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Single-Bunch Instabilities Driven by Space Charge During Low-Energy Cooling at Injection in the EIC Hadron Storage Ring

P. Baxevanis

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P. Baxevanis, A. Blednykh and M. Blaskiewicz

Brookhaven National Laboratory, Upton, NY

I. INTRODUCTION

This paper presents a simulation-based study of single-bunch dynamics at the injection energy of 23.8 GeV for protons in the EIC hadron storage ring, focusing on the impact of space-charge–driven instabilities. The analysis demonstrates that at this energy, the proton bunch experiences significant transverse space-charge forces, which can reduce the stability margin in the presence of the geometric and resistive wall impedance. Various collective effects were considered, with particular attention to the nonlinear nature of the transverse space charge. To stabilize the beam, high chromaticity and octupoles were introduced and their effects analyzed using the ELEGANT code. The results provide a quantitative assessment of the stability thresholds and offer guidance for the machine design and operational strategy at injection.

II. TRANSVERSE BEAM DYNAMICS

At injection energy (23.8 GeV for protons), a single bunch in the EIC hadron storage ring experiences a relatively strong space charge field that can facilitate the onset of deleterious instabilities, particularly in the transverse plane, in conjunction with other wake sources. In order to guard against this possibility, a systematic study of the underlying beam dynamics under the influence of the relevant collective effects is required. In particular, special emphasis needs to be placed on the impact of the transverse component of the space charge force, which is intrinsically nonlinear in character and can potentially reduce the stability margin for a given machine impedance. In what follows, we present the outline of a short, simulation-based study of the above-mentioned effects, with the aim of clarifying the fundamental single-bunch stability behavior of the protons.

To start with, a straightforward means of quantifying the impact of (linear) wakefields upon the transverse dynamics of the hadrons is through the familiar concept of the transverse impedance. The latter quantity includes both dipole and quadrupole-type contributions, signifying dependence of the corresponding wake with respect to the transverse coordinates of the leading and trailing particles (respectively). Assuming a Gaussian transverse profile for the beam, it can be shown that, for the case of linear transverse space charge, the beam-induced component of the dipole impedance per unit length is given by [1, 2]

$$\hat{Z}^{(d)}_{x,y}(\omega) = Z^{(d)}_{x,y}/L_R = -\frac{Z_0}{4\pi\gamma_0^2\Sigma_{x,y}^2}\,, \tag{1}$$

where $Z_0 \approx 377\,\Omega$ is the vacuum impedance, L_R is the ring circumference, γ_0 is the average relativistic factor and $\Sigma_{x,y} = (\sigma_{x,y}(\sigma_x+\sigma_y)/2)^{1/2}$ - with $\sigma_{x,y}$ being the proton beam transverse sizes, assumed constant in the simplified, smooth focusing model under consideration. As far as the corresponding quadrupole impedance per unit length is concerned, it is simply given by $\hat{Z}^{(q)}_{x,y}(\omega) = -\hat{Z}^{(d)}_{x,y}(\omega)$. An additional, but sometimes overlooked, source of dipole space charge impedance can be visualized by considering the image charges induced by an off-axis hadron beam upon the vacuum chamber, which (because of their asymmetrical distribution) cause a net deflection to an on-axis test particle. The relevant contribution to the dipole impedance is expressed by

$$\hat{Z}^{(d,i)}_{x,y}(\omega) = \frac{Z_0}{2\pi\gamma_0^2 b^2}\,, \tag{2}$$

where b is the vacuum chamber radius. Both the dipole and the quadrupole contributions to the transverse impedance can be fed directly into a particle tracking code such as ELEGANT [3]. Specifically, the latter code facilitates a transverse impedance input by means of the intrinsic ZTRANSVERSE element.

