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EIC hadron ring longitudinal stability simulations at injection

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I. INTRODUCTION

At the EIC injection energy (23.8 GeV for protons), the hadron beam experiences a relatively strong space charge field. The corresponding wakefield, in conjunction with the wakes from the RF cavities (i.e. the geometrical wakes), may become the cause of single-bunch instabilities, both longitudinal and transverse. In this report, we focus on the longitudinal stability aspect of this picture, using a simplified model for the ring.

II. LONGITUDINAL BEAM DYNAMICS

For the case of a single (regular) RF system, the longitudinal motion of the hadron beam under the influence of wakefields can be modeled by the following equations ([1–3]):

$$\frac{dz}{dt} = -c\alpha\eta \, , \frac{d\eta}{dt} = \frac{1}{\alpha}\frac{\omega_{s0}^2}{c}z - \frac{e^2}{\gamma_0 mcL_R}\int_z^{\infty} dz' n_l(z',t) w_l(z'-z) \, . \tag{1}$$

Here, e and m are the charge and mass of the hadrons, $\gamma_0 = (1-\beta_0^2)^{-1/2}$ is the relativistic factor, $z = z_{lab} - c\beta_0 t$ is the longitudinal position of a hadron relative to the bunch centroid, $\eta = \Delta\gamma/\gamma_0$ is the energy deviation variable, ω_{s0} is the synchrotron frequency and $\alpha = \alpha_c - 1/\gamma_0^2$ is the slippage factor (itself expressed in terms of the momentum compaction factor $\alpha_c = 1/\gamma_t^2$, where γ_t is the transition gamma). For a sinusoidal RF voltage profile of the form

$$V_{RF}(t) = V_0 \sin(\omega_{RF} t) \, , \tag{2}$$

where V_0 is the voltage amplitude and $\omega_{RF}/2\pi = f_{RF}$ is the RF frequency, the synchrotron frequency ω_{s0} is given by

$$\omega_{s0} = \left(\frac{2\pi\alpha f_{RF} eV_0}{mc^2\gamma_0 T_0 \beta_0}\right)^{1/2} ,$$

where $T_0 = L_R/(c\beta_0)$ is the revolution period for a ring of circumference L_R (assuming $\eta = 0$), the synchrotron tune being equal to $\nu_s = (\omega_{s0}T_0)/2\pi$. Moreover, $n_l(z,t)$ is the line density of the hadron bunch - which satisfies the relation $\int_{-\infty}^{\infty} dz' n_l(z',t) = N$, where N is the total number of hadrons - while $w_l(s)$ is the *longitudinal wake*. The latter is expressed in terms of the *longitudinal impedance* $Z_l(\omega)$ via the relation

$$w_l(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega Z_l(\omega) \exp(-i\omega s/c) \, . \tag{3}$$

From a physical point of view, in the context of our notation $-q_t q_s w_l(s)/c$ expresses the longitudinal momentum kick from a source particle with charge q_s to a trailing (test) particle with charge q_t located a distance s behind the source. It is also worth noting that, in the second term on the RHS of the energy equation in Eq. (1) - which expresses the energy change due to the wakefields, the lower integration limit has to be switched from z to $-\infty$ for a wake $w_l(s)$ that does not vanish for negative s (such as the space charge wake to be discussed later).

In the case of zero wake, the phase space density of a bunch with a Gaussian energy profile that is matched to the RF system is of the form

$$f(z,\eta) \propto \exp\left(-\frac{z^2}{2\sigma_{z0}^2} - \frac{\eta^2}{2\sigma_\eta^2}\right) , \tag{4}$$

where σ_η is the rms energy spread and $\sigma_{z0} = \alpha c \sigma_\eta / \omega_{s0}$ is the unperturbed rms bunch length. Under the influence of the wake, the equilibrium longitudinal profile of the beam (but not its energy profile) is distorted. The new longitudinal profile $F_0(z)$ satisfies the *Haïssinski equation*, namely

$$F_0(z) = \exp\left(-\frac{z^2}{2\sigma_{z0}^2} + \frac{e^2}{\gamma_0 mc^2 \alpha \sigma_\eta^2 L_R} \int_0^z dz' \int_{z'}^\infty dz'' n_l(z'') w_l(z''-z')\right) , \tag{5}$$

where the equilibrium line density n_l is given by $n_l(z) = NF(z) = NF_0(z)/\int_{-\infty}^\infty dz' F_0(z')$.

