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On the Computation of the Beam Position in the Arcs of the Hadron Storage Ring

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DISCLAIMER

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To compensate for the velocity differences of the hadrons and electrons in the Electron-Ion Collider, the hadron beam will be operated in a large range of the radial displacement [1] (±20 mm in the 64 mm wide vacuum chamber). Machine protection and beam optics measurement require high-accuracy calculations of the beam position from the signal induced on the pickup electrodes. For the HSR, we need at least the fifth-order polynomial fit, which provides 70 microns r.m.s. error in the 40 by 20 mm region of interest as shown in Fig. 1. The calculations are made using a MATLAB script solving the 2D electrostatic problem in the manner described in [2].

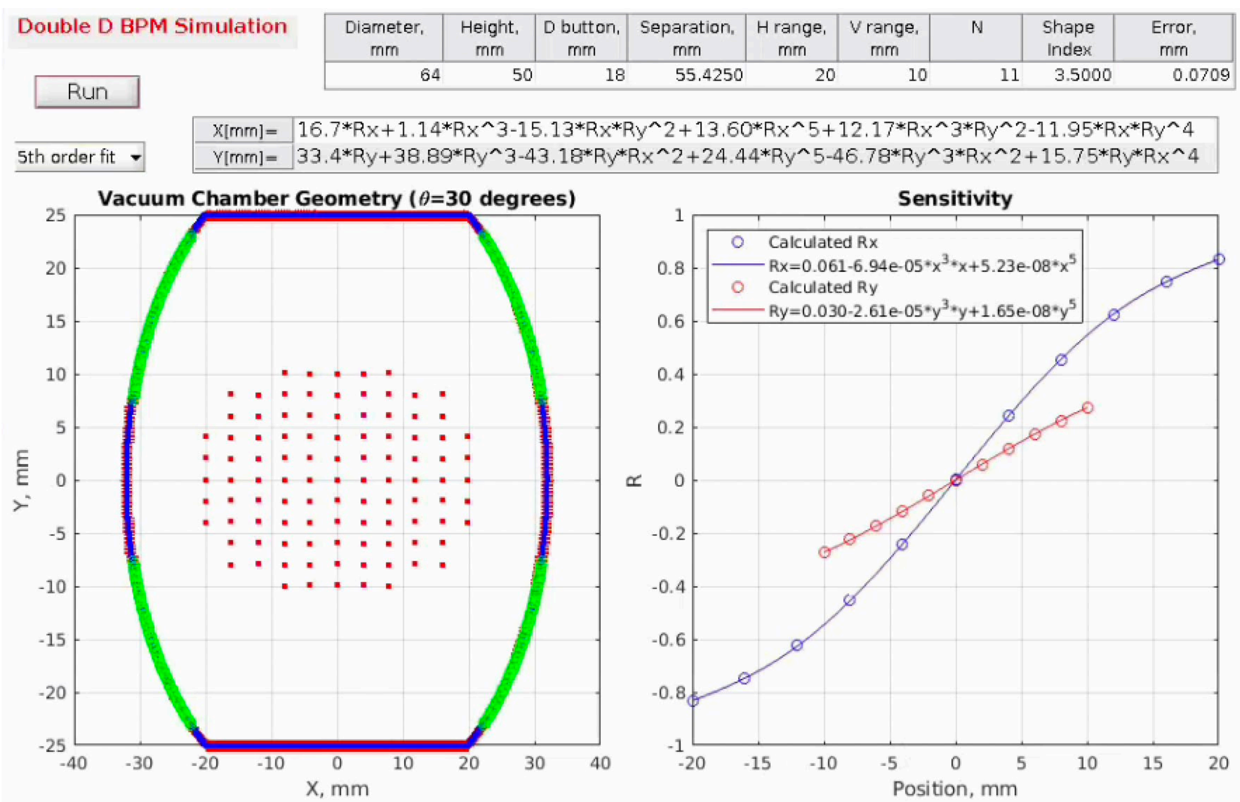

Figure 1: Fifth-order polynomial fit for HSR arc BPM.