The dynamics impact of the above-described linear space charge impedances can be elucidated by means of the corresponding tune shifts, which are given by

$$\Delta\nu^{\rm linear}_{x,y} = -\nu_{x,y}\beta^2_{x,y}\frac{Ne^2cZ_0\lambda(z)}{8\pi\gamma_0^3 m_h c^2\Sigma^2_{x,y}}\,, \tag{3}$$

where $\nu_{x,y} = L_R/(2\pi\beta_{x,y})$ are the original tunes of the ring, $\beta_{x,y}$ are the (constant) beta function values, Ze and m_h the charge and mass of the hadrons (respectively), N is the total number of hadrons and $\lambda(z)$ is the scaled longitudinal profile of the beam (normalized according to $\int dz\lambda(z) = 1$, where z is the position within the bunch). As expected for the defocusing space charge force, the tune shifts are negative and vary along the bunch according to the local current value.

All the results mentioned so far have assumed that the transverse space charge force is linear in nature. However, for the Gaussian model under consideration, this is merely an approximation. A more rigorous analysis reveals that the effect of the neglected nonlinear terms is to add a dependence of the particles tunes with respect to the transverse action variables $J_{x,y}$, so that the tune shifts now become $\Delta\nu_{x,y} = \Delta\nu^{\rm linear}_{x,y}F(J_x, J_y)$. In fact, the

TABLE I: Stability study parameters

Parameter	**Value**
Energy spread σ_δ	7.4×10^{-4}
RMS bunch length σ_{z0} (m)/double RF system	1.5
RF voltage V_0 (kV)	40
RF frequency f_{RF} (MHz)	24.61
Transition gamma γ_t	22.7
Injection gamma γ_0	25.36
Momentum compaction factor $\alpha_c = 1/\gamma_t^2$	1.94×10^{-3}
Proton normalized emittance $\gamma_0 \epsilon_{x,y}$ (mm-mrad)	3.0 / 0.3
Proton average beta function $\beta_{x,y}$ (m)	22.4 / 23.3
Proton transverse rms beam size $\sigma_{x,y}$ (mm)	1.62 / 0.52
Ring circumference L_R (km)	3.83
Average beam pipe radius b (mm)	24

modification factor F is such that the transverse actions only *reduce the absolute value of the tune shifts*. Moreover, it can be shown that the action-averaged values of the tune shifts can be approximated by $\langle \Delta\nu_{x,y} \rangle \approx 0.5 \Delta\nu_{x,y}^{\text{linear}}$ [4]. Accordingly, one can simply halve the beam-induced components of the linear-model dipole and quad space charge impedances (though not their image counterpart) in order to take some account of the nonlinearity. Apart from this rather heuristic way of dealing with this complication, a more self-consistent analysis involves the use of the Basseti-Erskine formula ([5]) in order to rigorously calculate the transverse fields of the hadron beam, assuming the latter remains Gaussian in profile. This particular method is implemented in the ELEGANT tracking code via the SCMULT element [6], which facilitates the automatic insertion of transverse space charge into a lattice.

III. NUMERICAL DATA

Confining our attention to protons from now on - the main parameters for the configuration under study are listed in Table I. Given a Gaussian energy profile, a double RF system is being considered with the aim of generating a longer bunch and raising the sta-

