BNL-229513-2026-TECH
EIC-ADD-TN-161

Rigid-Mode Limit of the Yokoya Matrix Formalism and the Burov-Lebedev Dispersion Equation

A. Blednykh

March 2026

Electron-Ion Collider
Brookhaven National Laboratory

U.S. Department of Energy
USDOE Office of Science (SC), Nuclear Physics (NP)

DISCLAIMER

Rigid-Mode Limit of the Yokoya Matrix Formalism and the Burov–Lebedev Dispersion Equation

A. Blednykh,[1] P. Baxevanis,[1] M. Blaskiewicz,[1] D. Xu,[1] and R. Lindberg[2]
[1] *Brookhaven National Laboratory, Upton, NY 11973, USA*
[2] *Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA*

Abstract
Transverse single-bunch instabilities of space-charge-dominated coasting beams with round and flat transverse geometries are studied using a unified dispersion-relation framework. The analysis combines the Burov-Lebedev formalism, which captures space-charge tune spread, Landau damping, and instability threshold behavior, with Yokoya's projection method for representing coherent transverse mode structure and its dependence on beam aspect ratio. In the rigid-beam limit, the formulation reduces to a scalar dispersion relation of Burov-Lebedev paper. For non-rigid transverse oscillations, truncation of Yokoya's Hermite-based expansion yields a finite-dimensional matrix eigenvalue problem in which space-charge and coupling impedance effects enter through Burov-Lebedev–type denominators. This approach provides a consistent basis for comparing rigid and non-rigid instability behavior in round and flat beams and for assessing the role of beam ellipticity in modifying coherent mode structure and stability thresholds.

INTRODUCTION

The Electron–Ion Collider (EIC) project includes a low-energy electron cooler designed to reduce the transverse emittances of the hadron beam to values as low as

$$\varepsilon_x = \varepsilon_y = 0.3\ \mu\text{m}. \tag{1}$$

While such emittance reduction is beneficial for luminosity performance, it substantially modifies the collective stability at injection energy of the Hadron Storage Ring (HSR).

Numerical studies indicate that for the round-beam configuration the beam remains longitudinally stable but becomes vertically unstable at 23.8 GeV [1–3]. The instability is driven predominantly by transverse space-charge impedance, enhanced by the combined contribution of geometric and resistive-wall impedances. Simulations performed with ELEGANT [4] and TRANFT [5] confirm that the beam is unstable when realistic transverse impedance models are included.

In contrast, a flat-beam configuration with

$$\varepsilon_x = 3\ \mu\text{m}, \qquad \varepsilon_y = 0.3\ \mu\text{m}, \tag{2}$$

can be stabilized using high chromaticity,

$$\xi_x/\xi_y = +6/+6, \tag{3}$$

together with full octupole strength [6]. However, operational constraints impose significant limitations. Experience from APEX studies at RHIC indicates that injection is limited to approximately 25% of the available octupole strength due to dynamic aperture considerations. At such reduced octupole settings, the beam becomes unstable.

These observations highlight the sensitivity of stability thresholds to transverse beam geometry and to the interplay between space-charge and broadband impedance effects. While a consistent round-beam formalism has been established, the treatment of flat beams and strong space-charge effects remains under investigation. In particular, the modification of coherent mode structure and stability boundaries in the presence of beam ellipticity and large space-charge tune spread is of direct relevance for EIC operation with ultra-low emittance beams.

The theoretical description of transverse instabilities in beams with strong space charge has been developed in several complementary approaches. Blaskiewicz derived a self-consistent Vlasov formulation for transverse collective motion that avoids the rigid-beam approximation and yields an integral equation for the perturbation of the phase-space distribution [7]. Pestrikov extended this framework to include the effects of space-charge tune shift and lattice detuning, deriving dispersion relations and stability diagrams for coasting beams with chromatic and nonlinear tune spreads [8, 9]. Building on these developments, Burov and Lebedev clarified the domain of validity of the rigid-beam approximation and obtained analytical expressions for Landau damping and instability thresholds, emphasizing the role of the separation between coherent and incoherent frequencies induced by space charge [10]. The formulation used in this note follows the Burov–Lebedev approach and establishes its connection with the non-rigid transverse mode expansion introduced by Yokoya.

