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Effect of magnetic hysteresis on the dipole field in the EIC Rapid Cycling Synchrotron

H. Singh

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Brookhaven National Laboratory

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Harshita Singh and Boris Podobedov

1 Introduction

The soft iron material, of which the yoke of a normal conducting magnet is made of, is subject to magnetic hysteresis under a time varying excitation. The hysteresis loop is influenced by the flow of induced eddy currents, which further affects the magnet field.

The Rapid Cycling Synchrotron (RCS) at the planned Electron Ion Collider (EIC) facility will accelerate polarized electrons up to 18 GeV at 1 Hz. The impact of hysteresis in the magnet yoke on the magnet field during the ramp, including potential effects on cycle-to-cycle reproducibility, need to be evaluated. This will be the subject of discussion of this tech note in the context of the RCS dipoles.

2 Underlying theory

In this section, basic mathematical relations to determine the time rate of change of the dipole field are derived. These will then be used to examine answers to the following questions:

- How do the magnet air gap width and the mean yoke length decide the extent of the impact of hysteresis on the magnet field?
- How do the ramp rate and the field magnitude affect the hysteresis loop?
- Are there any related effects that may affect the field reproducibility?

To this end, let us consider the example of a C-shaped dipole magnet shown in Figure 1. The area of cross-section of the yoke along the field path is assumed to be the same. The excitation winding is split into two coils of N turns each. The coils are wound around the poles and are connected in series. For simplicity, the effect of leakage and fringe fields will be neglected in the analysis presented herein.

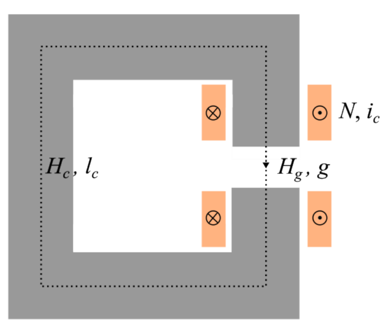


Figure 1: Field path in a C-dipole.

Using Ampere's Law, the relationship between the fields in different parts of the magnet may be written as

$$H_c l_c + H_g g = 2N i_c \tag{1}$$

where H_c and H_g are the mean field intensities in the yoke and air gap respectively, l_c is the mean field path length inside the yoke, g is the air gap width and i_c is the excitation current. Taking the time derivative of these quantities yields

$$l_c \frac{dH_c}{dt} + g \frac{dH_g}{dt} = 2N \frac{di_c}{dt} \tag{2}$$

Fields in the yoke at a given moment in time will be a function of the material properties, excitation current magnitude and also the field history. Using the relationship between field intensity and flux density, and applying Gauss' Law ($B_c = B_g$) gives

$$\frac{l_c}{\mu_0} \frac{d}{dt}\left(\frac{B_g}{\mu_r}\right) + \frac{g}{\mu_0} \frac{dB_g}{dt} = 2N \frac{di_c}{dt} \tag{3}$$

where B_g and B_c are the mean flux densities (fields) in the air gap region and yoke, and μ_0 is the permeability of free space. The variable μ_r denotes the relative permeability of the magnet yoke (given by B_g/H_c) and is a function of B_g, change in the field δB_g and the material. Simplifying (3) yields

$$\frac{dB_g}{dt} = \begin{cases} \frac{2\mu_0 N}{\frac{l_c}{\mu_r}(1-\frac{B_g}{\mu_r}\frac{d\mu_r}{dB_g})+g} \frac{di_c}{dt}, & \text{if } B_g, H_c \neq 0 \\ \frac{2\mu_0 N}{g}\frac{di_c}{dt}, & \text{if } B_g = 0 \text{ or } H_c = 0 \end{cases} \tag{4}$$

A normal conducting magnet is usually designed employing a single valued non-linear B-H characteristic for the yoke. In this case, the quantities μ_r and $\frac{d\mu_r}{dB_g}$ in the denominator are unique for a given value of B_g and Eq. (4) gives a unique expected value of $\frac{dB_g}{dt}$. In a real hysteretic material, the rising and falling branches of the B-H trajectory generally have different local values of μ_r and $\frac{d\mu_r}{dB_g}$ at the same B_g. Therefore, the denominator in (4), and thereby $\frac{dB_g}{dt}$, differs from the value predicted using the single valued design characteristic. This difference will be referred to as the deviation in $\frac{dB_g}{dt}$ herein.