It was proposed to utilize an analytical formula for beam position calculations [3]. The case was limited to the cylindrical vacuum chamber with straight electrode orientation (top-bottom and left-right). The calculations were based on the formulas for the distribution of the image current on the walls [4]. The geometry of the problem is shown in Fig. 2 with the vacuum chamber having a radius a, when the beam is displaced by $x = r\cos\theta$ and $y =$

$r \sin\theta$. The density of the induced charge vs. azimuthal angle φ is shown in the formula below:

$$\sigma(\theta) \sim \frac{a^2 - r^2}{a^2 + r^2 - 2ar\,cos(\phi - \theta)} \tag{1}$$

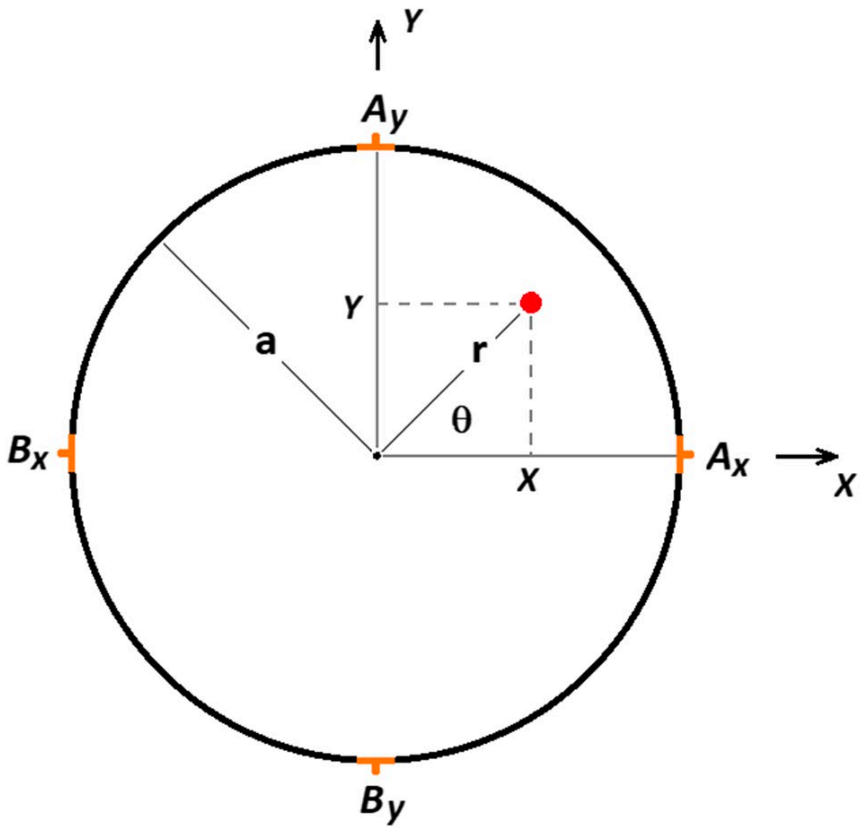

Figure 2: BPM geometry for straight configuration (adopted from [2]).

The beam position is calculated from the two values of difference over the sum

$$\begin{gathered} Q_x = \frac{A_x - B_x}{A_x + B_y} = \frac{2\rho \cos\theta}{\rho^2 + 1} \\ Q_y = \frac{A_y - B_y}{A_y + B_y} = \frac{2\rho \sin\theta}{\rho^2 + 1} \end{gathered} \tag{2}$$

where

$$\begin{gathered} \rho = r/a = \frac{1}{Q} - \sqrt{\frac{1}{Q^2} - 1} \\ Q = \sqrt{Q_x^2 + Q_y^2} \end{gathered} \tag{3}$$

Then the position can be found

$$x = a\rho \frac{Q_x}{Q}$$
$$y = a\rho \frac{Q_y}{Q} \quad (4)$$

The calculation using these formulas reaches 230 microns r.m.s. deviation from the nominal beam position. After introducing empirical corrections, the accuracy was improved to 23 microns.

