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Parameters of double-RF system for cooling EIC protons at injection energy

S. Seletskiy

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Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

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S. Seletskiy*, A. Fedotov, D. Kayran

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Abstract

Cooling Electron Ion Collider (EIC) protons at injection energy requires reducing the proton peak current to eliminate potential collective instabilities. This can be achieved by employing a double-RF system to produce flat-top proton bunches. In this note, we calculate the expected proton peak current for a range of RF parameters.

1 Protons in EIC Hadron Storage Ring at injection energy

Protons are injected into the EIC Hadron Storage Ring (HSR) at $\gamma = 25.4$. Electron cooling reduces the vertical emittance of the proton bunches by a factor of ≈ 7 prior to the ramp-up to collision energy.

The HSR parameters relevant to the following considerations are listed in Table 1.

Table 1: HSR parameters at injection energy

relativistic γ	25.4
transition γ_{Tr}	23
RF number h	315
RF main harmonic frequency f_{RF} [MHz]	24.6
number of protons per bunch N_p	$2.8 \cdot 10^{11}$

*seletskiy@bnl.gov

2 Some useful analytic expressions

For a single RF system with voltage given by $V = V_{RF} \sin(\omega_{RF} t)$, the synchrotron tune is given by:

$$Q_s = \sqrt{\frac{eV_{RF}h|\eta|}{2\pi\beta^2 E}} \tag{1}$$

where $\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{Tr}^2}$, $\beta \approx 1$ is a relativistic factor, and $E = \gamma m_p c^2$ is the proton's energy.

The resulting longitudinal distribution of a proton bunch $f = f_t \cdot f_\delta$ is well approximated by:

$$\begin{aligned} f_t = & \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{t^2}{2\sigma_t^2}\right) \\ f_\delta = & \frac{1}{\sqrt{2\pi}\sigma_\delta} \exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right) \end{aligned} \tag{2}$$

The relation between σ_t and σ_δ is given by:

$$\sigma_t = \frac{|\eta|}{\omega_s}\sigma_\delta \tag{3}$$

where the synchrotron frequency is $\omega_s = Q_s \frac{\omega_{RF}}{h}$, and $\omega_{RF} = 2\pi f_{RF}$.

Analytical expressions for the case of a double-RF system were derived in [1].

The HSR RF system employs the 2nd harmonic of the main RF, with an overall RF voltage $V = V_{RF} \sin(\omega_{RF} t) - \frac{V_{RF}}{2} \sin(2\omega_{RF} t)$. In this case, the resulting bunch distribution can be approximated by:

$$\begin{aligned} f_t = & \frac{2}{\Gamma(1/4)\varsigma_t} \exp\left(-\frac{t^4}{\varsigma_t^4}\right) \\ f_\delta = & \frac{1}{\sqrt{2\pi}\sigma_\delta} \exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right) \end{aligned} \tag{4}$$

where $\Gamma(1/4) \approx 3.6256$.

The relation between the distribution parameters is:

$$\varsigma_t = \frac{2^{3/4}}{\omega_s}\sqrt{\frac{|\eta|Q_s}{h}}\sqrt{\sigma_\delta} \tag{5}$$

For electron cooling, the rms relative momentum of the protons is important. Therefore, it is useful to express the peak current of the proton bunch in terms of σ_δ.

For a single RF system, this expression is:

$$I_{1RF} = \frac{1}{\sqrt{2\pi}} \frac{eN_p \omega_{RF} Q_s}{|\eta| h \sigma_\delta} \tag{6}$$

For a double RF system, the peak current becomes:

$$I_{2RF} = \frac{2^{1/4}}{\Gamma(1/4)} eN_p \omega_{RF} \sqrt{\frac{Q_s}{|\eta| h \sigma_\delta}} \tag{7}$$

Finally, the ratio of the peak currents is:

$$\frac{I_{1RF}}{I_{2RF}} = \frac{\Gamma(1/4)}{2^{3/4}\sqrt{\pi}} \sqrt{\frac{Q_s}{|\eta| h \sigma_\delta}} \tag{8}$$

3 Simulations

We use dedicated Python code to simulate the resulting distribution of the proton bunch in a flat RF bucket.

In the simulations, we "inject" a proton bunch containing 10^5 macro-particles, with parameters matched to a single-RF bucket, into a double-RF one. We let the bunch develop for $N_t = 2 \cdot 10^5$ turns. It was found that using a larger N_t does not change the resulting distribution, even though at $N_t = 2 \cdot 10^5$, the remnant filamentation pattern can still be clearly observed.