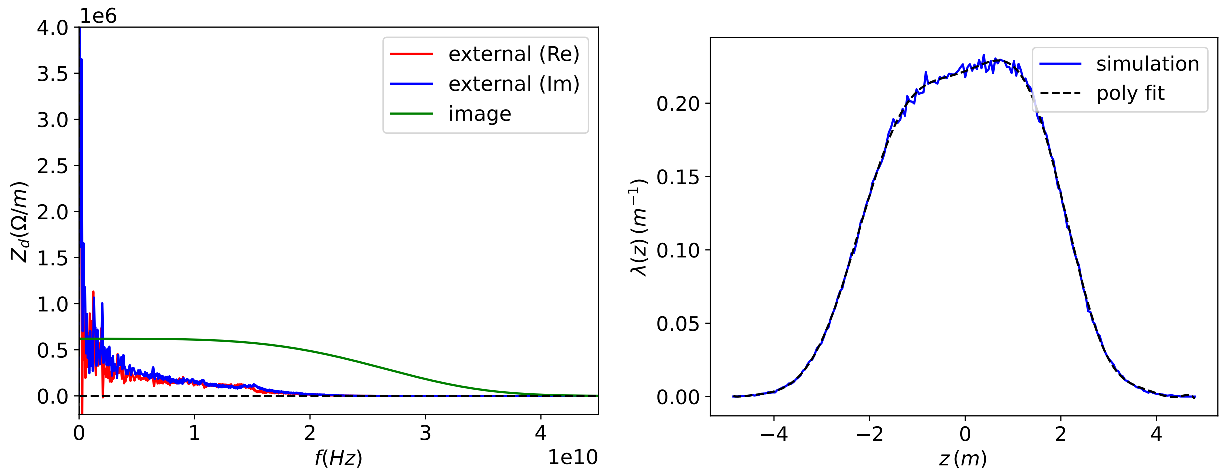

FIG. 1: Various components of the dipole impedance (left) and scaled longitudinal profile of the proton bunch for an average current of 3 mA (right).

bility threshold (for a more detailed discussion of the longitudinal dynamics, based on the Haïssinski solution, we refer the reader to Ref. [7]). Using the stated parameters, the left panel of Fig. 1 plots the real and imaginary parts of the external (i.e. geometric plus resistive wall-related) transverse impedance $Z_d(\omega)$, while also comparing them to the dipole space charge impedance component due to the image charges. As is evident, these contributions are of a similar magnitude (a few MΩ/m), but they are dwarfed by the beam-related dipole and quad impedances, which are given by $Z_x^{(q)} = -Z_x^{(d)} = 90$ MΩ/m and $Z_y^{(q)} = -Z_y^{(d)} = 330$ MΩ/m. On the other hand, the right-hand-side panel of Fig. 1 plots the scaled longitudinal profile of the bunch under consideration. For the latter, we note that one must use all sources of longitudinal impedance (resistive, geometric and space charge) in order to generate the distorted super-Gaussian profile shown here.

Before we move on to the main simulation results, it is instructive to consider the output of some simple tracking studies that aim to provide a benchmark for the intrinsic ELEGANT algorithm for transverse space charge calculation. In particular, Fig. 2 plots the extracted horizontal and vertical particle tunes for three consecutive turns, assuming tracking under the influence of nonlinear transverse space charge alone (the external dipole impedance is excluded here). As expected from our earlier discussion, the simulated tunes lie predominantly within the area defined by the original tune values and the linear space charge tune shifts, the latter defined by Eq. (3).

