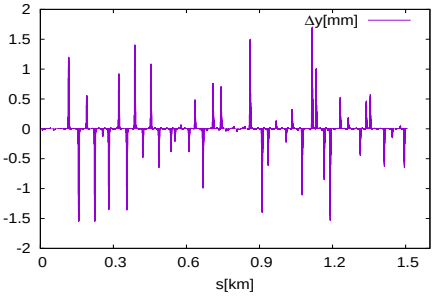


Figure 2: Vertical closed orbit variation due to the 40 harmonic bumps.

results of the Tao [6] calculation at 17.8465 GeV (ν_0=40.5). P_z, with $z = x, y, s$, is the

Table I: Results of 40 bumps correction

	$\delta\hat{n}_{0,rms}$[mrad]	P_∞[%]	P_x[%]	P_y[%]	P_s[%]	α_d [a.u.]
no bumps	8	72.3	81.9	78.6	70.0	2e-4
with 40 bumps	7	87.4	48.7	43.3	77.7	4e-5

polarization in presence of motion purely in the z direction. The spin diffusion term, $\alpha_d \propto \frac{11}{18} \oint \frac{1}{\rho^3} \left(\frac{\partial \hat{n}}{\partial \gamma}\right)^2$, calculated by Tao in linear spin motion approximation, is reduced and asymptotic polarization improved. Polarization along the RCS ramp in presence of these bumps is shown in Figure 3. The polarization at 18 GeV is 91%.

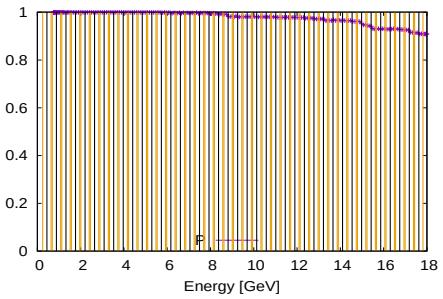


Figure 3: Expected polarization vs. energy during the RCS ramp for a particular error realization with harmonic correction through 40 closed orbit bumps. The final polarization is 91%.

B. Harmonic correction using all arc vertical correctors.

The orbit variation due to some of the 40 bumps is relatively large. Although a more careful choice of the bumps could maybe reduce the orbit perturbation, it is worth trying a different approach. We may use all 320 vertical correctors in the arcs for correcting the same harmonics using a SVD analysis which provides a pseudo-inverse matrix minimizing the corrector kicks. The resulting orbit variation is shown in Figure 4. The largest kick is just 17 μrad. Table II shows the results of the Tao calculation at 17.8465 GeV. Polarization vs. energy along the RCS ramp in presence of the SVD correction is shown in Figure 5. The polarization at 18 GeV is 93.6%.

The fact that some kicks are very small rises the question whether they are doable. Ignoring the 51 correctors with a kick change smaller than 1 μrad the correction quality

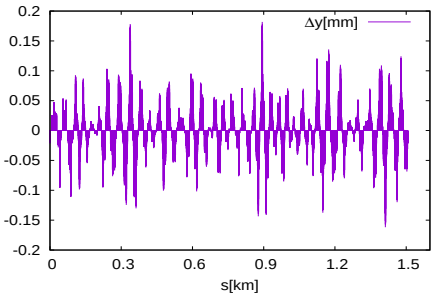


Figure 4: Vertical closed orbit variation due to the 320 vertical correctors used for harmonics correction.

