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The Sparks tracking code

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This note introduces a new tracking code, `Sparks`, built on the foundations laid by `hemod4` and designed for fast tracking of large ensembles of single particles over thousands of turns. The code is intended for the rapid evaluation of nonlinear lattice behavior through observables such as detuning, Poincaré sections, and smear, providing a computationally efficient alternative to large-scale dynamic aperture studies during lattice design and optimization.

CONTENTS

I. CONTEXT, CONVENTIONS AND TRACKING SETUP

When evaluating the lattice designs of the Electron-Ion Collider (EIC) [1] Hadron Storage Ring (HSR) [2], there are multiple accelerator modeling and tracking tools one can use to study their particular field of interest: MAD-X [3], Xsuite [4], Bmad [5], SIMTRACK [6] are all established codes that enable a wide spectrum of simulations.Very often the dynamic aperture (DA) is picked as the critical parameter that validates a given lattice setup – but such studies require a significant amount of CPU time.

There is a need for a more rapid process for evaluating nonlinear lattice performance. The work presented here describes an approach that consists of two complementary steps: selecting observables that characterize the onset of nonlinear dynamics without relying on long-term survival studies, and developing a dedicated tracking code that can evaluate these observables efficiently (in terms of CPU time) for large ensembles of particles and lattice configurations.

A. Alternative observables to dynamic aperture

Certain studies require large-scale simulation campaigns spanning multiple lattice descriptors and parameter configurations. One example is the determination of tolerances on higher-order field errors for the design of the new superconducting magnets in the HSR colliding insertion (IR6). For such studies, conventional DA tracking simulations are prohibitively expensive from a computational standpoint, with execution times often reaching several days for machines of the scale of the EIC.

Dynamic aperture remains an essential benchmark for validating the long-term stability of a given accelerator lattice; however it is not always the most appropriate observable for large parameter scans aimed at comparing nonlinear behavior. Instead, the present work focuses on diagnostics that characterize the onset and evolution of nonlinear motion directly. The resulting trajectories are analyzed both qualitatively through Poincaré sections, which reveal resonance islands, phase-space distortions, and chaotic structures, and quantitatively using the *smear* S defined as:

$$S = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(I_i - \langle I\rangle\right)^2}}{\langle I\rangle}, \qquad \langle I\rangle = \frac{1}{N}\sum_{i=1}^{N} I_i, \qquad I_i = Z_i^2 + PZ_i^2. \qquad Z \in \{X, Y\} \tag{1}$$

where I_i denotes the normalized Courant–Snyder invariant evaluated at turn i, and N is the total number of tracked turns.

Because smear can be evaluated from comparatively short tracking simulations, it provides a computationally efficient quantitative observable for assessing and comparing the nonlinear performance of lattice configurations over large parameter spaces.

B. Developing a faster code

Recent work on resonance trajectories [7] and resonance island jump theory [8] motivated a re-examination of the tools used during earlier studies on tune modulation theory [9]. The latter relied on Hénon-like lattices featuring only a single sextupole or octupole element: such simplified lattices were nevertheless demonstrated to *exhibit all the typical properties of more complicated mappings and dynamical systems* [10]. Building upon this framework, a realistic representation of a collider lattice can be constructed by extending the simple Hénon-like model to include multiple sextupole and octupole elements located at their physical positions around the ring.

`Sparks` symplectically propagates single particles in normalized coordinates (X_i, PX_i), with i the turn number. The tracking algorithm is implemented as follows, under the assumption that the relevant Twiss parameters are provided as input:

- the nonlinear kicks are derived from the Hamiltonian formulation in normalized coordinates:

$$\begin{aligned} &\textbf{Sextupole} \quad && \Delta PX_i = -\frac{1}{2}\, K_2 L \left(\beta_x \sqrt{\beta_x}\, X_i^2 \;-\; \beta_y \sqrt{\beta_x}\, Y_i^2\right) \\ & && \Delta PY_i = K_2 L\, \beta_y \sqrt{\beta_x}\, X_i\, Y_i \\ &\textbf{Octupole} \quad && \Delta PX_i = -\frac{1}{6}\, K_3 L \left(\beta_x^2\, X_i^3 - 3\, \beta_x\, \beta_y\, X_i\, Y_i^2\right) \\ & && \Delta PY_i = \frac{1}{6}\, K_3 L \left(3\, \beta_x\, \beta_y\, X_i^2\, Y_i - \beta_y^2\, Y_i^3\right) \end{aligned} \tag{2}$$

  where K_2L and K_3L are the integrated multipole strengths of the sextupole and octupole elements, respectively, at the location where the normalized coordinates are updated.