LINEARIZED VLASOV EQUATION IN BUROV–LEBEDEV NOTATION (COASTING BEAM)

We follow the notation of Burov and Lebedev [10] and compare it with the non-rigid generalization introduced by Yokoya [11].

$$f(J_x, J_y, \hat{p}), \qquad \int f\, dJ_x\, dJ_y\, d\hat{p} = 1. \tag{4}$$

The incoherent betatron tune of a particle is written as

$$Q(J_x, J_y, \hat{p}) = Q_0 + Q_l(J_x, J_y, \hat{p}) + Q_{sc}(J_x, J_y), \tag{5}$$

where Q_l represents lattice-induced tune shifts (chromaticity, octupoles, etc.) and Q_{sc} is the direct space-charge tune shift, which may depend on the transverse actions.

A.1 Linearized Vlasov Equation

The Vlasov equation

$$\frac{\partial f}{\partial t} + \{f, H\} = 0 \tag{6}$$

is linearized by writing $f \to f + \delta f$ and retaining only first-order terms in δf. In action–angle variables (J_x, ϕ_x), the Poisson bracket is

$$\{f, H\} = \frac{\partial f}{\partial \phi_x}\frac{\partial H}{\partial J_x} - \frac{\partial f}{\partial J_x}\frac{\partial H}{\partial \phi_x}. \tag{7}$$

We decompose the distribution function and Hamiltonian into equilibrium and perturbation parts,

$$f = f_0(J_x, J_y, \hat{p}) + \delta f, \tag{8}$$

$$H = H_0(J_x, J_y, \hat{p}) + H_1. \tag{9}$$

This representation assumes that the unperturbed Hamiltonian H_0 depends only on the invariants (actions), implying integrable single-particle motion. Within this standard Vlasov framework, collective stability is described while effects such as emittance growth from nonlinear single-particle diffusion are not included.

The equilibrium distribution satisfies

$$\{f_0, H_0\} = 0, \tag{10}$$

since f_0 depends only on the invariants (actions), and H_0 depends only on $J_x, J_y, \hat{p}$.

Substituting the decompositions into the Vlasov equation,

$$\frac{\partial (f_0 + \delta f)}{\partial t} + \{f_0 + \delta f, H_0 + H_1\} = 0, \tag{11}$$

and retaining only first-order terms in the perturbations, we obtain

$$\frac{\partial \delta f}{\partial t} + \{\delta f, H_0\} + \{f_0, H_1\} = 0. \tag{12}$$

Higher-order terms like $\{\delta f, H_1\}$ are neglected.

In action–angle variables (J_x, ϕ_x), the Poisson bracket gives

$$\{\delta f, H_0\} = \frac{\partial \delta f}{\partial \phi_x}\frac{\partial H_0}{\partial J_x} - \frac{\partial \delta f}{\partial J_x}\frac{\partial H_0}{\partial \phi_x}. \tag{13}$$

Since H_0 depends only on the actions,

$$\frac{\partial H_0}{\partial \phi_x} = 0, \qquad \frac{\partial H_0}{\partial J_x} = \omega_0 Q(J_x, J_y, \hat{p}), \tag{14}$$

and therefore

$$\{\delta f, H_0\} = \omega_0 Q(J_x, J_y, \hat{p})\frac{\partial \delta f}{\partial \phi_x}. \tag{15}$$

Similarly,

$$\{f_0, H_1\} = \frac{\partial f_0}{\partial \phi_x}\frac{\partial H_1}{\partial J_x} - \frac{\partial f_0}{\partial J_x}\frac{\partial H_1}{\partial \phi_x}. \tag{16}$$