Table II: Tao results of SVD analysis

	$\delta\hat{n}_{0,rms}$[mrad]	P_∞[%]	P_x[%]	P_y[%]	P_s[%]	α_d [a.u.]
no harmonics corr.	8	72.3	81.9	78.6	70.0	2e-4
with SVD corr.	4	89.4	81.0	78.6	88.1	2e-5

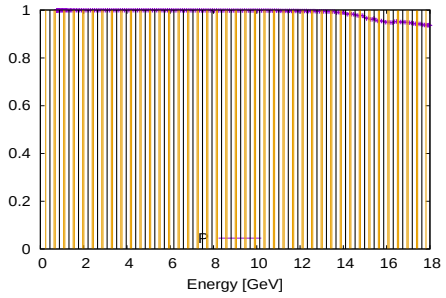


Figure 5: Expected polarization vs. energy during the RCS ramp for a particular error realization with harmonic correction through all 320 arc vertical correctors. Final polarization is 93.6%.

is unchanged, while increasing the threshold to 5 μrad the polarization at the end of the ramp is 88%. The magnetic field due to a RCS corrector is given by [7]

$$B = \frac{NI\mu_0}{g}$$

where N=340 is the number of winding, I is the current, g=0.15 m is the gap and μ_0=4πe-7 =1.26e-6 H/m is the vacuum permeability. The resulting kick is

$$\theta = \frac{e}{p} B\ell$$

where ℓ=0.308 m is the corrector length. The minimum safe current being 0.4 mA [8],

the minimum doable kick is 5.8 nrad at 18 GeV and 10.5 nrad at 10 GeV. Therefore the smallness of the kicks should not be an issue.

C. Impact of BPMs errors on harmonics correction

The impact of BPMs reading errors and misalignments has been studied for the SVD correction. We have assumed that the BPMs are mounted on the close-by quadrupole so that their misalignments are the same as those of the quadrupole, with in addition possible smaller extra misalignments as well as calibration errors. In the following simulations however those extra errors have been not added. The same BPMs errors are applied also to the MADX MICADO orbit correction. The Tao results of the SVD correction in presence of the BPMs errors considered are shown in Table III, while the results of the tracking are shown in Figure 6.

Table III: Tao results of SVD analysis in presence of BPMs misaligned as the close-by quadrupole

	$\delta\hat{n}_{0,rms}$[mrad]	P_∞[%]	P_x[%]	P_y[%]	P_s[%]	α_d [a.u.]
no harmonics corr.	8	72.3	81.9	78.6	70.0	2e-4
SVD (BPMs errs.)	7	80.9	85.8	84.5	80.9	1e-4

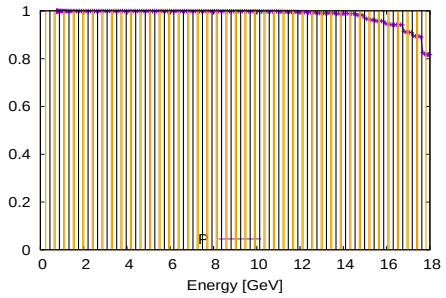


Figure 6: Expected polarization vs. energy during the RCS ramp for a particular error realization with harmonic correction through all 320 arc vertical correctors in presence of BPMs errors. Polarization at 18 GeV is 81.7%.

The BPMs errors have a large impact on the quality of the correction. This can be understood with the fact that the relative errors on the small orbit harmonics far from the betatron tune is large. In fact the orbit harmonics u_q and v_q are related to the harmonics a_p and b_p of the dipole error fields by

$$\left\{ \begin{array}{c} u_q^0 \\ v_q^0 \end{array} \right\} = \frac{Q^2}{Q^2 - q^2} \left\{ \begin{array}{c} a_q \\ b_q \end{array} \right\}$$

The relative error on the computed harmonics is

$$\left\{ \begin{array}{c} \Delta u_q/u_q^0 \\ \Delta v_q/v_q^0 \end{array} \right\} = \frac{Q^2 - q^2}{\pi Q^2} \sum_i^M \Delta y_i \Delta\alpha_i \left\{ \begin{array}{c} \cos q\alpha_i/a_q \\ \sin q\alpha_i/b_q \end{array} \right\}$$

Assuming that the dipole error fields has a white spectrum, the effect of the orbit measurement error is relatively larger for the harmonics far from the betatron tune.