- between two nonlinear elements of interest E_1 and E_2, linear transport in normalized phase space is represented by simple rotation matrices. The transport matrix M_{12} is therefore obtained by concatenating the rotations associated with the phase-advance increments $\Delta\Phi_{x,y}$ of all intervening lattice elements. This matrix-based formulation readily accommodates more general transport maps in the future;

- a tune modulation (`tunemod`) element is available at the end of each turn to apply a small additional rotation $\Delta\phi$ in normalized phase space due to the combination of synchrotron motion and nonlinear chromaticity terms [11].

To enable efficient tracking of large particle ensembles over many turns, `Sparks` relies on a Numba-compiled kernel. The core tracking loop is implemented as an `@njit(parallel=True, fastmath=True)` function, removing Python overhead and enabling parallel execution over particle ensembles where supported. Preallocated arrays are used to store normalized coordinates and all turn-by-turn quantities, ensuring memory-efficient execution under just-in-time compilation.

C. Setting up the initial distribution

An example `Sparks` input file, written in YAML format, is provided in Appendix A. It shows that the initial distribution of single particles can be specified either through a dedicated input file or constructed on a grid using additional input variables[1]. If `grid` is selected, the code parameterizes the distribution in action-phase $(J_{x,y}, \Phi_{x,y})$ space based on the `coordinate` option chosen[2]:

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[1] Future versions will extend the supported initial conditions to include, for example, Gaussian and halo distributions.
[2] The variable names used in the text follow the conventions illustrated in Appendix A.

- with `action`, the numpy arrays `Jx_vals`, `Jy_vals` are constructed directly:

```
Jx_vals = Jx_min + Jx_step * np.arange(nx_range)
Jy_vals = Jy_min + Jy_step * np.arange(ny_range)
```

- if instead `amplitude` is picked, these same arrays are derived via:

```
Ax_vals = Ax_min + Ax_step * np.arange(nx_range)
Ay_vals = Ay_min + Ay_step * np.arange(ny_range)
```

The transformation between action and amplitude variables is defined as $A_z = \sqrt{2J_z}$, leading to:

```
Jx_vals = 0.5 * Ax_vals**2
Jy_vals = 0.5 * Ay_vals**2
```

The distribution arrays are then populated using a nested loop over horizontal and vertical actions:

```
idx = 0

for xx in range(nx_range):
       Jx = Jx_vals[xx]
       ax = np.sqrt(2.0 * Jx)

       for yy in range(ny_range):
              Jy = Jy_vals[yy]
              ay = np.sqrt(2.0 * Jy)

              # horizontal
              xn_dist[idx] = ax * math.cos(2.0 * math.pi * phix_init)
              pxn_dist[idx] = -ax * math.sin(2.0 * math.pi * phix_init)

              # vertical
              yn_dist[idx] = ay * math.cos(2.0 * math.pi * phiy_init)
              pyn_dist[idx] = -ay * math.sin(2.0 * math.pi * phiy_init)

              idx += 1
```

D. Resonance island tracking

Support for resonance-island tracking was one of the primary motivations behind the development of `Sparks`. To determine the island betatron tune Q_I, a modified version of the core tracking loop is combined with a fixed-point finder algorithm. This mode is triggered when the `fixed_point` option is selected in the YAML input. In this case, a single particle is tracked using initial coordinates $\left(J_x^0, \Phi_x^0\right)$ provided as input along with the integer order R of the targeted resonance (e.g., $R = 4$ for a fourth-order resonance).