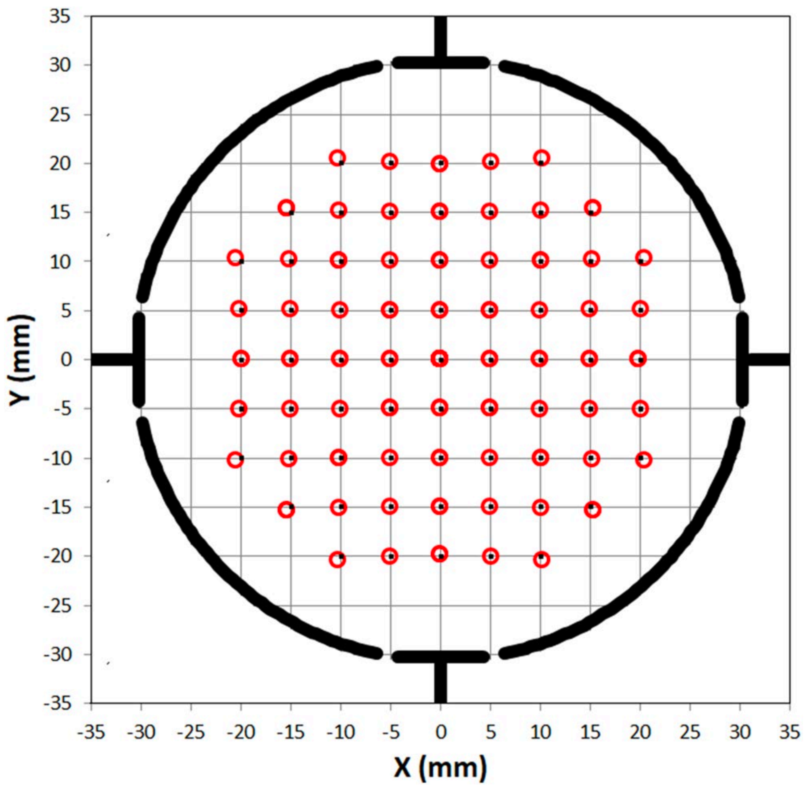

Figure 3: Comparison of nominal and calculated beam positions for the round chamber with the analytical formula (adopted from [2]).

The BPMs in the arcs of the hadron storage ring have a double-D shape with buttons placed diagonally, as shown in Fig. 1. The analytical formulas from [3] need to be modified for such geometry. We start with a cylindrical vacuum chamber and electrodes placed at an angle θ_0 as shown in Fig. 4.

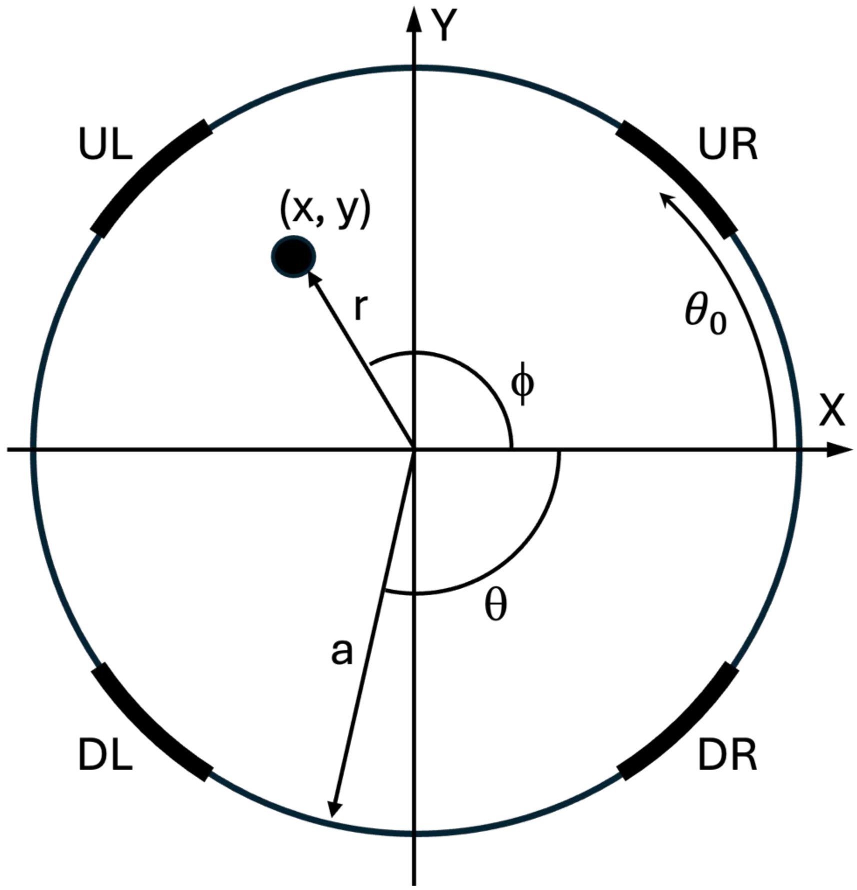

Figure 4: Rotated geometry of the BPM. There are four electrodes placed symmetrically. The vacuum chamber has a radius a, beam is located at (x, y) coordinates. We are looking for the charge density distribution vs. angle θ.