III. POLARIZATION IMPROVEMENT THROUGH "KICK MINIMIZATION".

In a planar ring without rotators, the polarization axis distortion is related to the vertical closed orbit by

$$F(s) = (1 + a\gamma)\Big(Ky + \frac{1}{B\rho}\Delta B_x\Big)$$

where K is the quadrupole strength and $\Delta B_x(s)$ is the horizontal dipole field on the design orbit.

At a quadrupole displaced of δ_y^Q wrt the design orbit it is

$$\frac{1}{B\rho}\Delta B_x = -K\delta_y^Q$$

At those locations where a BPM and a correction dipole are sitting nearby a quadrupole it holds

$$\begin{aligned} Ky + \frac{1}{B\rho}\Delta B_x = Ky - K\delta_y^Q + \frac{\theta^c}{\ell_c} &= K(y_M + \delta_y^M) - K\delta_y^Q + \frac{\theta^c}{\ell_c} \\ &= K(y_M + \delta_Q^M) + \frac{\theta^c}{\ell_c} \end{aligned}$$

where y_M is the actual BPM reading, δ_y^M is the BPM vertical offset wrt design orbit, δ_Q^M is the BPM offset wrt the quadrupole and θ^c is the vertical corrector kick.

The method attempts to minimise the orbit kick due to quadrupoles and correctors [9]. The quantity δ_M^Q is measurable by Beam Based Alignment techniques. At HERAe BBA

was done by scanning the beam position through the quadrupole after changing its strength. The kick minimization method, which became never operational, was tested during a single machine study [10]. It was possible to decrease the rms value of the vertical orbit and vertical slope from 1.1 mm and 110 μrad to 0.75 mm and 20 μrad respectively, improving polarization from 11% to 28%. Unlike HERAe, in the RCS also the horizontal focusing quadrupoles will be equipped with a BPM, which should be an advantage for the application of this method.

For this minimization is not possible to use MADX orbit correction. Therefore a code implementing the MICADO algorithm was written for the RCS. Between all correctors it is chosen that one which minimizes the rms value of $f(s) \equiv F(s)/(1 + a\gamma)$ at the BPMs. This corrector is retained, but its kick recomputed, in the successive iteration for finding the second best corrector, and so on. The Twiss functions are computed once with MADX for the unperturbed ring. The algorithm was first tested for closed orbit correction. Although it was able of correcting the closed orbit, the correctors found were not the same as those found by MADX MICADO. The algorithm was then tested for correcting $f(s)$ for the case that quadrupoles are misaligned only vertically with $\delta^Q_{y,rms}$=100 μm, without previous orbit correction. The correction was done in steps with 25 correctors each. Table IV shows the results of this test. The orbit foreseen by MADX with the kicks computed by the code is used for computing the values shown in the last column.

Table IV: Kick minimization algorithm test

# of correctors	f_{rms}[mrad]	f_{rms}[mrad] (MADX orbit)
0	1.0781	-
25	0.12707	0.19429
+25	0.10282	0.10446
+25	0.08367	0.08487
+25	0.06724	0.07427
+25	0.05525	0.05789
+25	0.04745	0.04934

Thus the method was applied for correcting $f(s)$ in presence of the same misalignments considered in the previous sections and after a first MADX orbit correction, which is what

one would do in operation. The correction was carried out in steps with 25 correctors each. The "measured" vertical position, $y^M = y - \delta_y^M$, with quadrupole-BPM pairs assumed to have the same offset (δy_Q^M=0), were used. The BPM misalignments were included in the MADX orbit correction. Before kick minimization it was x_{rms}=0.19 mm and y_{rms}=0.15 mm. The rms value of $f(s)$ at the BPMs was reduced by at least 3 order of magnitude. However, a Tao computation at 17.8465 GeV gave disappointing results (see Table V).