Once the fixed point has been identified, a small perturbation is applied in action–phase space to generate oscillations about the fixed point. The resulting trajectory is then tracked over many turns, and the island tune Q_I is extracted using three independent methods:

- phase-slope analysis,
- FFT-based frequency analysis,
- reconstruction of the local one-turn transport matrix, with Q_I obtained from its trace.

The latter method also enables the reconstruction of the effective linear optics functions ruling the motion within the resonance island. This application is currently under development.

II. PERFORMANCE AND OUTLOOK

The following presents representative tracking benchmarks for various radial shift configurations of the HSR lattice `260217` using `Sparks` on a MacBook Pro equipped with an Apple M1 processor. Tracking 51 independent particles over 1009 turns required an average CPU time of 0.035,s. The execution time scales approximately linearly with the total number of particle-turns; for example, tracking 510 particles over 100 periods of 1009 turns required 33.370,s.

Figures 1, 2, and 3 show the horizontal action–phase Poincaré sections obtained by tracking 51 single particles for 1009 turns in the nominal lattice and for positive and negative circumference changes of the `260217` HSR lattice. Rather than relying solely on dynamic aperture, these phase-space portraits provide a direct visualization of the nonlinear dynamics, revealing the extent of the linear region, the resonance-island topology, and the surrounding chaotic motion.