The analytical expressions presented above can be readily extended for the case of a double RF system, an option that may be used to generate a flatter longitudinal profile. In particular, for an RF voltage of the form

$$V_{RF}(t) = V_0 \sin(\omega_{RF} t) - \frac{V_0}{2} \sin(2\omega_{RF} t) , \tag{6}$$

which incorporates a second-harmonic contribution in order to generate a quartic potential well (to leading order), the longitudinal equations of motion become ([1])

$$\frac{dz}{dt} = -c\alpha\eta \, , \, \frac{d\eta}{dt} = \frac{eV_0}{2\gamma_0 mc^2 T_0}\left(\frac{\omega_{RF} z}{\beta_0 c}\right)^3 - \frac{e^2}{\gamma_0 mc L_R}\int_z^\infty dz' n_l(z',t) w_l(z'-z) , \tag{7}$$

while the new Haissinski equation is

$$F_0(z) = \exp\left(-\frac{z^4}{4\sigma^4} + \frac{e^2}{\gamma_0 mc^2 \alpha \sigma_\eta^2 L_R} \int_0^z dz' \int_{z'}^\infty dz'' n_l(z'') w_l(z''-z')\right) , \tag{8}$$

with $n_l(z) = NF(z) = NF_0(z)/\int_{-\infty}^\infty dz' F_0(z')$ and

$$\sigma = c\left(\frac{\gamma_0 mc^2 T_0 \alpha \sigma_\eta^2 \beta_0^3}{4\pi^3 e V_0 f_{RF}^3}\right)^{1/4} . \tag{9}$$

In the context of this study, the main impedance sources are the RF cavities and space charge, with the latter being particularly important at injection energy. The (purely inductive) longitudinal space charge impedance for the whole ring is given by

$$Z_{l,SC}(\omega) = -i\omega L = iZ_0\Lambda\frac{L_R\omega}{2\pi\gamma_0^2 c}\,, \tag{10}$$

where L is the total inductance and $Z_0 \approx 377\,\Omega$ is the vacuum impedance. The form factor Λ is given by $\Lambda = 1/2 + \log(b/a)$, if one assumes a round beam pipe of radius b and a flattop transverse profile for the beam with radius a. For a Gaussian transverse profile with an rms beam size a, the form factor becomes $\Lambda \approx -0.05 + \log(b/a)$. This simple analytical formula for $Z_{l,SC}$ is valid for $\omega \ll \gamma_0 c/b$. Moreover, the point charge - to - point charge wake associated with the inductive space charge impedance is given by $w_l(s) = Lc^2\delta'(s)$, where $\delta(s)$ is the delta function. For this particular wake, the Haissinski equation can be further simplified. For example, assuming a regular RF system and neglecting impedance sources other than space charge, the equilibrium longitudinal profile satisfies the relation

$$F_0(z) = \exp\left(-\frac{z^2}{2\sigma_{z0}^2} - \frac{e^2N\hat{L}}{\gamma_0 m\alpha\sigma_\eta^2}\frac{F_0(z)}{\int_{-\infty}^{\infty}dz'F_0(z')}\right)\,, \tag{11}$$

where $\hat{L} = L/L_R = -Z_0\Lambda/2\pi\gamma_0^2 c$ is the inductance per unit length along the ring. An entirely analogous relations is valid for the case of the double RF system.

Lastly, it needs to be clarified that, although this study does not consider transverse instabilities (such as those driven by transverse wakes), a simple, smooth focusing model has been assumed for the betatron oscillations of the hadrons. According to this approximation, the constant beam sizes $\sigma_{x,y}$ are given by $\sigma_{x,y} = (\epsilon_{x,y}\beta_{x,y})^{1/2}$, where $\beta_{x,y}$ are the average beta functions and $\epsilon_{x,y}$ are the transverse emittance values. Since the space charge impedance calculation that was presented earlier relies on a round beam, one can typically take $\sigma \equiv a \approx (\sigma_x\sigma_y)^{1/2}$, assuming that the size asymmetry between x and y is not too large.

III. STABILITY STUDIES

Confining our attention to protons from now on, the main parameters for the system configuration under study are listed in Table I. Fig. 1 plots the real and imaginary part of the geometrical impedance $Z_{l,g}(\omega)$, while also comparing them to the (purely imaginary) space charge impedance. As is evident, the space charge impedance contribution is the

TABLE I: Stability study parameters

Parameter	**Value**
Energy spread σ_η	7.4×10^{-4}
RMS bunch length σ_{z0} (m)/single RF system	1
RF voltage V_0 (kV)	40
RF frequency f_{RF} (MHz)	24.61
Transition gamma γ_t	22.7
Injection gamma γ_0	25.36
Momentum compaction factor α_c	1.94×10^{-3}
Proton normalized emittance $\gamma_0\epsilon_{x,y}$ (mm-mrad)	0.5 / 0.3
Proton average beta function $\beta_{x,y}$ (m)	29.8 / 26.7
Proton transverse rms beam size $\sigma_{x,y}$ (mm)	0.76 / 0.56
Ring circumference L_R (km)	3.83
Average beam pipe radius b (mm)	24
Average beam radius a (mm)	0.66

predominant one. However, one must still use both sources of impedance in order to generate the correct particle distribution that satisfies the Haïssinski profiles mentioned earlier in the