Table V: Tao results of kick minimization applied to the previously well corrected orbit

	$\delta\hat{n}_{0,rms}$	P	P_x	P_y	P_s	α_d
MICADO	8 mrad	72.3%	81.9%	78.6%	70.0%	2e-4
+ kick minimization	13 mrad	60.5%	83.5%	83.8%	60.4%	4e-4

To check the algorithm implementation, the code was used for the machine without previous orbit correction. At 17.8465 GeV Tao computed that $\delta\hat{n}_0$ was decreased from 80 mrad to 13 mrad and polarization increased from 9% to 60%. This means that the implementation was correct but the method is not better than the usual orbit correction at improving polarization.

IV. POLARIZATION IMPROVEMENT THROUGH DIRECT QUADRUPOLE OFFSETS COMPENSATION.

A. Results by using the close-by correctors.

The alignment of the quadrupoles wrt the design orbit is unavoidably not perfect. However, if the offsets are known with enough precision, one may attempt to compensate the resulting kicks by the close-by dual plane correctors. The orbit kicks due to the misalignments are

$$\left\{ \begin{array}{c} \theta_x^Q \\ \theta_y^Q \end{array} \right\} = \left\{ \begin{array}{c} +K\delta_x^Q \\ -K\delta_y^Q \end{array} \right\}$$

or, correcting for the possible quadrupole roll angle ψ ($\psi > 0$ is a clockwise rotation)

$$\left\{ \begin{array}{c} \theta_x^Q \\ \theta_y^Q \end{array} \right\} = \left\{ \begin{array}{c} +K\delta_x^Q \cos\psi + K\delta_y^Q \sin\psi \\ -K\delta_x^Q \sin\psi - K\delta_y^Q \cos\psi \end{array} \right\}$$

The kick that the corrector must provide is scaled for taking into account the possible difference in β function between the corrector and the quadrupole locations. The closed orbit computed by MADX after inserting the corrective kicks computed assuming a perfect knowledge of the quadrupole misalignments, is reduced from x_{rms}=1.24 mm and y_{rms}=2.68 mm to x_{rms}=0.30 mm and y_{rms}=0.22 mm. Table VI shows the results of orbit correction and Tao computations at 17.8465 GeV.

Table VI: Results with MICADO orbit correction and with direct quadrupole offsets compensation by close-by correctors with perfect offsets knowledge.

	x_{rms}	y_{rms}	$\delta\hat{n}_{0,rms}$	P_∞	P_x	P_y	P_s	α_d
	[mm]	[mm]	[mrad]	[%]	[%]	[%]	[%]	a.u.
MICADO	0.19	0.14	8	72.3	81.9	78.6	70.0	2e-4
quad comp.	0.22	0.22	8	88.7	91.4	91.0	88.3	3e-5

The first row in Table VI refers to a pure orbit correction by several MICADO iterations. Although MICADO achieved a smaller rms orbit, the resulting polarization is lower. The long term tracking results are shown in Figure 7. The polarization at 18 GeV is 90.2%.

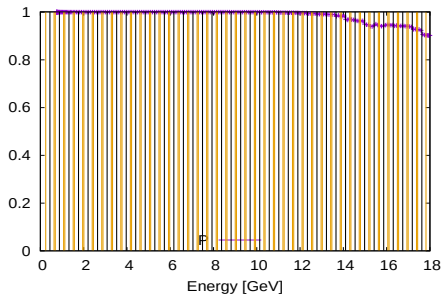


Figure 7: Expected polarization vs. energy during the RCS ramp for a particular error realization with quadrupole kicks compensation, assuming perfectly known misalignments. The correction is done by using the close-by correctors. Final polarization is 90.2%.

Table VII shows the orbit and the Tao computation results in presence of errors in the misalignments knowledge. Long term tracking results are shown in Figure 8.

