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Solenoidal Fields from Permanent Magnets for a Polarized Electron Preinjector

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Abstract

This technical note describes the design of a compact focusing solenoid constructed from permanent bar magnets, intended for the low-energy section of a spin-polarized electron injector. Typically, quadrupole triplets provide appropriate focusing for the low-energy spin rotator section; however, their large footprint precludes the insertion of sufficient beam diagnostics. The primary motivation of this study is to provide beam focusing while preserving the vertical spin orientation of the electron beam. We propose a solenoidal field composed of multiple permanent magnets arranged to naturally yield a zero longitudinal magnetic field integral ($\int B_z dz = 0$). This design serves as a compact, passive, and non-coupling alternative to traditional quadrupole triplets. We present the magnet arrangement, field calculation methodology, spin tracking simulations using space charge codes, and calculations demonstrating that the required field strength is achievable.

1 Motivation

The Electron-Ion Collider (EIC) pre-injector is designed to produce and transport a high-intensity, spin-polarized electron beam[1]. The electron source generates a beam with longitudinal spin polarization[2]. Subsequently, a pair of Wien filters rotates the spin from the longitudinal to the vertical direction at a beam energy of 300–320 keV, with each filter rotating the spin vector by 45°[3].

A critical requirement for all magnetic elements situated downstream of the first Wien filter is the preservation of this vertical spin orientation. A solenoidal field with a non-zero longitudinal field integral ($\int B_z dz \neq 0$) acts as a focusing lens but induces spin precession away from the vertical axis.

Polarized injectors typically employ solenoids with opposing windings to maintain vertical spin. This technique is effective in regions where the spin is purely vertical; even if a minor non-zero field integral exists, it can be compensated by an additional solenoid or by fine-tuning the power supplies. However, in the drift space between the two Wien filters, the spin direction is oriented at 45° relative to the longitudinal axis. An imperfect solenoid in this region would cause the spin direction at the exit of the second Wien filter to rotate within the x-z plane, necessitating dipole field corrections that are often impractical.Traditional focusing elements, such as quadrupole triplets, are a common alternative as they inherently preserve spin direction[4]. Although they can be configured for a zero field integral, they present several distinct disadvantages in this context:

- **Strong Focusing:** Quadrupoles provide strong focusing. Achieving the weak focusing required for this section necessitates low field gradients, which can make field stability challenging.
- **Space Charge Effects:** At low energies (300–320 keV), longitudinal space charge forces are significant. Minimizing drift space is crucial to prevent beam expansion and the growth of energy spread. Consequently, the inter-Wien filter distance must be kept as short as possible to maintain beam quality.

Given these constraints, a compact focusing element with a zero field integral is highly desirable. A permanent magnet solenoid, constructed from a single ring of magnets, meets these requirements by providing focusing without inducing beam rotation or net spin precession. Potential drawbacks of using a permanent magnet solenoid include:

- The Wien filter itself possesses intrinsic quadrupole field components, which require compensation using at least two quadrupoles after each filter. Therefore, the overall lattice complexity may not be significantly reduced.
- The solenoid's fringe field will extend into the Wien filter and cause transverse coupling. The impact on polarization requires further study.

The fundamental principle of this design is that the on-axis fringe fields on either side of the ring naturally exhibit a polarity opposite to the field in the center of the ring. This creates an effective triplet field profile from a single, compact magnetic component. The positive and negative lobes of the longitudinal field $B_z(z)$ ensure that the total field integral along the axis is automatically zero, as shown schematically in Figure 3.