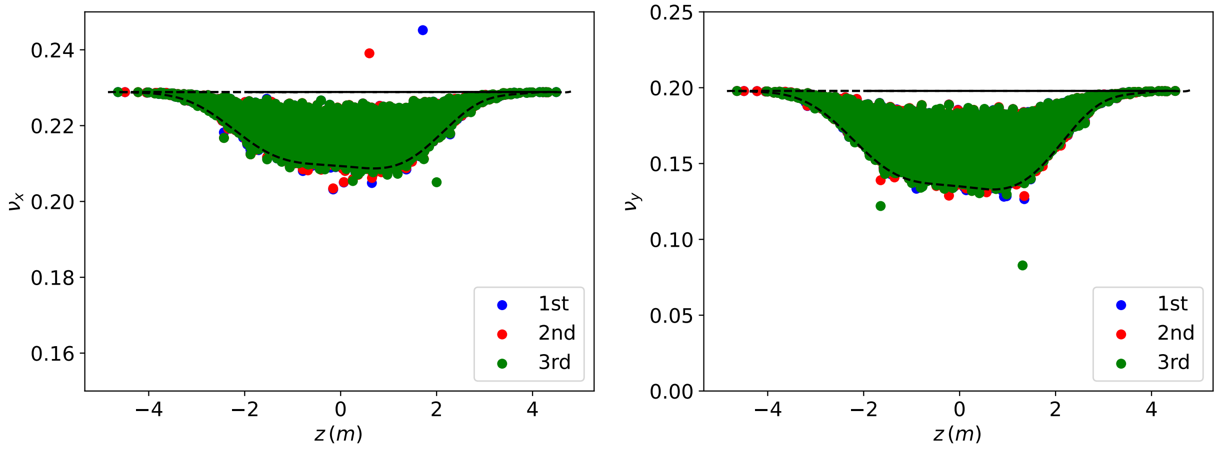

FIG. 2: Fractional particle tunes for three consecutive turns, for ELEGANT tracking under the influence of nonlinear transverse space charge (3 mA current). The flat lines represent the original tunes, while the dashed curves correspond to the linear space charge limits.

Having verified the space charge algorithm, we examine the stability of the parameter set under consideration by performing parallelized ELEGANT simulations. Unlike the previously-mentioned benchmark runs, the full-scale simulations take into account all sources of transverse impedance. Specifically, we follow two different approaches: for both, the external dipole impedance and its image charge-related counterpart are fed directly into the code as tabulated impedance files. However, as far as the remainder is concerned, we either

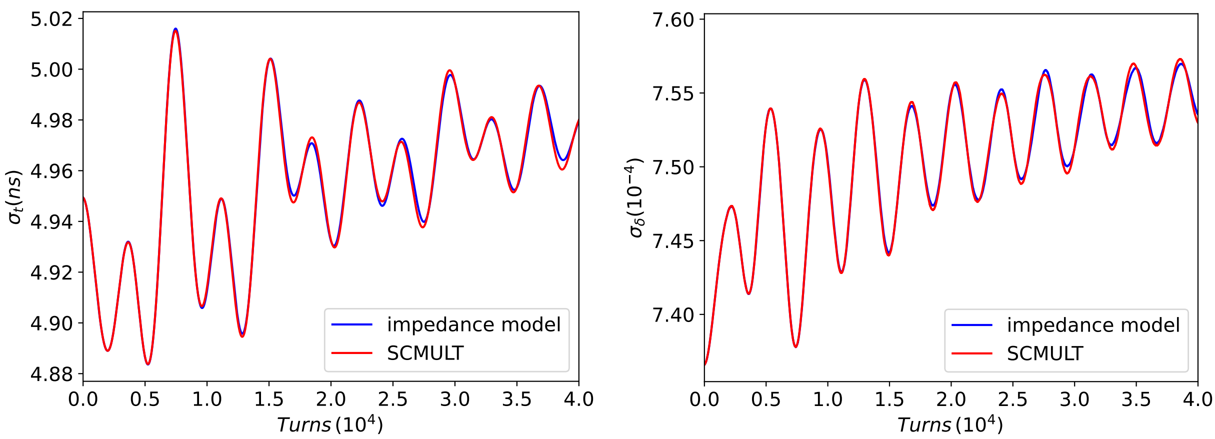

FIG. 3: ELEGANT simulation data for an average current of 3 mA/double RF system: the left-hand sub-figure shows the variation of the rms bunch duration σ_t over many revolutions, while the right-hand sub-figure plots the evolution of the rms energy spread.