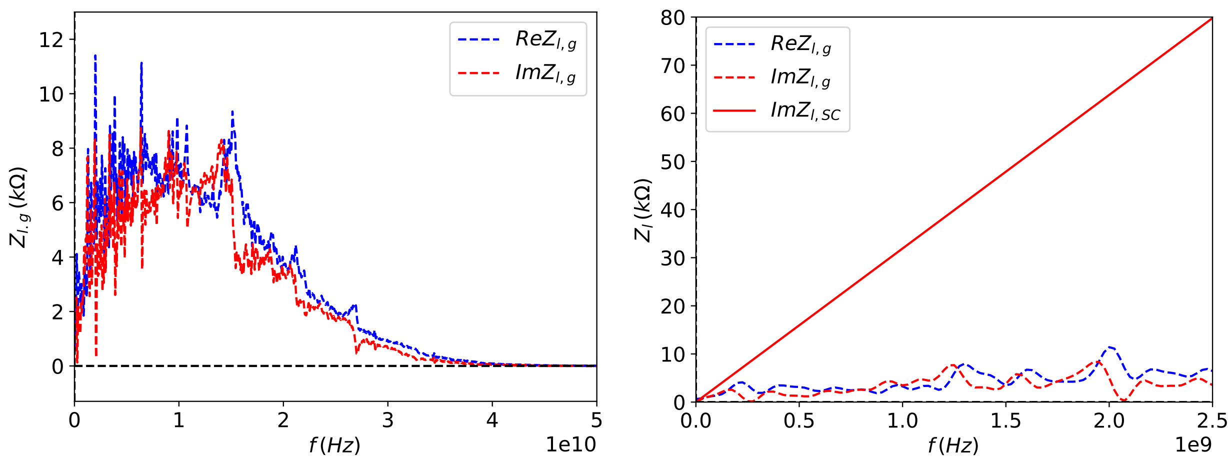

FIG. 1: Impedance profiles used for the longitudinal stability simulations: the left-hand panel shows the geometrical impedance, while the right-hand panel also plots the imaginary part of the space charge impedance.

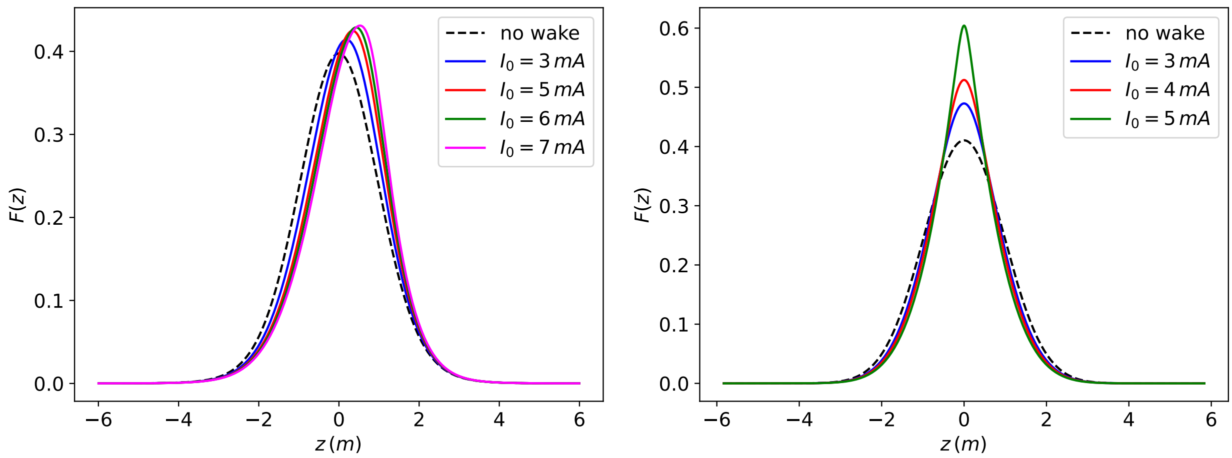

FIG. 2: Scaled longitudinal (Haïssinski) profiles for various average currents I_0 (single RF system, $\int_{-\infty}^{\infty} dz F(z) = 1$). The left-hand panel includes both the geometrical and the space charge impedance, while its right-hand counterpart only takes into account the space charge effect.

text. This becomes evident from Figs. 2-3, where we plot the scaled longitudinal profiles $F(z)$ for various values of the average bunch current $I_0 = eN\beta_0 c/L_R$, both for the single and the double RF system cases. The inclusion of the geometrical impedance qualitatively modifies the longitudinal equilibrium profile, adding a characteristic asymmetry and making the profile less sharp than in the case of pure space charge. Moreover, we note that - as