2 Bar Magnet Field Calculation

The magnetic field of a uniformly magnetized rectangular bar magnet is derived using the magnetic scalar potential formalism. The magnet is modeled by effective magnetic charge sheets on its pole faces, which act as sources for the magnetic field intensity, $\vec{H}$.

$$\vec{H} = -\nabla \Phi_m \tag{1}$$

The potential Φ_m is generated by the effective magnetic surface charge density, σ_m, where the magnitude of σ_m corresponds to the magnet's intrinsic magnetization, M. For a single, thin rectangular surface of dimensions $a \times b$ located in the $z = 0$ plane, the potential at a point (x, y, z) is given by the integral over the surface charge:

$$\Phi_m(x, y, z) = \frac{\sigma_m}{4\pi} \iint \frac{1}{|\vec{r} - \vec{r}'|} dS' = \frac{\sigma_m}{4\pi} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \frac{1}{\sqrt{(x - x')^2 + (y - y')^2 + z^2}} dy' dx' \tag{2}$$

A rectangular bar magnet of length L and magnetized along the z-axis is modeled as two such plates separated by a distance L:

1. A surface with positive magnetic charge, $+\sigma_m$, at $z = +L/2$.
2. A surface with negative magnetic charge, $-\sigma_m$, at $z = -L/2$.

By the principle of superposition, the total scalar potential for the bar magnet is the sum of the potentials from these two surfaces:

$$\Phi_{\text{total}}(x, y, z) = \Phi_{\text{plate}}(x, y, z - L/2, +\sigma_m) + \Phi_{\text{plate}}(x, y, z + L/2, -\sigma_m) \quad (3)$$

The integral in Equation 2 as an odd function, the integral of the field along the z is exactly equal to zero. And it possesses an analytical solution that can be expressed as a sum of contributions from the four corners of the rectangle. This leads to an expression for the total potential, Φ_{total}, as a sum over the eight corners of the bar magnet. The full analytical expressions for these components are lengthy and can be found in standard literature on magnetostatics, such as Furlani [5]. The calculation described above yields the $\vec{H}$-field, which is the field generated by the magnetic poles and is commonly referred to as the demagnetizing field. To determine the total magnetic field, $\vec{B}$, we use the constitutive relation that connects $\vec{B}$, $\vec{H}$, and the material's magnetization, $\vec{M}$.

- In the region outside the magnet, where $\vec{M} = 0$:

$$\vec{B} = \mu_0 \vec{H} \quad (4)$$

- In the region inside the magnet, where the magnetization is $\vec{M} = M\hat{z}$:

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \quad (5)$$

3 Magnet Arrangement and Field

To realize a solenoidal field profile satisfying $\int B_z dz = 0$, we propose two distinct configurations:

- A single coaxial ring comprised of a series of permanent magnets. The ring is assembled from multiple rectangular magnets arranged symmetrically around the beam path, with all magnets oriented with the same polarity (e.g., North poles facing radially inward) as shown in Figure 1. Rectangular magnets are commercially available and allow for easier configuration changes. This arrangement is the primary focus of this section.
- A dual coaxial ring configuration. Each ring is assembled from multiple rectangular magnets arranged symmetrically around the beam path, with all magnets oriented with their North poles facing radially, either toward or away from the beamline, as shown in Figure 6. By tuning the gap between the rings, one can control the focal length. As this is a variation of the single-ring setup, we will discuss this arrangement further in the section on field tuning.