The dependence of induced charges and, hence, signal amplitudes for the electrodes can be found using Eq. 1.

$$
\begin{aligned}
V_{UR} &\sim \frac{a^2-r^2}{a^2+r^2-2ar\cos(\phi-\theta_0)} \\
V_{DR} &\sim \frac{a^2-r^2}{a^2+r^2-2ar\cos(\phi+\theta_0)} \\
V_{UL} &\sim \frac{a^2-r^2}{a^2+r^2+2ar\cos(\phi+\theta_0)} \\
V_{DL} &\sim \frac{a^2-r^2}{a^2+r^2+2ar\cos(\phi-\theta_0)}
\end{aligned}
\tag{5}
$$

Then we redefine the difference over the sum ratio for calculating the beam position

$$
\begin{aligned}
Q_X &= \frac{1}{2}\left(\frac{V_{UR}-V_{DL}}{V_{UR}+V_{DL}} + \frac{V_{DR}-V_{UL}}{V_{DR}+V_{UL}}\right) = \frac{2ar\cos\phi\cos\theta_0}{a^2+r^2} \\
Q_Y &= \frac{1}{2}\left(\frac{V_{UR}-V_{DL}}{V_{UR}+V_{DL}} - \frac{V_{DR}-V_{UL}}{V_{DR}+V_{UL}}\right) = \frac{2ar\sin\phi\sin\theta_0}{a^2+r^2}
\end{aligned}
\tag{6}
$$

and

$$Q = \sqrt{(Q_X \sin\theta_0)^2 + (Q_Y \cos\theta_0)^2} = \frac{2ar\sin\theta_0\cos\theta_0}{a^2+r^2} \tag{7}$$

From Eq. 7, we can find the ratio of the beam displacement from the center to the pipe radius

$$\frac{r}{a} = \frac{\sin\theta_0\cos\theta_0 - \sqrt{(\sin\theta_0\cos\theta_0)^2 - R^2}}{R} \tag{8}$$

From Eq. 6 and Eq. 7

$$\begin{aligned} \cos\phi &= \frac{R_X}{R}\sin\theta_0 \\ \sin\phi &= \frac{R_Y}{R}\cos\theta_0 \end{aligned} \tag{9}$$

Now we can write the formula for beam position calculations

$$\begin{aligned} x &= \frac{a\left(1-\sqrt{1-R^2/(\sin\theta_0\cos\theta_0)^2}\right)}{R^2}\sin^2\theta_0\cos\theta_0\, R_X \\ y &= \frac{a\left(1-\sqrt{1-R^2/(\sin\theta_0\cos\theta_0)^2}\right)}{R^2}\cos^2\theta_0\sin\theta_0\, R_Y \end{aligned} \tag{10}$$

The results using these formulas are shown in Fig. 5.

For the large button size, we can utilize corrections like the one described in [3] – a small tweak of the scaling factor, adding cross terms. The adjustment of the azimuthal angle of the button center is included as well. The formulas are shown below:

$$\begin{aligned} R'_x &= R_x\left(1 + a_x R_x^2 + b_x R_y^2\right) \\ R'_y &= R_y\left(1 + a_y R_y^2 + b_y R_x^2\right) \end{aligned} \tag{11}$$

The fitting results are shown in Fig. 6. The r.m.s. difference between the nominal and calculated beam positions is 40 microns. It should be mentioned that the fitting parameters depend on the shape of the region of interest.

The same correction procedure was utilized for the HSR arc BPM. The results are shown in Fig. 7. The deviation of the calculated position from the nominal is 60 microns. Further improvement is possible by adding the higher order terms to the correction.