Because f_0 depends only on the actions, $\partial f_0/\partial \phi_x = 0$, we have

$$\{f_0, H_1\} = -\frac{\partial f_0}{\partial J_x}\frac{\partial H_1}{\partial \phi_x}, \tag{17}$$

so that the linearized equation for δf is

$$\frac{\partial \delta f}{\partial t} + \omega_0 Q(J_x, J_y, \hat{p})\frac{\partial \delta f}{\partial \phi_x} = -\frac{\partial f_0}{\partial J_x}\frac{\partial H_1}{\partial \phi_x}. \tag{18}$$

A.2 Harmonic Ansatz and Definition of ν

Assuming harmonic dependence in time and retaining the first betatron harmonic (dipole mode), we write the perturbation of the distribution function as

$$\delta f(J_x, J_y, \hat{p}, \phi_x, t) = \tilde{f}(J_x, J_y, \hat{p})\, e^{i\phi_x}\, e^{-i\omega t}. \tag{19}$$

Under a similar and related approximation, the transverse collective force on the right-hand side of (18) becomes

$$\frac{\partial H_1}{\partial \phi_x} = -\tilde{F}_x(J_x, J_y)\, e^{i\phi_x}\, e^{-i\omega t}. \tag{20}$$

Following Burov and Lebedev, the complex coherent tune shift is introduced as

$$\nu \equiv \frac{\omega}{\omega_0} - (n + Q_0), \tag{21}$$

where n is the azimuthal mode number.

Assuming harmonic dependence in time and retaining the first betatron harmonic, the single-particle response is

$$\delta f(J_x, J_y, \hat{p}) = \frac{\partial f_0}{\partial J_x}\frac{F_x(J_x, J_y)}{Q_l(J_x, J_y, \hat{p}) + Q_{sc}(J_x, J_y) - \nu - i0}, \tag{22}$$

where the infinitesimal term $-i0$ specifies the Landau prescription for analytic continuation of the dispersion integral and determines the sign of the imaginary contribution associated with Landau damping. Up to this point, no assumption has been made regarding the transverse structure of the coherent mode. In the following subsection we introduce the rigid-mode closure used by Burov and Lebedev.

B. Rigid-Mode Closure in Burov–Lebedev

B.1 Assumption on the Collective Force

The essential approximation introduced by Burov and Lebedev is the rigid-mode closure for the collective force,

$$F_x(J_x, J_y) = [Q_c(\omega) - Q_{sc}(J_x, J_y)]\, X, \tag{23}$$

where $Q_c(\omega)$ is the coherent tune shift produced by the transverse impedance and X is the coherent centroid amplitude. This assumption implies that the transverse force has the same action dependence for all particles, corresponding to a rigid dipole displacement.

B.2 Perturbed Distribution

Substituting Eq. (23) into Eq. (22) yields

$$\delta f(J_x, J_y, \hat{p}) = \frac{\partial f_0}{\partial J_x} \frac{Q_c(\omega) - Q_{sc}(J_x, J_y)}{Q_l(J_x, J_y, \hat{p}) + Q_{sc}(J_x, J_y) - \nu - i0} X. \tag{24}$$

Eq. (24) shows that within the Burov–Lebedev (BL) rigid closure, the action dependence of the dipole perturbation is fully determined by the equilibrium distribution derivative $\partial f_0/\partial J_x$ and the local incoherent detuning. In particular, the perturbation factorizes as

$$\delta f(J_x, J_y, \hat{p}) = f_x(J_x, J_y)\, \Phi(J_x, J_y, \hat{p}; \nu)\, X, \qquad f_x \equiv \frac{\partial f_0}{\partial J_x}, \tag{25}$$

where

$$\Phi(J_x, J_y, \hat{p}; \nu) = \frac{Q_c(\omega) - Q_{sc}(J_x, J_y)}{Q_l(J_x, J_y, \hat{p}) + Q_{sc}(J_x, J_y) - \nu - i0}.$$

Thus, in the BL approximation, the action dependence of the dipole eigenfunction is fixed and does not constitute an independent unknown.