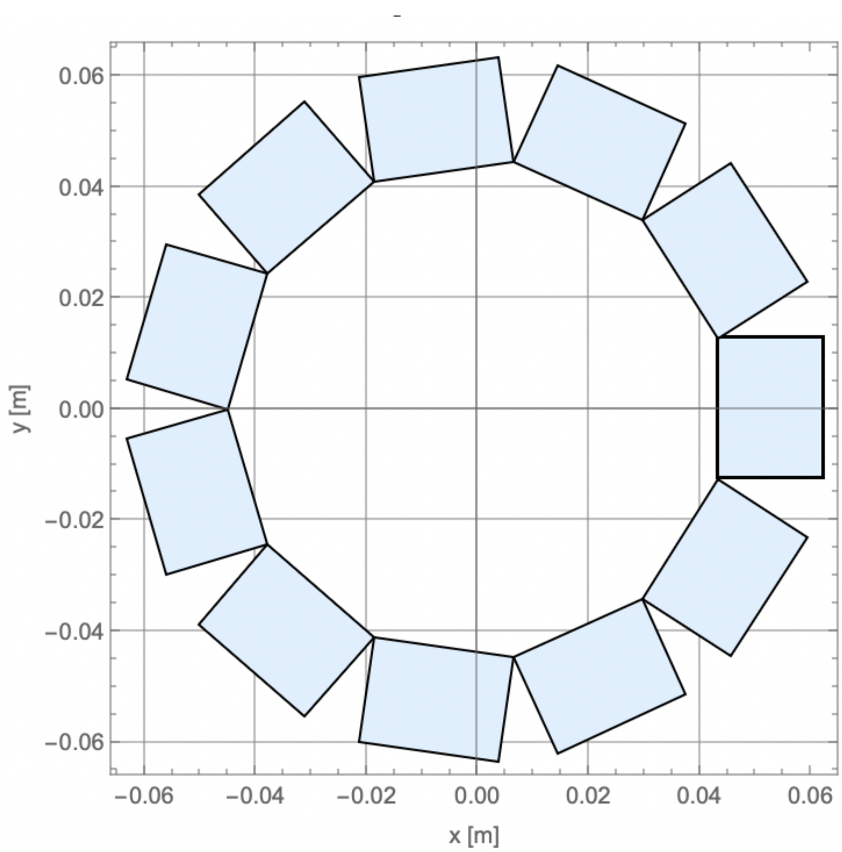


Figure 1: Schematic of a cross-section of a single-ring permanent magnet solenoid.

The magnetic fields for the bar magnet and the solenoid assembly are calculated using analytical formulas implemented in Mathematica 14.

The magnetic field of the assembly is calculated using the principle of superposition. Based on the magnet dimensions and beam pipe size, we can determine the number, position, and orientation of the magnets. By applying rotation and translation to the field of a single bar magnet, we can obtain the full field map. We selected the K&J Magnetics model **BX0X0C** [6] as an example. The relevant specifications are:

- **Material:** N42 Neodymium-Iron-Boron
- **Remanence (B_r):** 1.32 Tesla
- **Dimensions ($L \times W \times H$):** $1.0'' \times 1.0'' \times 0.75''$ (25.4 mm x 25.4 mm x 19.05 mm)

A photograph of the magnet is shown in Figure 2.

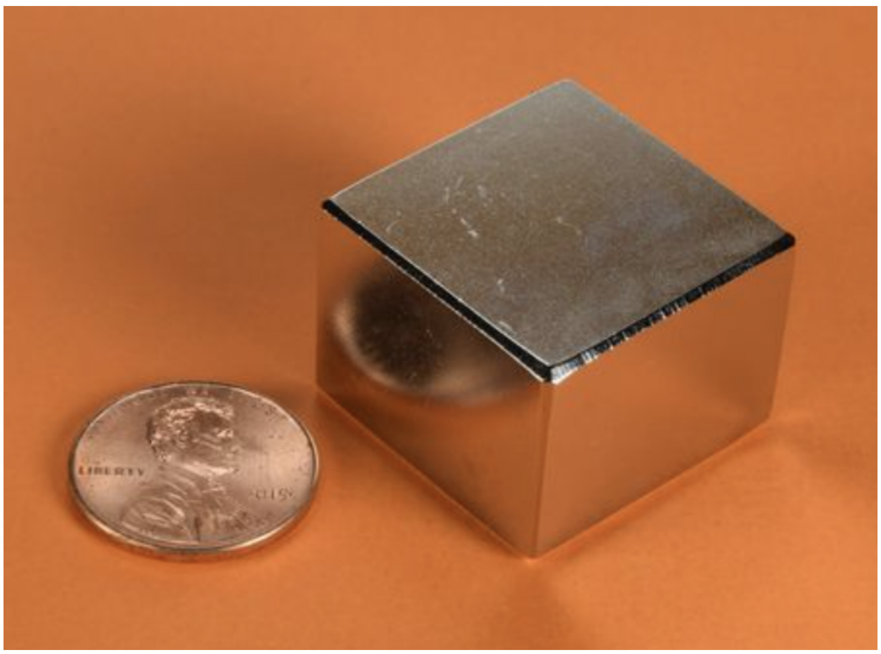


Figure 2: Photograph of the permanent magnet.