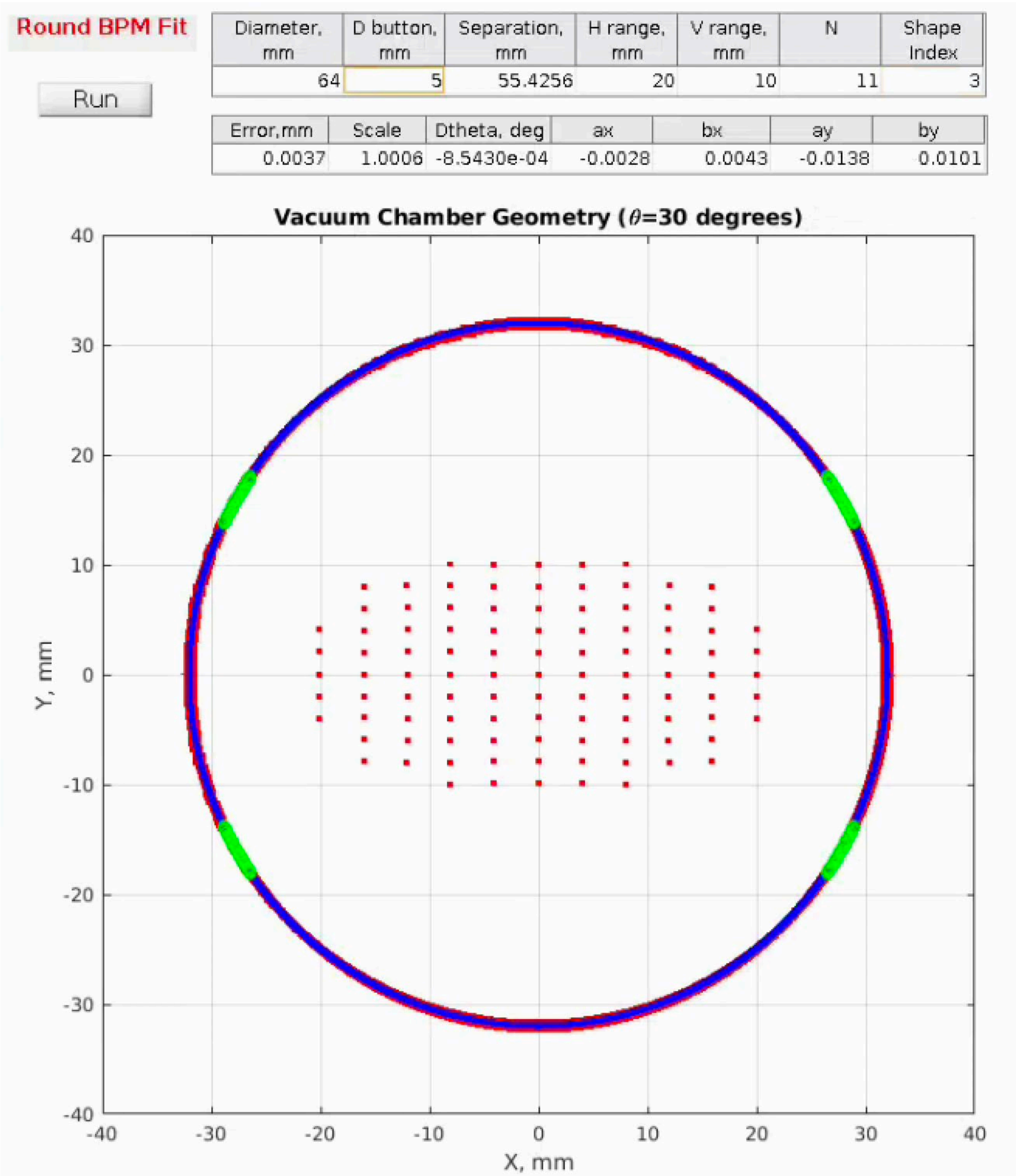

Figure 5: BPM geometry for rotated configuration in the cylindrical pipe and “uncorrected” analytical calculations.