B.3 Self-Consistency and Dispersion Relation

For a coasting beam, the centroid displacement is defined as

$$X = \int J_x\, \delta f(J_x, J_y, \hat{p})\, d\Gamma, \qquad d\Gamma = dJ_x\, dJ_y\, d\hat{p}. \tag{26}$$

Inserting Eq. (24) yields

$$X = X \int d\Gamma \frac{[Q_c(\omega) - Q_{sc}(J_x, J_y)]\, f_x(J_x, J_y) J_x}{Q_l(J_x, J_y, \hat{p}) + Q_{sc}(J_x, J_y) - \nu - i0}. \tag{27}$$

A non-trivial solution $X \neq 0$ exists if and only if

$$1 - \int d\Gamma \frac{[Q_c(\omega) - Q_{sc}(J_x, J_y)]\, f_x(J_x, J_y) J_x}{Q_l(J_x, J_y, \hat{p}) + Q_{sc}(J_x, J_y) - \nu - i0} = 0. \tag{28}$$

Equation (28) is exactly the Burov–Lebedev coasting-beam dispersion relation. Note that $Q_c(\omega)$ must be evaluated at $\omega = \omega_0(n + Q_0 + \nu)$, so that the dispersion relation is implicit in the eigenvalue ν.

C. Connection to the Yokoya Matrix Generalization

Equation (28) is a scalar dispersion equation because the transverse eigenfunction has already been fixed by the rigid-mode closure (23). In operator language, this corresponds to a rank-one truncation of the linearized Vlasov problem.

Yokoya does not modify Eqs. (18)–(22). Instead, he generalizes the closure by expanding the perturbed distribution as

$$\delta f(J_x, J_y, \hat{p}) = f_x(J_x, J_y) \sum_a A_a L_a(J_x, J_y), \tag{29}$$

The projection onto the basis functions L_a is performed with the natural BL weight $f_x(J)J_x$, which arises from the definition of the centroid moment. This guarantees that the rank-one truncation $L_{1,0} = 1$ reproduces the scalar dispersion equation.

Substituting Eq. (29) into the same linearized Vlasov equation, solving as in Eq. (22), and projecting onto each basis function L_b yields the linear system

$$\sum_a D_{ba}(\nu) A_a = 0, \tag{30}$$

with

$$D_{ba}(\nu) = \delta_{ba} - \int d\Gamma \frac{[Q_c(\omega) - Q_{sc}(J)]\, f_x(J)J_x}{Q_l(J,\hat{p}) + Q_{sc}(J) - \nu - i0} L_a(J) K_b(J). \tag{31}$$

The functions $K_b(J)$ denote the Yokoya kernel functions arising from the projection of the collective force, and are biorthogonal to the basis functions $L_a(J)$ under the BL weight $f_x(J)J_x$.

The dispersion condition becomes

$$\det D(\nu) = 0, \tag{32}$$

which is the non-rigid generalization of the Burov–Lebedev dispersion relation.

Retaining only a single basis function,

$$L_{1,0}(J) = 1, \qquad K_{1,0}(J) = 1, \tag{33}$$

reduces the matrix equation exactly to Eq. (28), demonstrating that the Burov–Lebedev result is the rank-one (rigid-mode) limit of the Yokoya formalism.

D. Round and Flat Beam Generalization

The derivation presented above is general and applies to both round and flat transverse beam distributions. The distinction enters only through the space-charge tune shift Q_{sc} and the Yokoya kernel functions K_b.

D.1 Round beams

For a round beam, the transverse distribution is symmetric,

$$\sigma_x = \sigma_y, \tag{34}$$

and the space-charge tune shift depends only on the total transverse action. In this case, the Yokoya kernel functions simplify significantly, and only a limited subset of basis functions contributes to the dielectric matrix. The rigid-mode approximation often provides a reasonable first estimate of stability thresholds.

D.2 Flat beams

For flat beams,

$$\sigma_x \neq \sigma_y, \tag{35}$$

the space-charge tune shift becomes explicitly dependent on both transverse actions J_x and J_y. In this case, the transverse eigenfunctions are generally non-rigid, and coupling between different basis modes becomes important.