We model a single ring composed of 11 BX0X0C magnets. The magnets are arranged in a circle in the x-y plane at $z = 0$. The longitudinal magnetic field B_z is calculated along the central axis. The resulting field profile is shown in Figure 3. The plot exhibits a strong central peak with two side lobes of opposite polarity, confirming the triplet-like nature of the field. Figure 4 shows the field distributions on the $z = 0$ plane and the $x = 0$ plane.

The simulation yields a peak central field of approximately 800 Gauss. With double or multiple layers, we can create a longitudinal field of nearly 2000 Gauss, which is suitable for electron beams up to 10 MeV. For the Wien filter section, the field generated using NdFeB magnets is more than sufficient for the required 100 Gauss focusing strength that was specified as an equivalent to the previous focusing scheme. The field integral is zero by design. The effective length of the magnetic assembly is less than 5 cm, making it

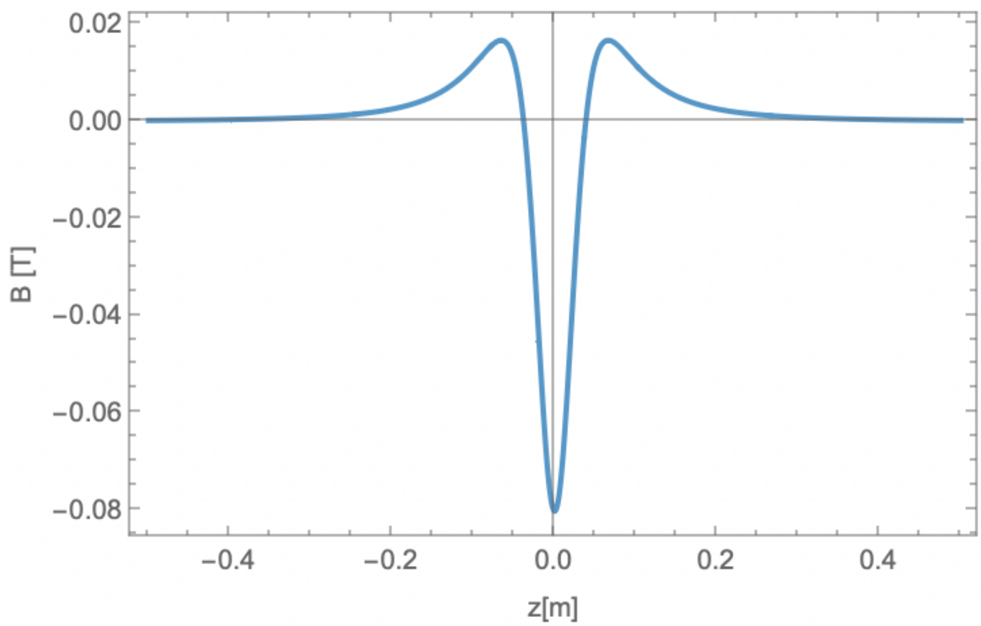
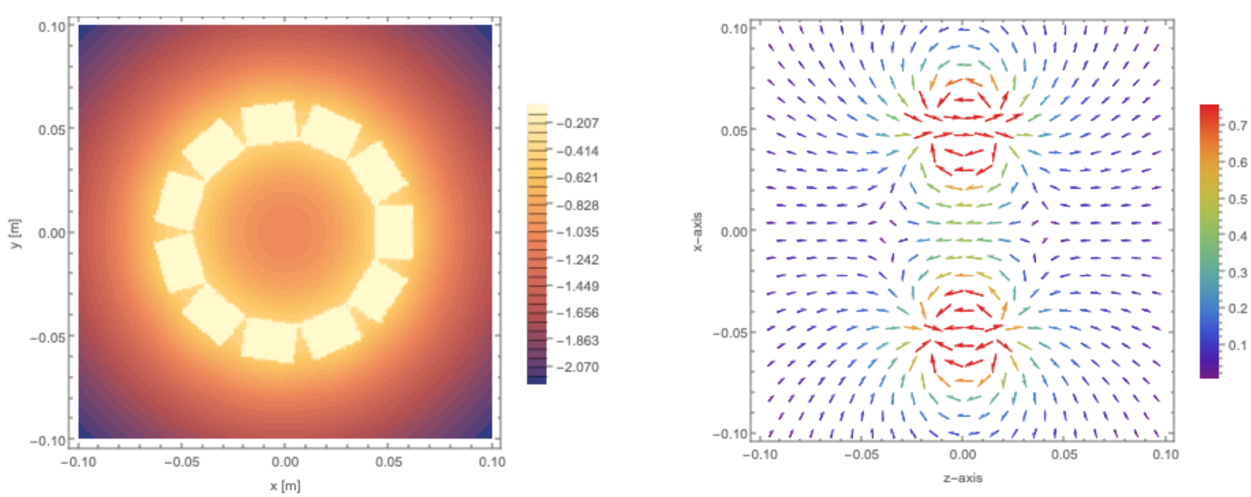