On each turn, each particle's phase-space coordinates (t, δ) are calculated according to:

$$\begin{aligned} \delta_{n+1} = &\quad \delta_n + \tfrac{eV_{RF}}{\beta^2 E} \sin(\omega_{RF} t_n) - \tfrac{eV_{RF}}{2\beta^2 E} \sin(2\omega_{RF} t_n) \\ t_{n+1} = &\quad t_n + \tfrac{\eta h}{f_{RF}} \delta_{n+1} \end{aligned} \tag{9}$$

In the code, we vectorize the algorithm given by Eq. (9). Therefore, the code runs rather fast—the outlined calculation takes about 10 minutes on a laptop with a 1.90 GHz processor.

4 Studies

4.1 Comparing analytic expressions to simulations

The calculations discussed in this section were performed for the double-RF bucket with the main harmonic voltage $V_{RF} = 40$ kV. The resulting bucket is shown in Fig. 1.

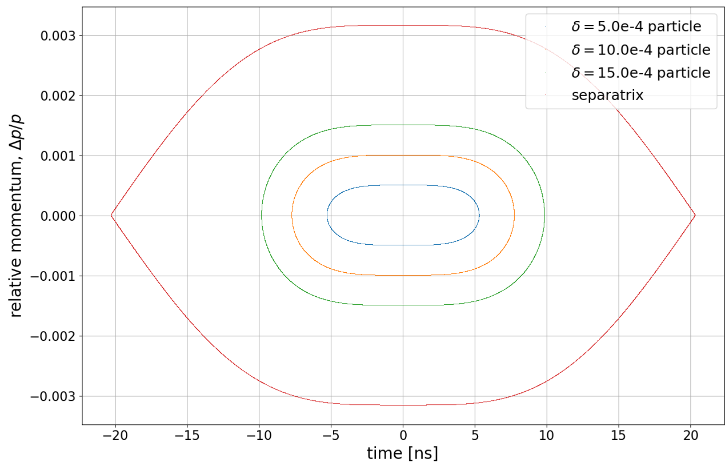

Figure 1: Double-RF bucket with main harmonic voltage $V_{RF} = 40$ kV.

Figure 2 compares the peak current calculated from the simulated distribution with the peak current predicted by Eq. (7) for various relative momentum spreads.

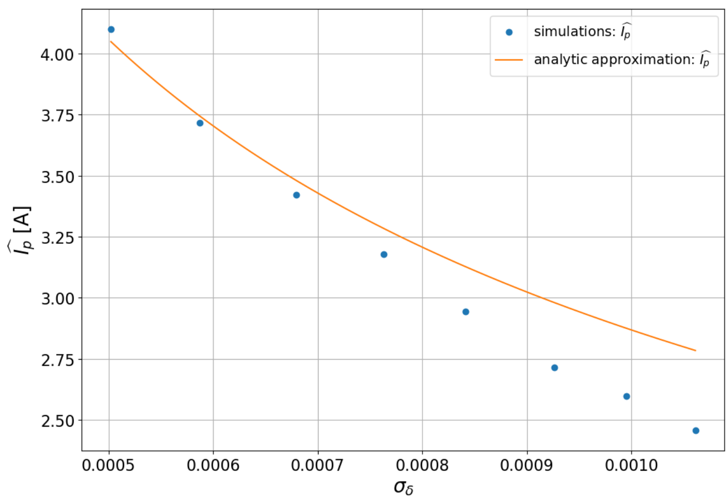

Figure 2: Peak current calculated from the simulated distribution (dots) and the peak current predicted by Eq. (7) (solid line). The calculations are performed for $V_{RF} = 40$ kV.

As one can see, the analytical approximation fits the simulations well when the particles in the bunch have small synchrotron amplitudes. This point is further illustrated by Fig. 3.