The Yokoya formalism naturally accommodates this situation through the kernel functions $K_{k,l}(J_x, J_y; R)$, where

$$R = \frac{\sigma_y}{\sigma_x} \tag{36}$$

is the beam aspect ratio [11]. The resulting dielectric matrix captures the modification of stability boundaries due to beam flatness.

Rigid limit as a special case

In both round and flat cases, retaining only a single basis function reduces the matrix dispersion relation to the Burov–Lebedev rigid-mode equation. Thus, the rigid approximation corresponds to the lowest-order truncation of the more general Yokoya expansion.

In practice, the calculation proceeds as follows:

1. Choose a truncation order $(k_{\max}, l_{\max})$ for the basis functions.
2. For a given intensity N, solve the eigenvalue problem for ν.
3. Track the same eigenmode as N is increased.
4. Identify the smallest N for which $\Im[\nu] > 0$.

The instability threshold obtained from the non-rigid (full Vlasov) solution is generally lower than that predicted by the rigid approximation for flat beams with strong space charge. This reflects the fact that additional transverse eigenfunctions beyond the rigid dipole mode can couple to the impedance and participate in the instability. In the non-rigid treatment the perturbation can be expanded in a complete set of transverse eigenfunctions,

$$\delta f = \sum_n a_n \, \psi_n(J_x, J_y), \tag{37}$$

leading to the eigenvalue problem

$$\det\Big(I - \hat{K}\Big) = 0, \tag{38}$$

where $\hat{K}$ is the impedance–Vlasov operator. In contrast, the rigid approximation restricts the perturbation to a single dipole-like eigenfunction,

$$\delta f \propto X \, \frac{\partial f_0}{\partial J_x}, \tag{39}$$

where X is the centroid displacement.

Appendix A: Mike Blaskiewicz Derivation (Beam–Beam Form and BL Dispersion)

This appendix reproduces the derivation presented in [12]. The approach follows the standard beam–beam (cosine/sine) decomposition, leading to a 2×2 system for the first harmonic. Under the single–sideband approximation, the result reduces to the Burov–Lebedev (BL) coasting–beam dispersion equation.

A.1 Cosine/sine decomposition and first harmonic

We start from an equilibrium distribution $f_0(J_x, J_y, \tau)$ and write a small perturbation at the first betatron harmonic in the horizontal plane as

$$f(J_x, J_y, \tau, \psi_x) = f_0(J_x, J_y, \tau) + \Big[f_s(J_x, J_y, \tau)\sin\psi_x + f_c(J_x, J_y, \tau)\cos\psi_x\Big]e^{-i\nu\theta}, \tag{40}$$

where ψ_x is the betatron phase, θ is the azimuthal variable, and ν is the complex coherent tune shift.

Following the beam–beam formalism, the first-harmonic dynamics can be expressed in terms of the complex amplitudes $\hat{x}_1$ and $\hat{p}_1$, which in general form a 2×2 system. Under a single–sideband approximation one may use

$$\hat{p}_1 = \pm i\, \hat{x}_1, \tag{41}$$

thereby reducing the system to a scalar equation for $\hat{x}_1$.

The functions f_s and f_c describe the first-harmonic modulation of the distribution in betatron phase space. Rather than solving explicitly for their action dependence, it is convenient to work with the corresponding first-harmonic moments, namely the complex centroid displacement $\hat{x}_1$ and its conjugate momentum $\hat{p}_1$. These quantities are obtained by projecting the perturbed distribution onto $\cos\psi_x$ and $\sin\psi_x$, respectively, and integrating over betatron phase. This procedure follows the standard beam–beam moment formalism and yields a closed set of equations for the lowest-order transverse moments.