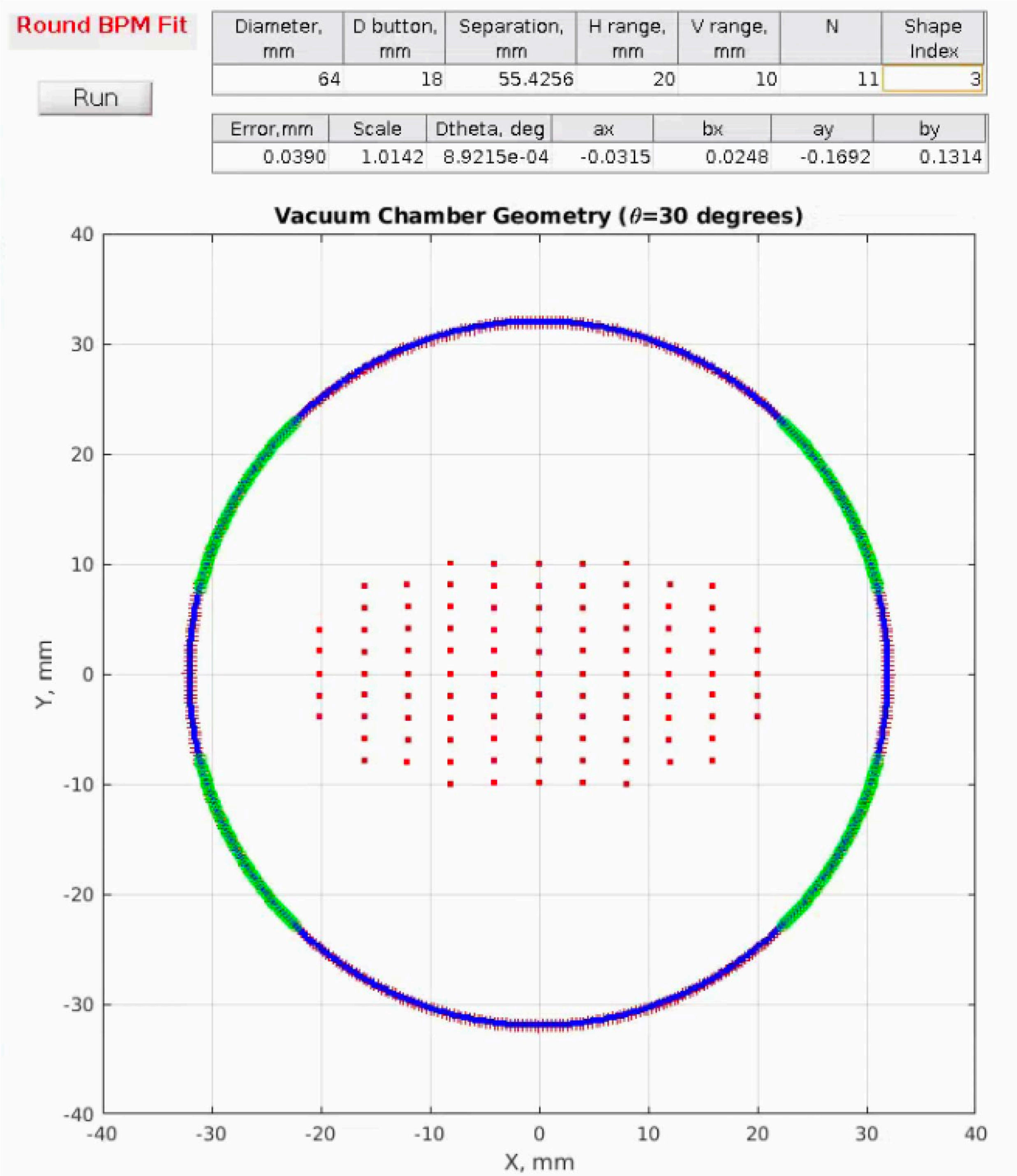

Figure 6: BPM with rotated configuration and corrected analytical calculations with a large button.