The relative deviation will depend on three factors:

(a) magnet dimensions l_c and g. It can be inferred from (4) that the larger the air gap width in comparison to the mean yoke length, the smaller the effect of hysteresis on the air gap field.

(b) ramp rate. Ferromagnetic materials, the laminations of which are generally used to construct the yoke, have an appreciable electrical conductivity. The faster the current is ramped, the larger are the induced eddy currents and losses (both classical and anomalous) in the material, wider is the hysteresis loop and the lower is the effective permeability, and thereby, attained field. Assembling a stack of laminations separated by a layer of insulation aids in reducing the eddy currents. As an example, the relative permeability of a 3% grain-oriented silicon steel of lamination thickness 0.305 mm at 0.6 T for a sinusoidal excitation drops by roughly 4% in going from 60 Hz to 100 Hz and by roughly 25% in going from 60 Hz to 400 Hz [1] (see Fig. 12(a)).

(c) field magnitude. The effective permeability initially increases as the field strength is increased from zero, reaches a peak value and then decreases.

Apart from the deviation in $\frac{dB_g}{dt}$, its reproducibility is also of interest. A phenomenon closely related to hysteresis that might affect reproducibility is the Barkhausen effect [2]. The interaction of the domain walls with the inhomogeneities in the material results in sudden changes in the local magnetization. This manifests itself as step changes in the field and as high frequency noise in the time derivative of the field. The Barkhausen noise depends on both the material microstructure and the ramp rate. Sudden change in the field is accompanied by localized eddy currents, the macroscopic aggregate of which contributes to anomalous loss [2].

The silicon steel laminations used to build an accelerator magnet are processed to remove inhomogeneities and so they produce little Barkhausen noise. In [3], it is shown that the peak of the

Barkhausen noise signal seen at the ends of a search coil wound around a sample of such a steel is roughly 1/2000 times the peak emf voltage signal (corresponding to the macroscopic hysteresis field without the noise) for a peak field of 0.4 T at a frequency of 50 Hz (see Fig. 4 of [3]). This is only 0.05% of the peak time rate of change of field. Note the material properties (for the tested material in the plot) are not clearly specified for this example. The noise will be different for a grain-oriented versus a non-oriented steel, lamination thickness and the test frequency.

Literature on the extent of the impact of Barkhausen effect on the field in accelerator magnets is unavailable which likely indicates this has not presented an issue in accelerators. Also, since this noise originates from stochastic microscopic events occurring throughout the material, it is expected that some of this noise seen by the particle beam will average out over the length of the magnet, affecting the integrated field to an even lesser extent.

An impact of hysteresis is that the fields in the material take some time to settle into a steady state. In practice, a magnet is repeatedly cycled between two known states to establish a repeatable B-H trajectory in the good field region [4] (about 3-4 cycles for quasi-static current variation, the exact number is dependent on the ramp rate). This is to say that although the deviation in $\frac{dB_g}{dt}$ from its expected value remains, cycle-to-cycle repeatability can be achieved after the initial few cycles.

In the next section, the deviation in $\frac{dB_g}{dt}$ for an RCS dipole will be approximated using (4).

3 Application to RCS dipoles

The dipoles in the RCS of EIC have the following specifications:

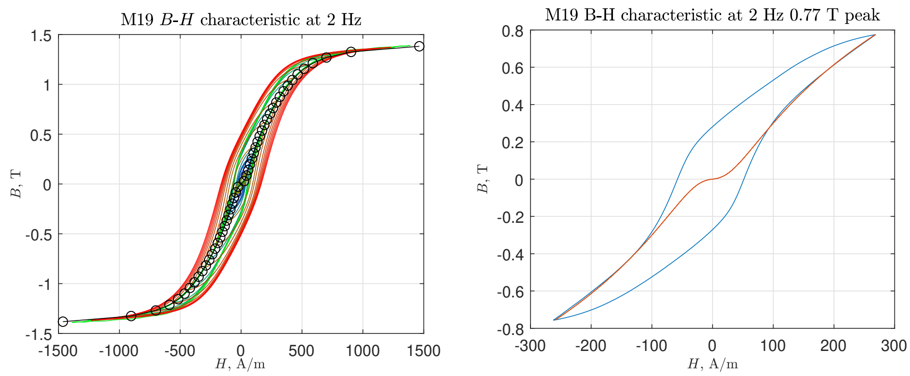
- An air gap g of 4 cm
- A peak field B_{max} of 0.655 T
- A turn count per pole N of 3
- A field ramp rate of 6.28 T/s (calculation based on the assumption of infinite permeability of the magnet yoke)