As shown in the semi-analytic Vlasov model of Ref. [12], the first-harmonic moments for beam 1 close as Eqs. (42)–(43).

$$\hat{\hat{x}}_1 - \int dJ_x\, dJ_y\, \frac{J_x \dfrac{\partial F_{10}}{\partial J_x}\dfrac{\partial H_{10}}{\partial J_x}}{\left(\dfrac{\partial H_{10}}{\partial J_x}\right)^2 - (\nu + i\epsilon)^2} \Big\{2\beta_1\, \hat{\hat{x}}_1 - 2\alpha_1\, \hat{\hat{p}}_1 - \xi_1\, \hat{\hat{x}}_2\, U_c(J_x, J_y) - \hat{D}_1\Big\} = 0. \tag{42}$$

and

$$\hat{\hat{p}}_1 - \int dJ_x\, dJ_y\, \frac{J_x \dfrac{\partial F_{10}}{\partial J_x}(\epsilon - i\nu)}{\left(\dfrac{\partial H_{10}}{\partial J_x}\right)^2 - (\nu + i\epsilon)^2} \Big\{2\beta_1\, \hat{\hat{x}}_1 - 2\alpha_1\, \hat{\hat{p}}_1 - \xi_1\, \hat{\hat{x}}_2\, U_c(J_x, J_y) - \hat{D}_1\Big\} = 0. \tag{43}$$

The common denominator in Eqs. (42)–(43) arises from combining the cosine and sine components into a second-order response, with the infinitesimal imaginary term ϵ enforcing the Landau prescription.

The moment equations are obtained by multiplying the linearized Vlasov equation by $\cos\psi_x$ and $\sin\psi_x$, integrating over the betatron phase, and retaining only the first harmonic. The resulting expressions involve integrals over the transverse actions and depend on the derivative of the unperturbed Hamiltonian, $\partial H_{10}/\partial J_x$, which represents the incoherent betatron frequency. As shown in the semi-analytic Vlasov model of Ref. [12], the first-harmonic moments for beam 1 close as Eqs. (42)–(43).

A.2 Response denominator and Hamiltonian derivative

The unperturbed horizontal frequency entering the response is written as

$$\frac{\partial H_{10}}{\partial J_x} = Q_{1x} + \xi_1\, \frac{\partial \langle U\rangle}{\partial J_x}, \tag{44}$$

where Q_{1x} is the bare (lattice) tune, ξ_1 is the (central) space-charge tune shift parameter (with $\xi_1 < 0$ for direct space charge), and $\langle U\rangle$ is the transverse potential averaged over betatron phases.

A.3 Single–sideband reduction

Using Eq. (41) and retaining the first harmonic, the coupled cosine/sine equations lead to the response denominator

$$\left(\frac{\partial H_{10}}{\partial J_x}\right)^2 - (\nu + i\epsilon)^2, \tag{45}$$

where $\epsilon \to 0^+$ implements the Landau prescription. Under the single–sideband condition $|\nu - Q_{1x}| \ll Q_{1x}$, the quadratic denominator factorizes and reduces to a first–order response of the form $\partial H_0/\partial J_x - (\nu + i\epsilon)$, thereby recovering the BL-type dispersion structure.

A.4 Gaussian space-charge potential derivative

For a Gaussian transverse distribution, the notes define

$$U_c(J_x, J_y) \;\equiv\; 2\,\frac{\partial\langle U\rangle}{\partial J_x}, \tag{46}$$

with the explicit representation

$$U_c(J_x, J_y) = \int_0^{1/2} 4\lambda\, d\lambda\; e^{-\lambda(J_x+J_y)}\, I_0(\lambda J_y)\, \Big[I_0(\lambda J_x) - I_1(\lambda J_x)\Big], \tag{47}$$

where I_0 and I_1 are modified Bessel functions of the first kind. The normalization used in the notes implies

$$U_c(0, 0) = 2. \tag{48}$$

A.5 Dispersion relation

For eigenmode analysis the external drive must be set to zero, $\hat{D}_1 = 0$. The dispersion relation becomes

$$1 - \int dJ_x dJ_y \frac{J_x \partial f_0/\partial J_x}{\partial H_0/\partial J_x - (\nu + i\epsilon)} \left(\beta + id - \xi \frac{\partial\langle U\rangle}{\partial J_x}\right) = 0. \tag{49}$$

If an external dipole drive is present, the response amplitude $\hat{x}_1$ satisfies

$$\hat{x}_1 = \chi(\nu)\, \hat{D}_1, \tag{50}$$

where $\chi(\nu)$ is the susceptibility defined by the integral kernel above.