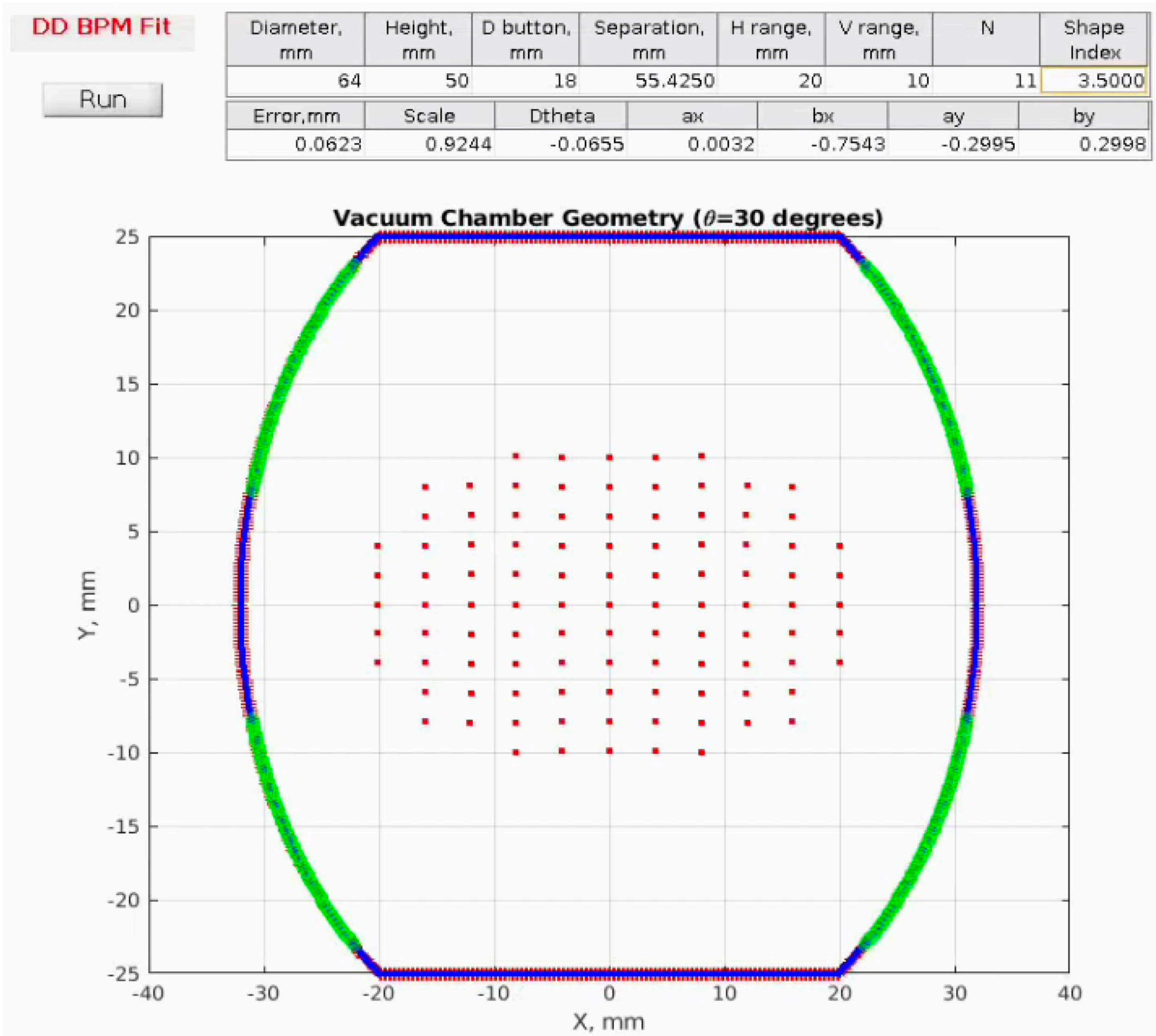

Figure 7: Corrected analytical calculations for the HSR arc BPM.

We have used the analytical fit small buttons in the round chamber to estimate the dependence of noise in the position vs. beam offset. Beam was supposed to have zero vertical offset. The signal on the buttons were estimated using Eq.1. Then the noise corresponding to the expected EIC conditions was added, and position was calculated using Eq. 10. These results are for single bunch single pass and only one ADC reading on the top of the pulse. Increasing sampling rate and number of bunches should improve precision of measurements.