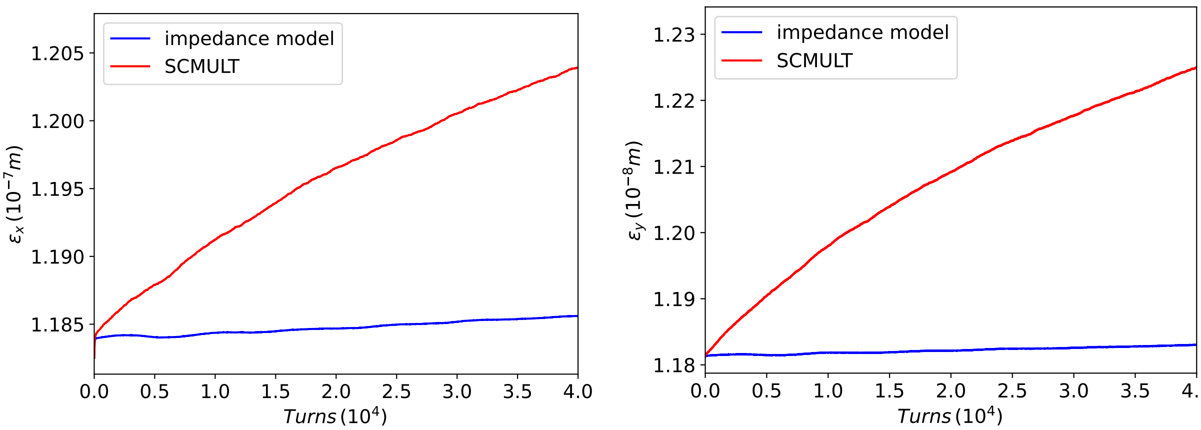

FIG. 4: ELEGANT simulation data for an average current of 3 mA/double RF system: the left-hand sub-figure shows the variation of the horizontal emittance ε_x over many revolutions, while the right-hand sub-figure plots the evolution of its vertical counterpart.

a) also feed it directly into the code in the form of dipole and quad impedance files, generated with the aid of Eq. (1) and incorporating the previously-discussed factor of 1/2 or b) model it explicitly through the SCMULT ELEGANT element. In both cases, to facilitate the transverse stabilization of the beam, an array of 80 octupoles (uniformly-distributed around the ring) is assumed, each with a 30 cm length and a strength of $K_3 = -150$ m^{-4} ([8, 9]). Finally, a lattice chromaticity of $\xi_{x,y} = d\nu_{x,y}/d\delta = 6.0$ has also been used in the simulations. The tracking extends up to 4×10^4 turns for 5×10^5 macro-particles, and key quantities such as the rms bunch duration, energy spread and the hadron transverse emittances are monitored in order to check for longitudinal and transverse instabilities. From the combined results, which are presented in Figs. 3-4, one observes i) a typical fluctuation of the longitudinal quantities due to the synchrotron motion and ii) a slow, relatively modest growth in terms of the transverse emittances, possibly numerical in nature. Overall, one may reasonably conclude that this system configuration is both longitudinally and transversely stable for average currents up to 3 mA.

The eventual transition to instability can be highlighted by increasing the current of the proton bunch. The corresponding simulation results are presented in Figs. 5-6, which show the evolution of the transverse emittances over a large number of turns for various single-bunch current values, using both the scaled impedance model and the SCMULT method. It is evident that both methods agree in predicting transverse stability for currents up to 4 mA.