Equation (49) is consistent with the coasting-beam dispersion equation of Burov–Lebedev in the rigid-mode approximation; the quantity in parentheses represents the coherent driving term (impedance/external drive) together with the direct space-charge contribution through $\partial\langle U\rangle/\partial J_x$.

SUMMARY

This Tech Note establishes the formal relationship between the scalar coasting-beam dispersion equation derived by Burov and Lebedev and the matrix formalism introduced by Yokoya for non-rigid transverse modes. Starting from the linearized Vlasov equation, the Burov–Lebedev dispersion relation is obtained under a rigid-mode closure assumption, which fixes the transverse eigenfunction *a priori*. It is then shown that Yokoya's projection method, based on a truncated Hermite–Laguerre expansion of the perturbed distribution, leads to a finite-dimensional matrix dispersion equation. The scalar Burov–Lebedev result is recovered exactly as the rank-one (rigid-mode) limit of the Yokoya matrix formalism. The framework provides a formal basis for treating both round and flat transverse beam geometries, with geometric effects entering through the space-charge tune shift and the Yokoya kernel functions. An equivalent derivation using the beam–beam moment formalism confirms consistency with the rigid-mode dispersion

equation under the single-sideband approximation.

This result clarifies that the rigid-mode Burov–Lebedev equation corresponds to the lowest-order truncation of the more general Yokoya matrix formalism, thereby establishing a direct theoretical connection between rigid and non-rigid treatments of transverse collective modes in space-charge-dominated beams.

[1] P. Baxevanis, A. Blednykh, and M. Blaskiewicz. Single–bunch instabilities driven by space charge during low–energy cooling at injection in the eic hadron storage ring. Technical Report BNL-229155-2025-TECH, Brookhaven National Laboratory, 2025.

[2] P. Baxevanis, A. Blednykh, and M. Blaskiewicz. Single–bunch instabilities driven by space charge during low–energy cooling at injection in the eic hadron storage ring. In *Proceedings of the 6th North American Particle Accelerator Conference (NAPAC2025)*, Upton, NY, USA, 2025. JACoW Publishing. Paper TUP090.

[3] A. Blednykh. Collective effects: Challenges and solutions for the eic project. In *Proceedings of the 6th North American Particle Accelerator Conference (NAPAC2025)*, Upton, NY, USA, 2025. JACoW Publishing. Paper THYD01.

[4] M. Borland. elegant: A flexible code for accelerator simulation. Advanced Photon Source, Argonne National Laboratory, 2000. https://ops.aps.anl.gov/manuals/elegantlatest/elegant.html.

[5] M. Blaskiewicz. The tranft user's manual. Technical Report BNL-77074-2006-IR, Brookhaven National Laboratory, 2006.

[6] S. Peggs. EIC-ADD-TN-077. Technical Report BNL-224979-2023-TECH, Brookhaven National Laboratory, 2023. EIC Accelerator Development Department Tech Note.

[7] M. Blaskiewicz. Fast head-tail instability with space charge. *Phys. Rev. ST Accel. Beams*, 4:044202, 2001.

[8] D. Pestrikov. Transverse stability diagrams with space charge. *Nucl. Instrum. Methods Phys. Res. A*, 562:65, 2006.

[9] D. Pestrikov. Landau damping and transverse instabilities with space charge. *Nucl. Instrum. Methods Phys. Res. A*, 578:65, 2007.

[10] A. Burov and V. Lebedev. Transverse instabilities of coasting beams with space charge. *Phys. Rev. ST Accel. Beams*, 12:034201, 2009.

[11] K. Yokoya. Limitation of the gaussian approximation in beam-beam simulations. *Phys. Rev. ST Accel. Beams*, 3:124401, 2000.

[12] Ji Qiang and Michael Blaskiewicz. Strong–strong simulations of combined beam–beam and wakefield effects in the electron–ion collider. *Nuclear Instruments and Methods in Physics Research Section A*, 1069:169942, 2024.