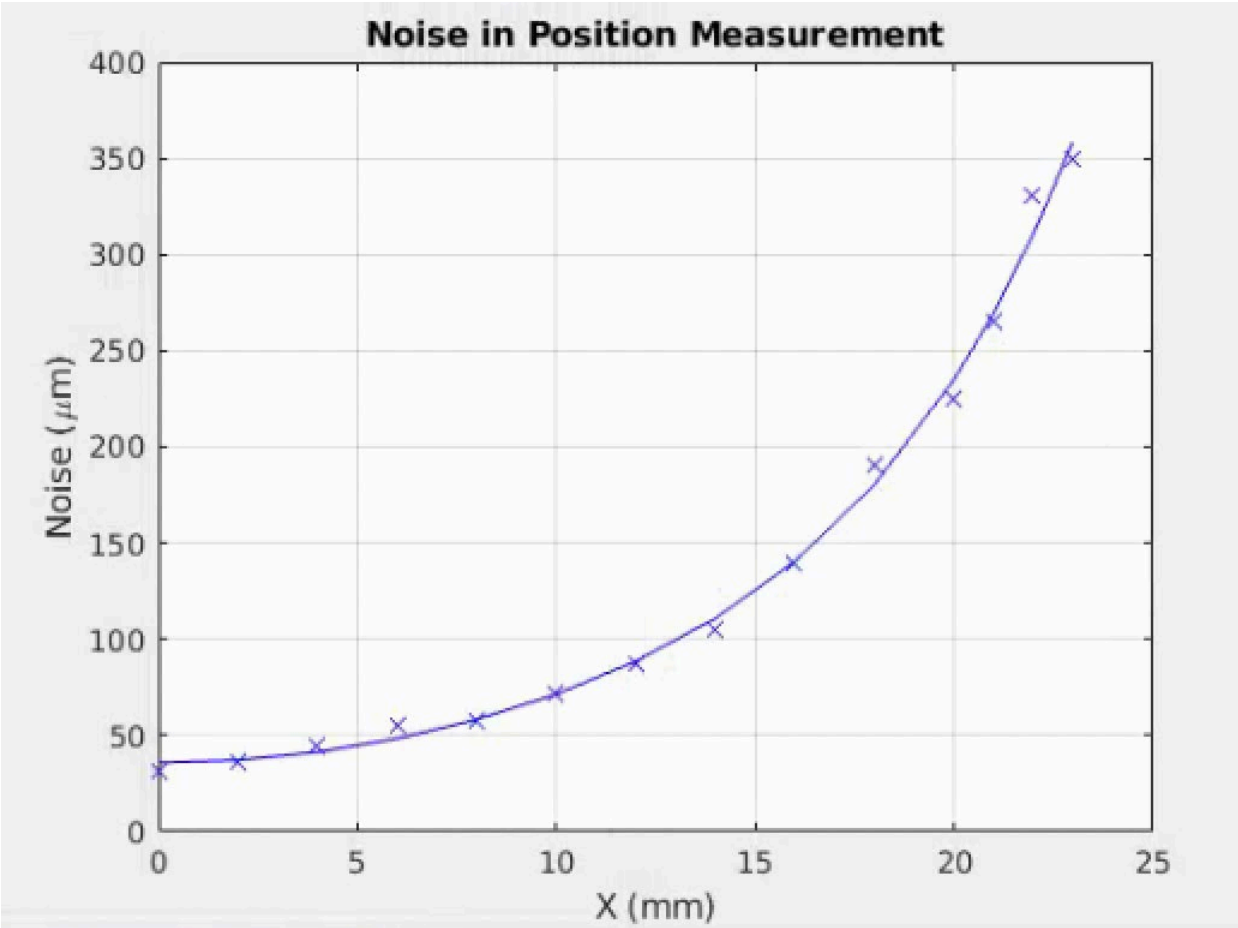

Figure 8: Estimate of the noise in the horizontal plane vs. orbit shift. The polynomial fit is described by formula $\sigma(x) \approx \sigma_0 \left(1 + \left(\frac{x}{10.2}\right)^2 + \left(\frac{x}{18.3}\right)^6\right)$.

References

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[2] T. Shintake et al., "Sensitivity Calculation of Beam Position Monitor Using Boundary Element Method," Nucl. Instr. Meth. A254 (1987), pp. 146-150.

[3] P. Thieberger *et al.*, "Fast readout algorithm for cylindrical beam position monitors providing good accuracy for particle bunches with large offset", Rev. Sci. Instrum. 89, 043303 (2018)

[4] R.T. Avery, A. Faltens, and E.C. Hartwig, "Non-intercepting Monitor of Beam Current and Position", Proc. of PAC'71, pp. 920-922.