Figure 3: Calculated on-axis longitudinal magnetic field $B_z(z)$ for the single-ring permanent magnet solenoid.



(a) The B_z field on the $z = 0$ plane

(b) The B field on the $y - z$ plane at $x = 0$

Figure 4: Magnetic field on cross-sectional views

exceptionally compact. The strong field generated by NdFeB magnets may also be useful for focusing the beam in downstream beamlines with higher energy or density.

For the beam from the gun to the bunching section, where the energy is only 300–320 keV, weaker magnets are preferred.

Table 1: Properties of Common Permanent Magnet Materials

Magnet Type	**Typical B_r (Tesla)**	Note
NdFeB (Neodymium-Iron-Boron)	1.0 – 1.4	Strongest commercially available
SmCo (Samarium-Cobalt)	0.8 – 1.1	Corrosion resistant
AlNiCo	0.6 – 1.3	Easily demagnetized
Ferrite (Ceramic magnets)	0.2 – 0.4	Lower strength, cost-effective

Both NdFeB and SmCo have high B_r values, making them more suitable for higher energy applications. Ferrite magnets are a viable option for this application. For a Ferrite magnet with $B_r = 0.3$ T, the peak field at the center is 160 G, which is closer to the required value.

4 Permanent Magnet Focusing Length Tuning

Tuning the field of a permanent magnet is inherently more difficult than that of an electromagnet. For this specific solenoid design, we consider several methods:

1. Adjusting the field by modifying the magnet radius or by adding another layer of magnets and changing the inter-layer distance or polarity. This has been used in permanent quadrupole magnets [7, 8].

2. Using trim coils to fine-tune the field, although this may disrupt the zero-integral nature of the B_z field. This method has been used in the CBETA project [9].

3. Changing the spacing between two solenoids to fine-tune the focusing [10]. This approach is space-consuming and offers only a limited tuning range.

In this section, we propose a compact design with a sufficient tuning range. Figure 5 shows the focal length as a function of the field reduction. To achieve a sufficient tuning range of 3.2 ± 2 m, we need to adjust the field strength by 1% to 30%. Regarding Method 1, this can be achieved using passive shielding, such as adjusting the gap of an iron tube to control field leakage, or varying the gap between an outside layer of magnets with opposite polarity. While feasible, this approach is mechanically complex. Furthermore, if the iron tube gap is not well-centered, it may cause the B_z integral to be non-zero. Method 2 is unsuitable as maintaining a zero B_z integral is difficult.

For Method 3, the focal length of the solenoid is given by:

$$f = \frac{4 \cdot (B\rho)^2}{\int B_z(z)^2 dz} \tag{6}$$

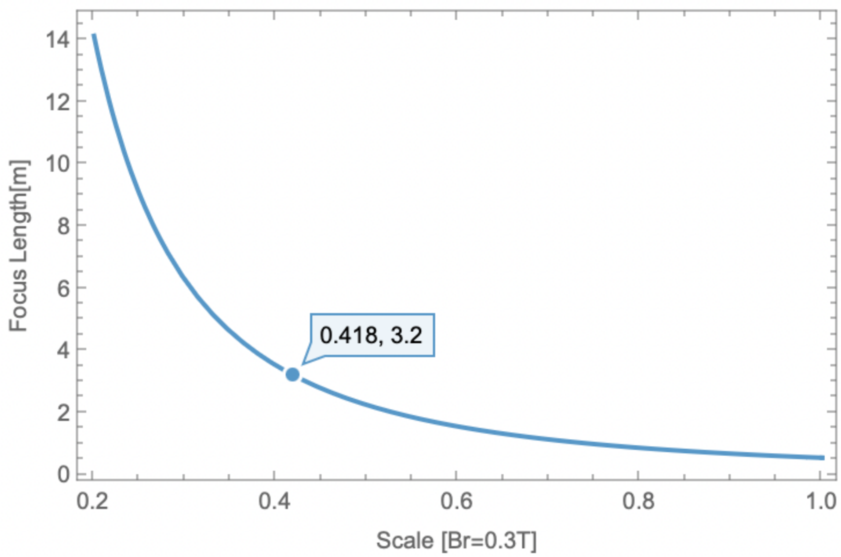


Figure 5: Focal length as a function of the field for a single solenoid.

By placing two solenoids with opposite polarity, a triplet-like focusing profile is created. A primary advantage is the ability to control the focal length. The longitudinal field along the solenoids and the focal length as a function of the gap are shown in Figure 7. By mechanically adjusting the gap between the two solenoids from 12 mm to 4 mm, the focal length can be tuned from 0.9 to 7.3 meters. This range comfortably covers the nominal focal length of 3.2 m with sufficient tuning sensitivity. With this configuration, the total length of the assembly is less than 5 cm, which is significantly more compact than a standard quadrupole triplet design (∼14 cm). This compactness preserves valuable space for diagnostics.

5 Spin Dynamics in the Fringes of the Solenoid

The General Particle Tracer (GPT) code is a well-established simulation platform for studying charged particle dynamics in electromagnetic fields. GPT includes spin tracking capabilities and has been previously employed to study beam dynamics for the CEBAF Injector[11].

In our simulation setup, we incorporated the 3D electric field map from the BNL 320 kV polarized HVDC gun[12], 2D Poisson field maps of the electromagnetic solenoid situated between the gun and the Wien filter, and the 3D electric and magnetic fields of the Wien filter (including fringe regions), also generated in OPERA.

To evaluate beam focusing before and between the Wien filters, we performed GPT simulations using both the permanent magnet solenoid and electromagnetic quadrupoles in a triplet configuration. The thin electromagnetic quadrupole lens was designed in Opera 3D with an aperture matching the beam pipe, and the resulting 3D fields were imported into GPT.

For the two permanent magnet solenoids, fields were generated using the code developed in this work. The focal length was tuned to match that of the quadrupole triplet before importing the fields into GPT. The beam envelope and polarization were tracked from the