The excitation current waveform $i_c(t)$ for these dipoles is trapezoidal in shape with a non-zero average value - it is ramped up from a non-zero minimum value to its peak, stays constant for some time, and is then ramped down to the minimum, where it stays until the end of the 1 s period. In the literature, however, the effective permeability is often calculated/depicted as a function of the excitation frequency pertaining to a sinusoidal excitation with a zero average value. In order to co-relate the case at hand to what is found in the literature, the RCS field ramp rate is used to calculate a corresponding frequency of roughly 2.5 Hz (0 to peak field in 100 ms) for a sinusoidal excitation.

To make a rough approximation of $\frac{dB_g}{dt}$ for an RCS dipole without formally running a finite element method study incorporating hysteresis in the yoke, let us consider a simple example. The yoke is assumed to be made from 24 gauge (0.64 mm) M19 C5 electrical steel as in [5]. A mean path length in the magnet yoke l_c of 1 m is also assumed. Because of the availability of hysteresis data (symmetric loop) at 2 Hz for a B_{max} of 0.77 T and not 0.655 T, the calculation is carried out for a sinusoidal current of amplitude $i_{c0} = 4.08$ kA (as opposed to 3.28 kA for 0.655 T). Note that the hysteresis curve obtained at 2.5 Hz is close to that obtained at 2 Hz as was previously mentioned. Also note for a dc component superimposed on the current sinusoid, the answer will be somewhat different. However, this analysis should give a good idea of the expected deviation.

The B-H loops for M19 at varying amplitudes and solely at 0.77 T peak field are shown in Figure 2. The single-valued characteristic is shown in black circles connected by a black line in Figure 2 (a) and in red in Figure 2 (b). The flux density and relative permeability from the 0.77 T peak loop are set as B_g and μ_r, and used to calculate the time rate of change of field for a triangular excitation current with a peak of 4.08 kA according to (4). The quantity $\frac{dB_g}{dt}$ for the two cases is depicted in Figure 3. The noise in $\frac{dB_g}{dt}$ around time stamps of 0 s, 0.255 s and 0.5 s stems from the measurement error in the B-H loop where the incremental permeability was calculated to be a very small negative value.

The maximum deviation in $\frac{dB_g}{dt}$ (from the expected value in blue) is approximated to be ~0.3 T/s, which is ~5% of the field ramp rate of 6.1 T/s (corresponding to the single-valued B-H characteristic). For $l_c > 1$ m, the maximum difference will increase accordingly. No issues are expected from the ramp rate deviations of this magnitude, because they are completely reproducible, and can be easily compensated with the magnet power supply.



(a) (b)

Figure 2: Symmetric B-H loops.

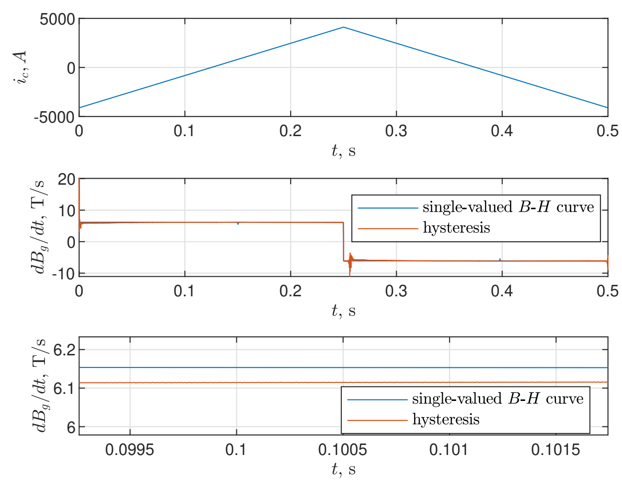


Figure 3: Time-domain waveforms.

4 Conclusion

For the RCS dipole parameters considered here, hysteresis in the magnet yoke is expected to produce a deviation of the order of a few percent in the air gap field ramp rate from that predicted using a single-valued B-H characteristic. This estimate should be regarded as approximate, because it is based on a simplified magnetic circuit model and available symmetric loop data rather than the exact biased trapezoidal excitation waveform. With appropriate pre-cycling between fixed current limits, the field trajectory is nevertheless expected to become repeatable from cycle to cycle. Barkhausen-noise-induced fluctuations are not expected to make a significant contribution to the integrated dipole field.

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