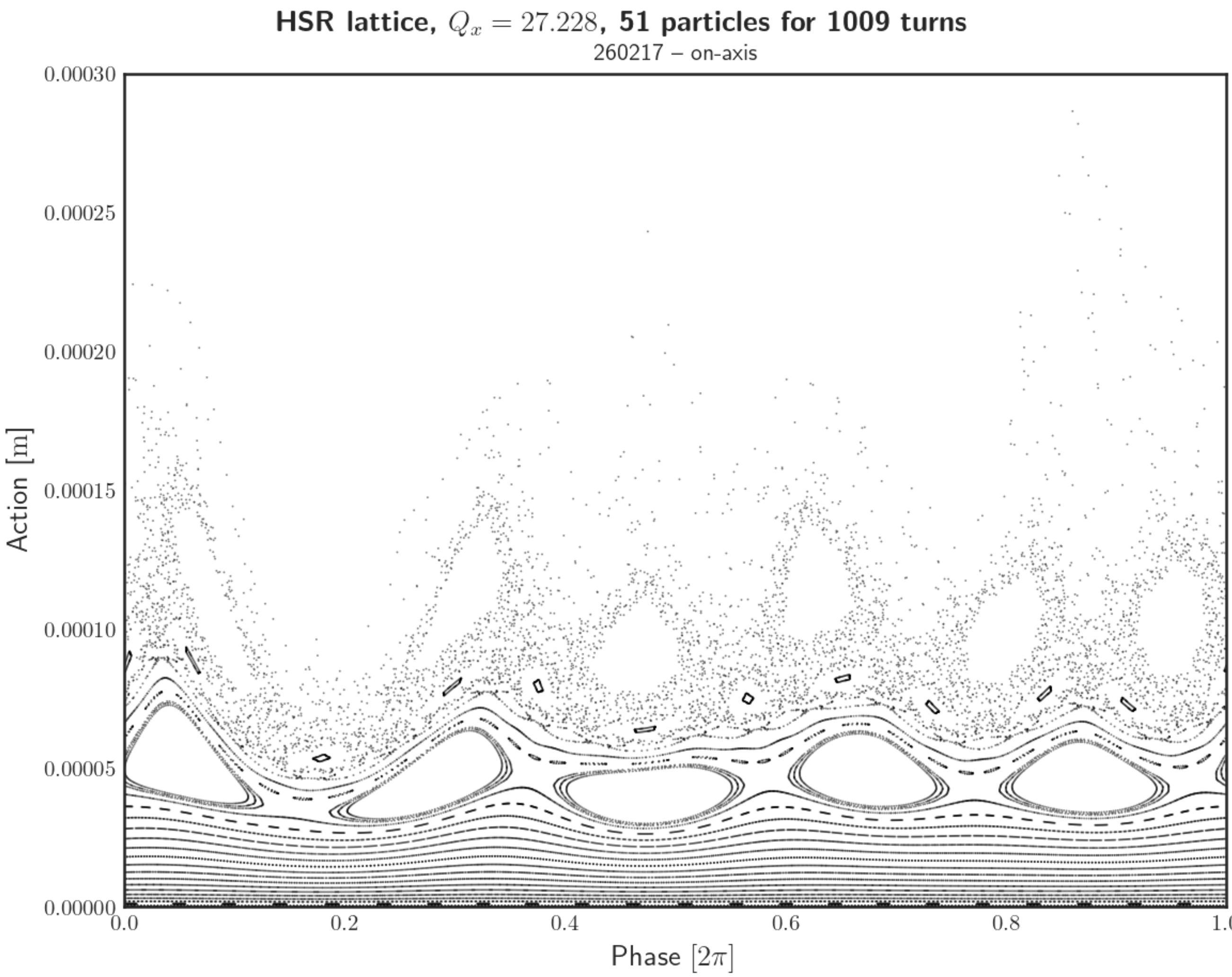

FIG. 1: Horizontal action–phase Poincaré sections for 51 single particles tracked over 1009 turns in the `260217` HSR lattice in the nominal on-axis configuration.