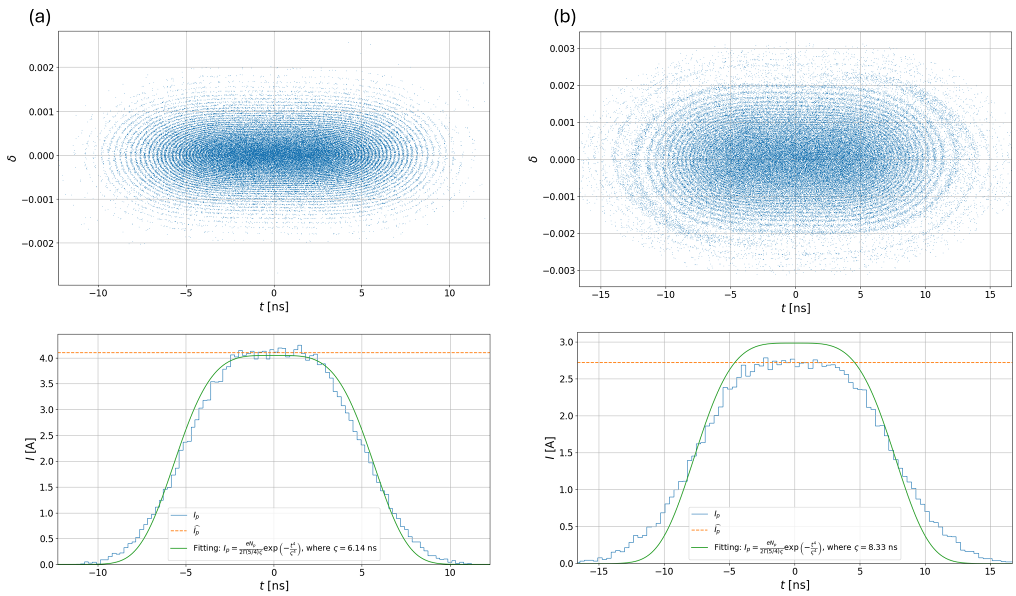

Figure 3: Bunch's longitudinal distribution and the corresponding fit provided by Eqs. (4)-(5). The results are shown for case (a), with $\sigma_\delta = 5.02 \cdot 10^{-4}$, and case (b), with $\sigma_\delta = 9.26 \cdot 10^{-4}$.

4.2 Choice of parameters

Figure 2 shows that, for a double-RF system with a main voltage of 40 kV, one can expect a peak current of $\widehat{I_p} \approx 3.1$ A at an rms relative momentum spread of $\sigma_\delta \approx 8 \cdot 10^{-4}$. Further reduction of the peak current requires proton bunches with a larger momentum spread.

The peak current can be reduced while keeping the momentum spread around $(7-8) \cdot 10^{-4}$ by reducing the RF voltage. Figure 4 shows the dependence of the peak current on the choice of RF voltage, calculated from Eqs. (1) and (7).

Next, we explore the resulting peak current as a function of the relative momentum spread at $V_{RF} = 25$ kV. The results are shown in Fig. 5.

The double-RF system, with a main voltage of 25 kV, allows us to obtain a proton bunch with a momentum spread of $\sigma_\delta = 7.5 \cdot 10^{-4}$ and a peak current of $\widehat{I_p} = 2.7$ A.

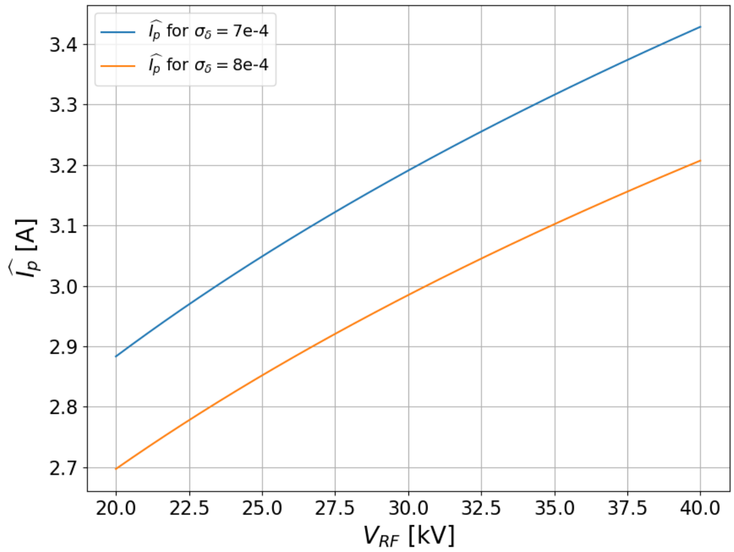

Figure 4: Peak current depending on the RF voltage for $\sigma_\delta = 7 \cdot 10^{-4}$ (blue line) and $\sigma_\delta = 8 \cdot 10^{-4}$ (orange line).