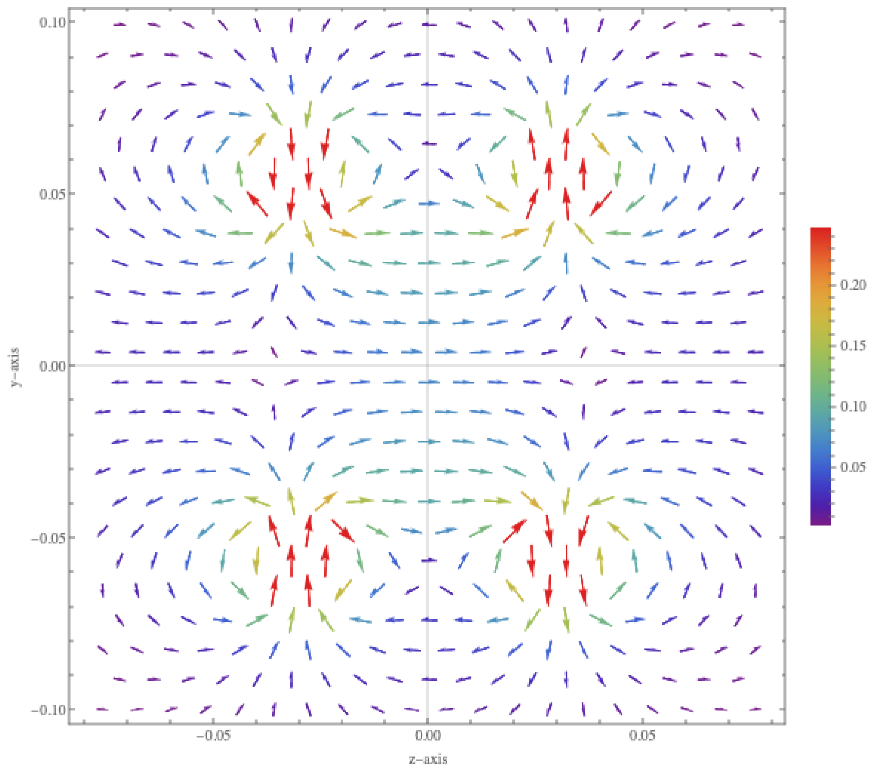
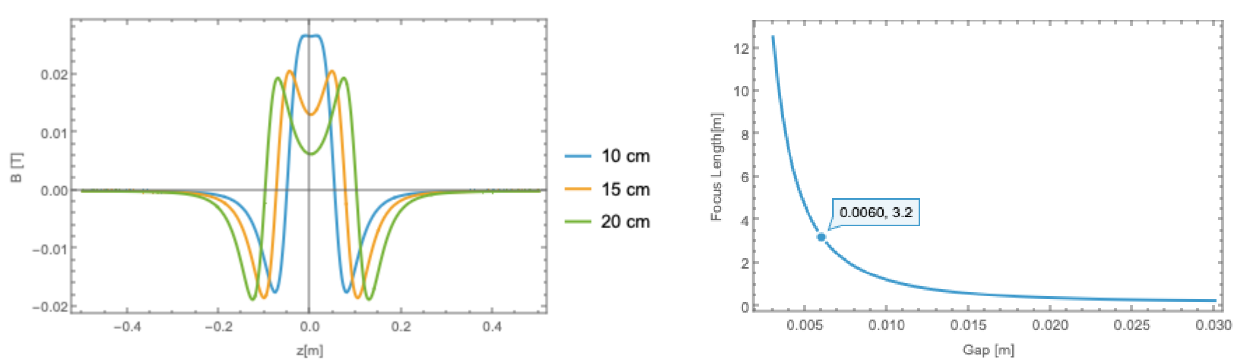


Figure 6: Magnetic field distribution of the two-solenoid setup.



(a)

(b)

Figure 7: Two-solenoid focal length tuning. a) Longitudinal B field along two solenoids; b) Focal length as a function of the distance between two solenoids.

dipole bend to the exit of the Wien filter. The initial polarization at the gun was 100%. At the dipole exit, the polarization remained approximately 99.95%. Figure 8 shows a typical X and Y trajectory of the beam from bending to the end of the Wien filters.