Table VII: Results of orbit correction with direct quadrupole offsets compensation in presence of errors in the offsets knowledge

$\Delta\delta^Q_{x,rms}$	$\Delta\delta^Q_{y,rms}$	$\Delta\delta^Q_{\psi,rms}$	x_{rms}	y_{rms}	$\delta\hat{n}_{0,rms}$	P_∞	α_d
[μm]	[μm]	μrad]	[mm]	[mm]	[mrad]	[%]	a.u.
5	5	20	0.36	0.25	8	85.3	6.0e-5
10	10	40	0.42	0.48	15	79.9	1.1e-4
15	15	60	0.76	0.76	23	73.2	1.9e-4
20	20	80	1.11	1.06	31	65.5	3.0e-4

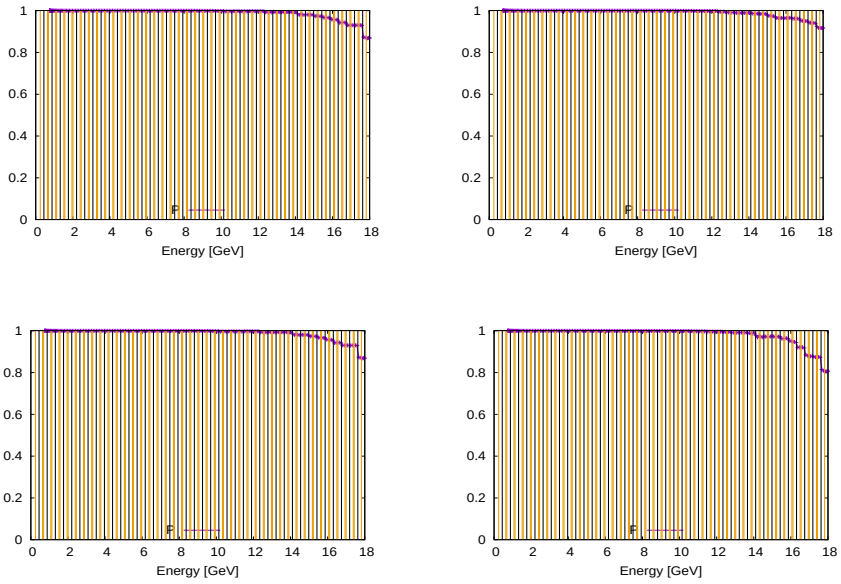


Figure 8: Expected polarization vs. energy during the RCS ramp for a particular error realization with quadrupole kicks compensation by close-by correctors, assuming errors in the knowledge of the misalignments. Top left: $\Delta\delta^Q_{x,rms}$=$\Delta\delta^Q_{y,rms}$=5 μm and $\Delta\delta^Q_{\psi,rms}$=20 μrad; P=90.2% at 18 GeV. Top rigth: $\Delta\delta^Q_{x,rms}$=$\Delta\delta^Q_{y,rms}$=10 μm and $\Delta\delta^Q_{\psi,rms}$=40 μrad; P=91.7% at 18 GeV. Bottom left: $\Delta\delta^Q_{x,rms}$=$\Delta\delta^Q_{y,rms}$=15 μm and $\Delta\delta^Q_{\psi,rms}$=60 μrad; P=86.9% at 18 GeV. Bottom right: $\Delta\delta^Q_{x,rms}$=$\Delta\delta^Q_{y,rms}$=20 μm and $\Delta\delta^Q_{\psi,rms}$=80 μrad; P=80.7% at 18 GeV.

B. Results by using "fake" correctors

It is worth trying to see whether the correction can be improved by introducing a corrector on top of the quadrupole. For this purpose all the quadrupoles were divided

into two halves and a zero-length dual plan corrector inserted in the middle, while keeping misalignments of the two halves the same as before dividing the quadrupoles.

Table VIII shows the results of orbit correction and Tao computations at 17.8465 GeV for various misalignment measurement errors.

Table VIII: Results of orbit correction with direct quadrupole offsets compensation by correctors in the middle of the quadrupole, in presence of errors in the misalignments knowledge

$\Delta\delta^Q_{x,rms}$	$\Delta\delta^Q_{y,rms}$	$\Delta\delta^Q_{\psi,rms}$	x_{rms}	y_{rms}	$\delta\hat{n}_{0,rms}$	P_∞	α_d
[μm]	[μm]	μrad]	[mm]	[mm]	[mrad]	[%]	a.u.
0	0	0	0.025	0.036	1	92.0	3e-6
5	5	20	0.362	0.240	7	90.6	1e-5
10	10	40	0.711	0.518	14	86.7	5e-5
15	15	60	1.068	0. 807	22	80.7	1e-4
20	20	80	1.430	1.110	31	73.1	2e-4

Figure 9 shows the long term tracking results for the case that the quadrupole misalignments are perfectly known. The polarization transmission is almost 100%.