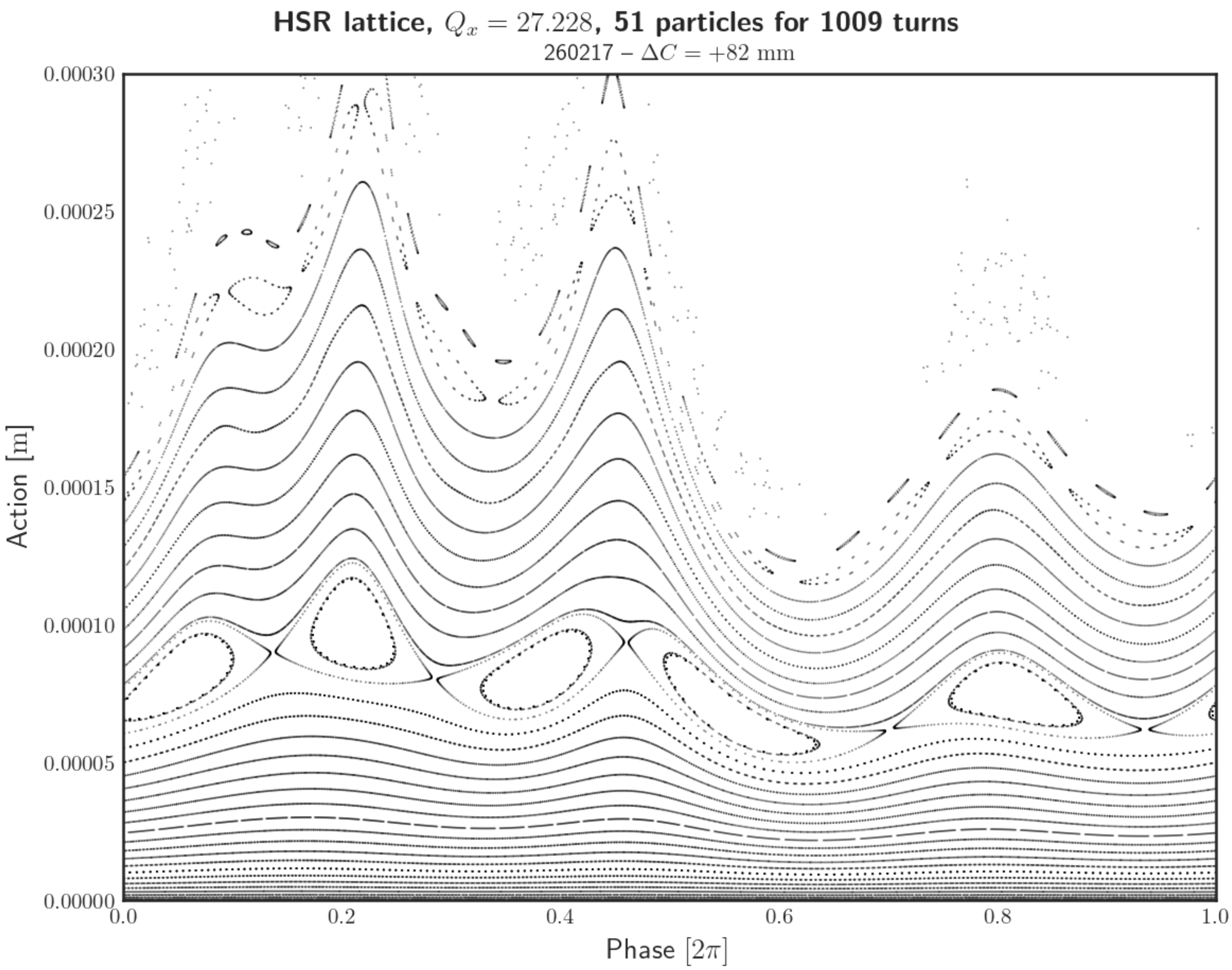

FIG. 2: Horizontal action–phase Poincaré sections for 51 single particles tracked over 1009 turns in the `260217` HSR lattice with a radial shift corresponding to $\Delta C = +82$ mm.