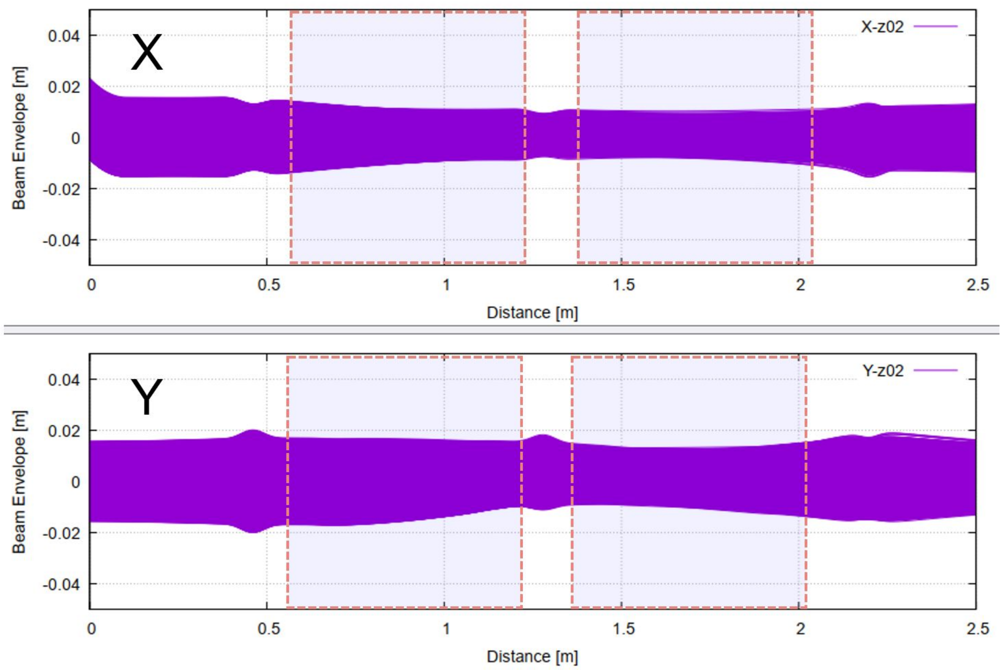


Figure 8: Beam envelope from bending to the end of the Wien filters. For focusing before and in-between the Wien filters, electromagnet quads or a permanent magnet solenoid could be used. This particular beam envelope shows the use of electromagnet quadrupoles in triplet configurations.

The quadrupole are triplet configuration, while the two solenoids center is aligned with the same center point as the triplet. We didn't add a quadrupole correction at the solenoids configulation. Figure 9 shows a comparison of polarization evolution with respect to electromagnetic quads and permanent magnet-based solenoids. The polarization evolution shows that the permanent magnet solenoid has similar polarization at the Wien filter exit to that of the electromagnet ones, and thus could be an efficient way of beam focusing with a wide range of focal lengths and a relatively simpler setup.

6 Conclusion

We have discussed how a solenoid created from permanent magnets is suitable for focusing a vertically spin-polarized electron beam. The structure is simple, compact, and requires no power supplies. The net longitudinal field integral is ensured to be zero, thus preserving the electron's vertical spin polarization. Various magnetic materials can be selected to achieve a field strength of 100–2000 Gauss. This approach offers a passive, robust, and cost-saving alternative to traditional quadrupole magnets. After reviewing and discussing focusing tuning

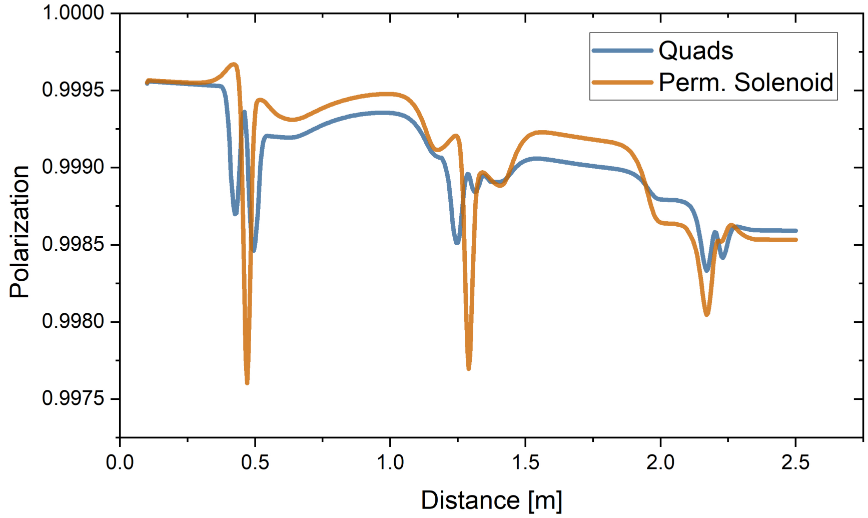


Figure 9: Evolution of beam polarization from dipole bend to the end of the Wien filter with a permanent magnet solenoid as the focusing element vs electromagnetic quads.

methods, we conclude that using two solenoids can achieve a large focusing range by adjusting the gap when optimal parameters are chosen. The spin tracking simulation shows results comparable to those of triplet quadrupole magnets and the permanent solenoid. Higher strength magnets could be used for higher energy electron beams to the MeV range.

Acknowledgments

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