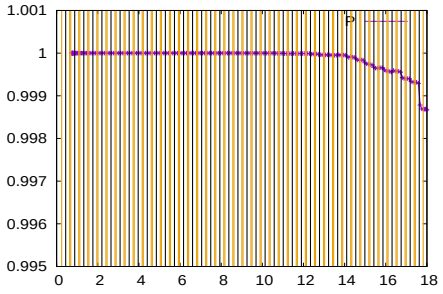


Figure 9: Expected polarization vs. energy during the RCS ramp for a particular error realization with quadrupole kick compensation by corrector in the quadrupole middle, assuming perfect knowledge of the misalignments (note the different ordinate axis scale).

Figure 10 shows the results of the long term tracking in presence of misalignment measurement errors.

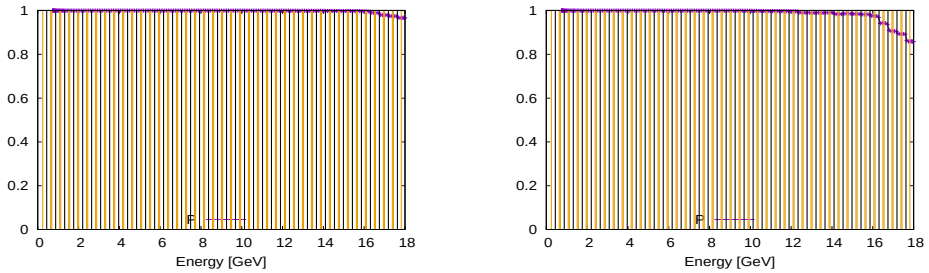


Figure 10: Expected polarization vs. energy during the RCS ramp for a particular error realization with quadrupole kicks compensation by correctors in the quadrupole middle, assuming errors in the knowledge of the misalignments. Left: $\Delta\delta^Q_{x,rms}$=$\Delta\delta^Q_{y,rms}$=10μm and $\Delta\delta^Q_{\psi,rms}$=40 μrad; P=96.7% at 18 GeV. Rigth: $\Delta\delta^Q_{x,rms}$=$\Delta\delta^Q_{y,rms}$=20 μm and $\Delta\delta^Q_{\psi,rms}$=80 μrad; P=85.8% at 18 GeV.

V. CONCLUSIONS

By correcting all the relevant harmonics at once the best results are obtained by applying a SVD analysis using all 320 vertical correctors in the arcs. However in presence of BPMs errors the polarization transmission decays from 93.6% to 81.7% when assuming BPMs perfectly aligned with the close-by quadrupole.

The compensation of the quadrupole kicks by the close-by corrector gives a polarization transmission of 90.2% if their misalignment is perfectly known, which decays to 80.7% when the measurement errors are $\Delta\delta^Q_{x,rms}$=$\Delta\delta^Q_{y,rms}$=20 μm and $\Delta\delta^Q_{\psi,rms}$=80 μrad.

Assuming a dual corrector in the center of each quadrupole the transmission is 99.9% if the quadrupole misalignments are perfectly known. With measurement errors $\Delta\delta^Q_{x,rms}$=$\Delta\delta^Q_{y,rms}$=20 μm and $\Delta\delta^Q_{\psi,rms}$=80 μrad, the transmission is 85.8.

Above results have been obtained for one particular error realization only, with rms quadrupole misalignments $\delta^Q_x = \delta^Q_y$=100 μm and rms quadrupole roll angles δ^Q_ψ=400 μrad.

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