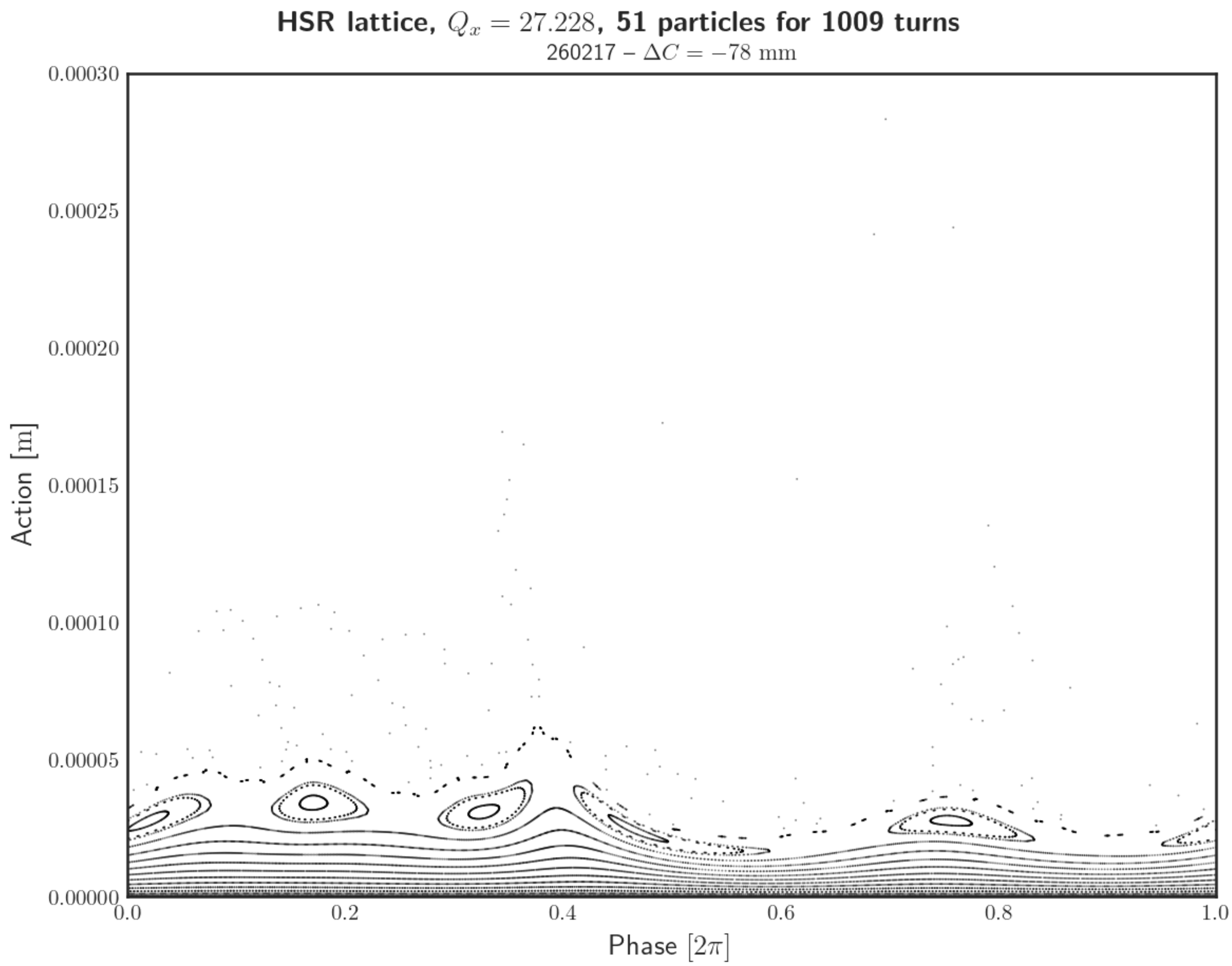

FIG. 3: Horizontal action–phase Poincaré sections for 51 single particles tracked over 1009 turns in the `260217` HSR lattice with a radial shift corresponding to $\Delta C = -78$ mm.

Figure 4 presents the horizontal detuning as a function of action for the three radial-shift configurations. A clear variation in the linear detuning slope is observed, with the magnitude of the slope directly correlated with the applied circumference change.

Although the examples presented in this note focus on horizontal dynamics, `Sparks` performs full 4D tracking in normalized transverse phase space. The current implementation includes nonlinear coupling arising from sextupole and octupole fields, while skew elements are not yet supported. Future development will extend the transport matrix formalism to fully coupled four-dimensional linear optics.

