

Trapping Two Dissimilar Rigidity Ions with Identical Average Dynamics

S. Brooks

January 2025

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy
USDOE Office of Science (SC), Nuclear Physics (NP)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Trapping Two Dissimilar Rigidity Ions with Identical Average Dynamics

Stephen Brooks

January 13, 2025

1 Introduction

It may seem obvious that ions with different charge-to-mass ratios (q/m) would have different dynamics in a trap or periodic focussing system, leading to different equilibrium bunch shapes and sizes. However, the ponderomotive acceleration in an oscillating field is proportional to $(q/m)^2$ rather than q/m for the direct field, leading to the possibility of combining the two effects to cancel any differences in the average dynamics between two chosen species.

2 Ponderomotive Acceleration

Nonrelativistically, the ponderomotive acceleration is given by

$$\mathbf{a}_p = -\frac{q^2}{4m^2\omega^2}\nabla|\mathbf{E}_p|^2,$$

for a oscillating electric field $\mathbf{E}(\mathbf{x}, t) = \mathbf{E}_p(\mathbf{x})\sin(\omega t)$. Compare this with the direct acceleration

$$\mathbf{a}_d = \frac{q}{m}\mathbf{E}_d.$$

2.1 Equal Acceleration of Two Species

For two ion species, consider the case where

$$\nabla|\mathbf{E}_p|^2 = \frac{4\omega^2}{\frac{q_1}{m_1} + \frac{q_2}{m_2}}\mathbf{E}_d.$$

The total (average) acceleration for ion species i is then

$$\begin{aligned}\mathbf{a}_i &= \frac{q_i}{m_i}\mathbf{E}_d - \frac{q_i^2}{4m_i^2\omega^2}\nabla|\mathbf{E}_p|^2 \\ &= \frac{q_i}{m_i}\mathbf{E}_d - \frac{q_i^2}{4m_i^2\omega^2}\frac{4\omega^2}{\frac{q_1}{m_1} + \frac{q_2}{m_2}}\mathbf{E}_d \\ &= \left(\frac{q_i}{m_i} - \frac{q_i^2}{m_i^2}\frac{1}{\frac{q_1}{m_1} + \frac{q_2}{m_2}}\right)\mathbf{E}_d \\ &= \frac{\frac{q_i}{m_i}\frac{q_1}{m_1} + \frac{q_i}{m_i}\frac{q_2}{m_2} - \frac{q_i^2}{m_i^2}}{\frac{q_1}{m_1} + \frac{q_2}{m_2}}\mathbf{E}_d.\end{aligned}$$

One of the terms on the numerator is also $\frac{q_i^2}{m_i^2}$, so it cancels, leaving only the mixed term. The average acceleration is therefore the same for both ions:

$$\mathbf{a}_i = \frac{\frac{q_1}{m_1} \frac{q_2}{m_2}}{\frac{q_1}{m_1} + \frac{q_2}{m_2}} \mathbf{E}_d.$$

2.2 Arbitrary Acceleration Ratio

In general,

$$\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} = \begin{bmatrix} \frac{q_1}{m_1} & -\frac{q_1^2}{4m_1^2\omega^2} \\ \frac{q_2}{m_2} & -\frac{q_2^2}{4m_2^2\omega^2} \end{bmatrix} \begin{bmatrix} \mathbf{E}_d \\ \nabla|\mathbf{E}_p|^2 \end{bmatrix}.$$

Inverting this system gives

$$\begin{bmatrix} \mathbf{E}_d \\ \nabla|\mathbf{E}_p|^2 \end{bmatrix} = \frac{1}{\frac{q_1}{m_1} - \frac{q_2}{m_2}} \begin{bmatrix} -\frac{m_1}{q_1} \frac{q_2}{m_2} & \frac{q_1}{m_1} \frac{m_2}{q_2} \\ -\frac{4m_1\omega^2}{q_1} & \frac{4m_2\omega^2}{q_2} \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix}.$$