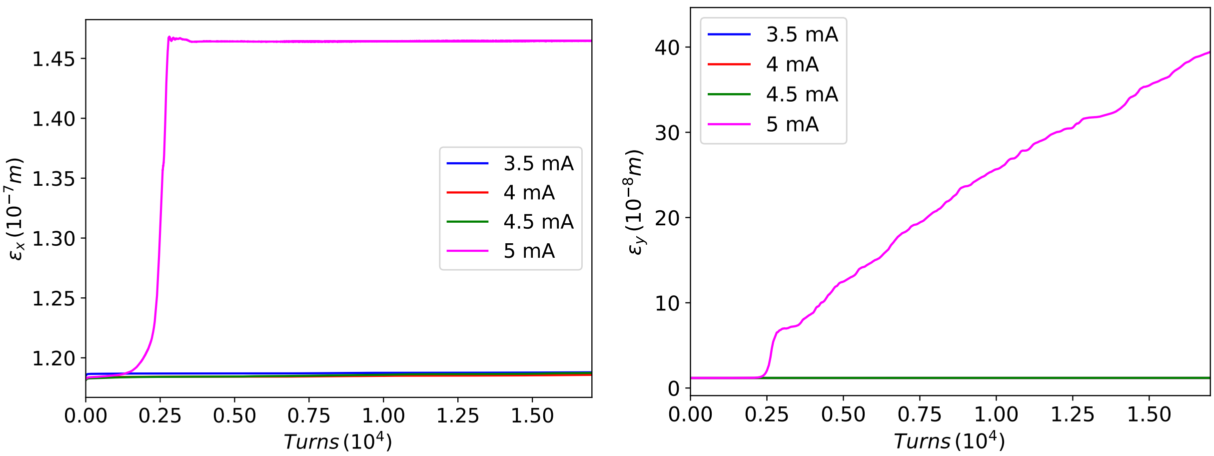

FIG. 5: ELEGANT simulation data for various average currents using the scaled transverse impedance model: the left-hand sub-figure shows the variation of the horizontal emittance ε_x over many revolutions, while the right-hand sub-figure plots the evolution of its vertical counterpart.

Of course, the two approaches remain qualitatively different: unlike the impedance model, SCMULT more faithfully takes into account the action dependence of the space charge tune shift. As a result, it is to be expected that the instability threshold and the actual pattern of transition into an unstable mode would be different for each method.

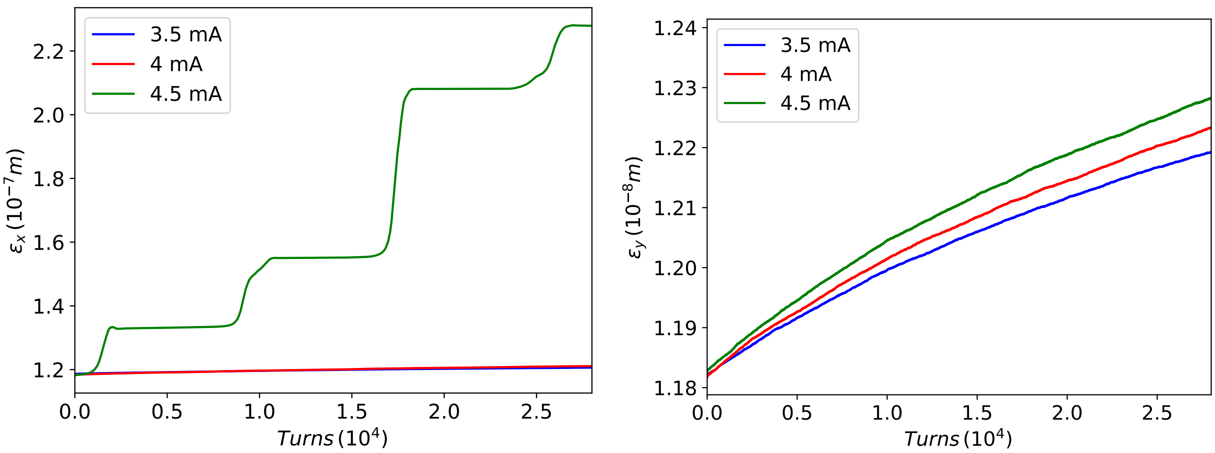

FIG. 6: ELEGANT simulation data for various average currents using the SCMULT method: the left-hand sub-figure shows the variation of the horizontal emittance ε_x over many revolutions, while the right-hand sub-figure plots the evolution of its vertical counterpart.

IV. CONCLUSIONS

In this study, we have investigated the single-bunch stability behavior (longitudinal and transverse) of the EIC proton beam at injection energy (23.8 GeV). Using a simplified model for the ring and taking into account all the major sources of impedance (space charge, as well as geometric and resistive wall-related), we performed parallelized simulation runs using the ELEGANT tracking code in order to ascertain the long-term evolution of the various beam quantities. Particular attentions was paid to the strong transverse space charge effect, with two different methods being used in order to model it as accurately as possible. The results of our simulations so far show that the proton beam can be made longitudinally and transversely stable for single-bunch currents of up to 4 mA, with the proper choice of lattice chromaticity and octupole strength. Of course, more optimization work remains to be done in order to, potentially, push the stability threshold to higher current values, as well as explore the synergy with other important topics such as dynamic aperture.

V. ACKNOWLEDGMENTS

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