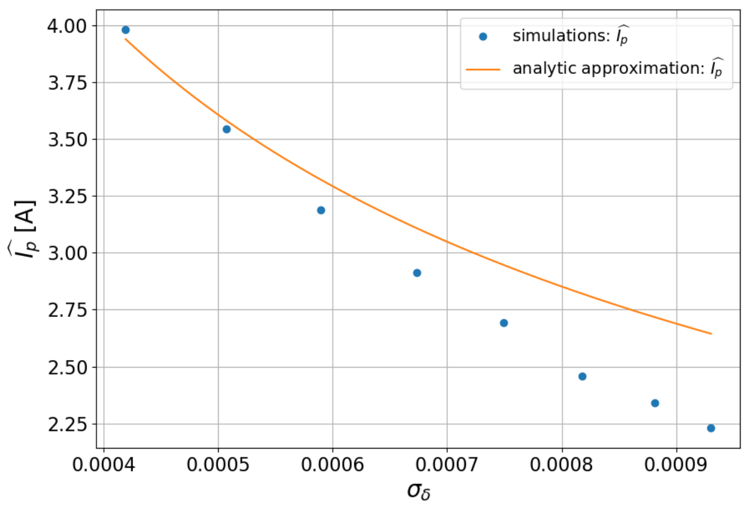

Figure 5: Peak current calculated from the simulated distribution (dots) and the peak current predicted by Eq. (7) (solid line). The calculations are performed for $V_{RF} = 25$ kV.

Finally, we show the ratio of the peak currents for the single-RF and double-RF buckets, assuming that the momentum spread is the same in both cases (Fig. 6). Although this is not directly relevant to the question at hand—namely, how to achieve proton bunches with both low energy spread and low peak current in the HSR at injection energy—the ratio of the currents has been discussed extensively on several occasions in the context of the HSR RF system. It would therefore be useful to clarify this issue once and for all.

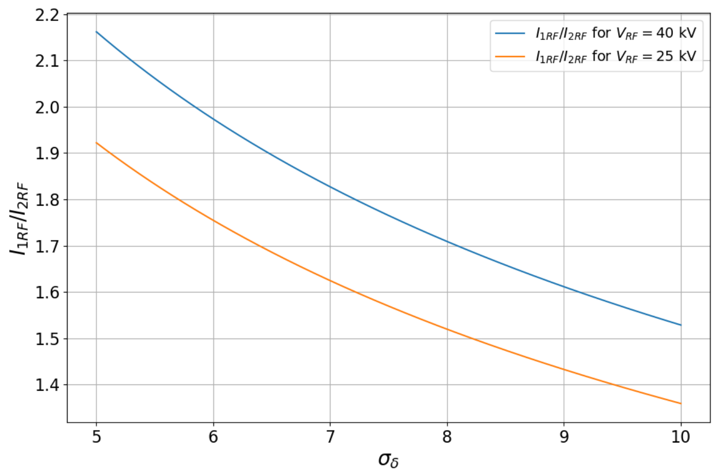

Figure 6: Ratio of the peak currents for the single-RF and double-RF buckets for $V_{RF} = 40$ kV (blue line) and $V_{RF} = 25$ kV (orange line).

5 Conclusion

The double-RF system for the EIC Hadron Storage Ring is required to reduce the peak current of the proton bunches. In this note, we consider a double-RF system employing a second harmonic.

For a main RF voltage of 40 kV, a proton bunch with a relative momentum spread of $\sigma_\delta = 7.6 \cdot 10^{-4}$ has a peak current of $\widehat{I_p} = 3.2$ A.

For a main RF voltage of 25 kV, a proton bunch with a relative momentum spread of $\sigma_\delta = 7.5 \cdot 10^{-4}$ has a peak current of $\widehat{I_p} = 2.7$ A.

Both of the outlined options require an RF system with accurate control of the voltage and high phase stability. The final choice should be guided by the accelerator-physics requirements for suppressing collective instabilities in the proton bunch, as well as by the availability of engineering solutions for achieving the required RF stability.

References

[1] A. Hofmann and S. Myers, Beam dynamics in a double RF system, CERN ISR-TH-RF/80-25, Proc. of XI International Conference on High Energy Accelerators, CERN, Geneva, July 7-11, 1980.