If a ratio $\mathbf{a}_2 = R\mathbf{a}_1$ is required, by setting $\nabla|\mathbf{E}_p|^2 = S\mathbf{E}_d$ for some scalar S , then

$$\begin{aligned} \left(\frac{q_1}{m_1} - \frac{q_2}{m_2}\right) \mathbf{E}_d &= \left(-\frac{m_1}{q_1} \frac{q_2}{m_2} + R \frac{q_1}{m_1} \frac{m_2}{q_2}\right) \mathbf{a}_1 \\ \left(\frac{q_1}{m_1} - \frac{q_2}{m_2}\right) S\mathbf{E}_d &= \left(-\frac{4m_1\omega^2}{q_1} + R \frac{4m_2\omega^2}{q_2}\right) \mathbf{a}_1. \end{aligned}$$

Dividing the second equation's \mathbf{a}_1 -directed component by the first,

$$S = \frac{-\frac{4m_1\omega^2}{q_1} + R \frac{4m_2\omega^2}{q_2}}{-\frac{m_1}{q_1} \frac{q_2}{m_2} + R \frac{q_1}{m_1} \frac{m_2}{q_2}} = 4\omega^2 \frac{R \frac{q_1}{m_1} - \frac{q_2}{m_2}}{R \frac{q_1^2}{m_1^2} - \frac{q_2^2}{m_2^2}}.$$

3 Quadrupole Field

Consider the direct field $\mathbf{E}_d = (kx, ky, -2kz)$. It has the potential $V_d = -\frac{k}{2}x^2 - \frac{k}{2}y^2 + kz^2$ such that $\mathbf{E}_d = -\nabla V_d$. For a particular acceleration ratio, it is required that

$$\nabla|\mathbf{E}_p|^2 = S\mathbf{E}_d = -S\nabla V_d,$$

for S defined in the previous section. This would be satisfied if

$$|\mathbf{E}_p|^2 = -SV_d + C,$$

for any constant C . The solution cannot fill all of space as $|\mathbf{E}_p|^2 \geq 0$ and V_d can attain unbounded positive and negative values. C should generally be positive so that the region around the origin is feasible.

Linear electric fields cannot attain the required saddle shape of $|\mathbf{E}_p|^2$, so consider a combined dipole+sextupole electric field with

$$\mathbf{E}_p(0, 0, z) = (0, 0, \sqrt{C} - sz^2)$$

on the z axis. This extends symmetrically about the z axis to the full sextupole field

$$\mathbf{E}_p = \left(sxz, syz, \sqrt{C} + \frac{s}{2}x^2 + \frac{s}{2}y^2 - sz^2 \right)$$

that satisfies Maxwell's equations in free space. Its modulus squared is

$$|\mathbf{E}_p|^2 = C + s\sqrt{C}(x^2 + y^2 - 2z^2) + \text{terms in } (x, y, z)^4.$$

Ignoring the higher order terms that are small near the origin, this has the correct form. Equating coefficients of $x^2 + y^2 - 2z^2$ gives

$$s\sqrt{C} = -S \left(-\frac{k}{2} \right) \quad \Rightarrow \quad s = \frac{kS}{2\sqrt{C}} = \frac{2k\omega^2}{\sqrt{C} \left(\frac{q_1}{m_1} + \frac{q_2}{m_2} \right)} \text{ for equal focussing.}$$

4 Alternating Quadrupole Trap

The field in the previous section gives the two ion species identical accelerations near the origin, but it is not a trap because the effective potential is saddle-shaped. This can be rectified by modulating the entire potential at a frequency $\omega_a \ll \omega$, yielding an alternating effective quadrupole that is overall focussing by the alternating gradient principle. This can be done by setting $k(t) = k_0 \sin(\omega_a t)$ in the previous section, which is a simple modulation of \mathbf{E}_d but only changes the sextupole part of \mathbf{E}_p and not the dipole. This idea uses the ponderomotive motion principle *twice* because alternating gradient focussing reduces to ponderomotive focussing for small phase advances.

5 Accelerator Beamlines

This method can also be used to transport two species of different rigidity through a particle accelerator with the same optics. In this case, a 2D effective quadrupole is formed by a rapidly alternating dipole+sextupole field, superimposed with the correct amount of constant quadrupole and then this combination is alternated more slowly to give alternating gradient focussing.