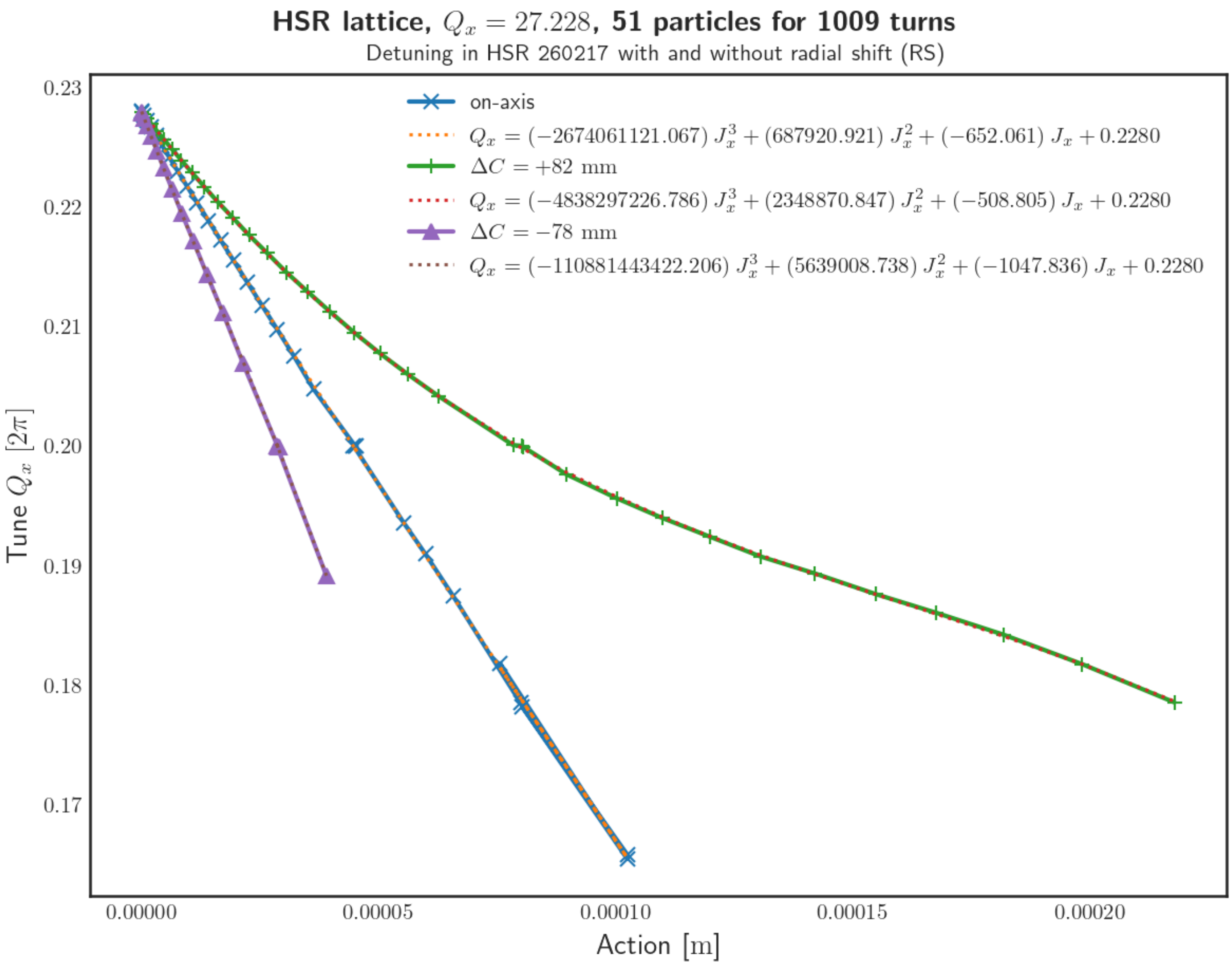

FIG. 4: Tune as a function of action for the three radial shift configurations of HSR lattice `260217`. The detuning coefficients are shown in the equations for the fit functions in the legend.

The next stage of development will focus on applying `Sparks` to systematic studies of HSR lattice configurations over a range of beam energies and radial-shift settings. Particular emphasis will be placed on the rapid evaluation of these lattices based on the design of the new superconducting magnets for the IR6 interaction region, where multipleparameter scans are required to quantify the impact of higher-order field errors on nonlinear beam dynamics, all through the accelerator ramp. By combining computationally efficient tracking with observables such as detuning, Poincaré sections, and smear, `Sparks` will provide a practical tool for guiding lattice and ramp optimization during the accelerator design process.

Appendix A: Sample Sparks input file

```
beamline:
       ring: Yellow
       twiss:
              file: twiss.baseline.20250730a_wB2.tfs

modulation:
       file: none
       mode: all # options: single | all
       amplitude: 0.

magnets:
       sextupoles:
              - all
       #octupoles:
       # - all

fixed_point:
       action: 2.488e-3
       phase: 0.421
       resonance_order: 4

particles:
       type: grid # options: grid | external
       coordinate: amplitude # options: action | amplitude
       Ax:
              min: 0.00001
              step: 0.0005
       Ay:
              min: 0.00001
              step: 0.0005
       Jx:
              min: 0.5e-10
              step: 5e-8
       Jy:
              min: 0.5e-10
              step: 5e-8
       Nx: 51
       Ny: 51
       phix: 0.0
       phiy: 0.0

tracking:
       mode: classic # options: classic | fixed_point
       n_periods: 1
       turns_per_period: 1009

#diagnostics:
# eicex:
#  - yi2-bh9
#  - yi2-bh11
```

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[4] G. Iadarola *et al.*, in *Proc. HB'23*, ICFA Advanced Beam Dynamics Workshop on High-Intensity and High-Brightness Hadron Beams No. 68 (JACoW Publishing, Geneva, Switzerland, 2024) pp. 73–80.
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[7] S. Peggs and T. Satogata, *Resonance Trajectories and Island Tune Spectra in the Tune Domain*, Tech. Rep. (Brookhaven National Laboratory (BNL), Upton, NY (United States), 2024).
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