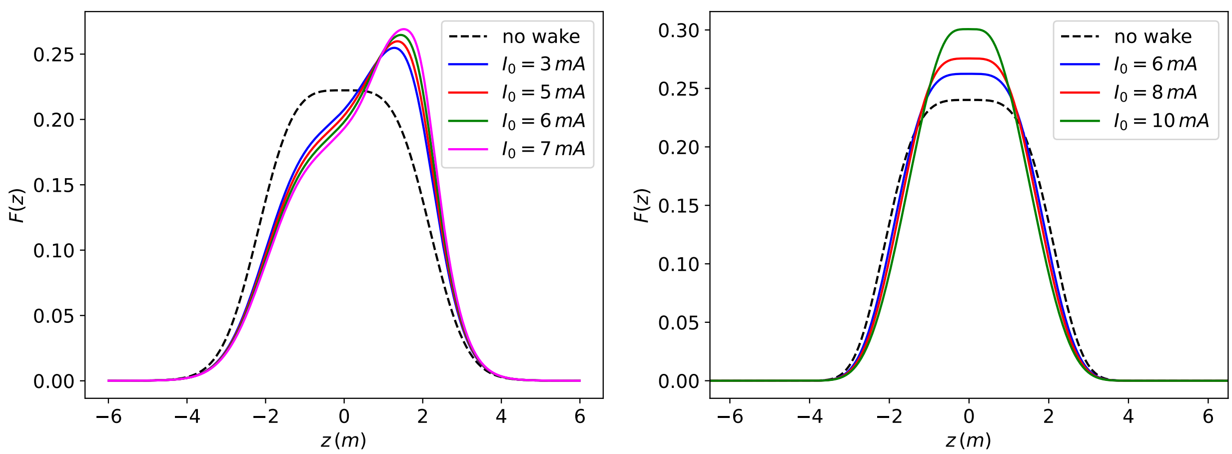

FIG. 3: Scaled longitudinal (Haïssinski) profiles for various average currents I_0 (double RF system, $\int_{-\infty}^{\infty} dz F(z) = 1$). The left-hand panel includes both the geometrical and the space charge impedance, while its right-hand counterpart only takes into account the space charge effect.

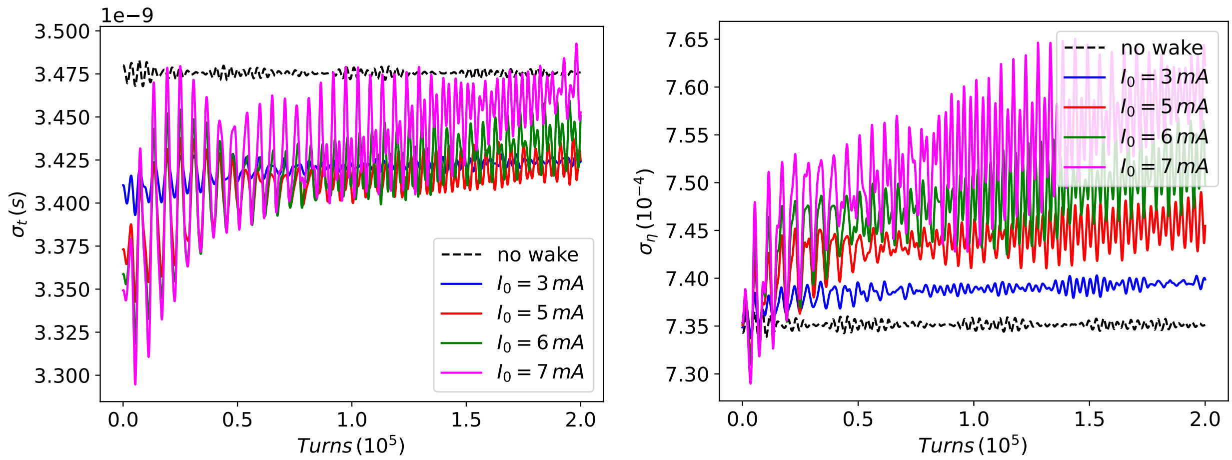

FIG. 4: ELEGANT simulations for various average current values/single RF system: the left-hand panel shows the variation of the rms bunch duration σ_t over many revolutions, while the right-hand panel plots the evolution of the rms energy spread.

expected - the double RF system produces a longer bunch with a flatter longitudinal profile.

To examine the stability of these equilibrium distributions, we perform parallel ELEGANT simulations (Ref. [4]) using the previously-described Haïssinski solutions as input (specifically, those which include both sources of impedance). The tracking is done for up to 2×10^5 turns, and key quantities such as the rms bunch duration and the energy spread

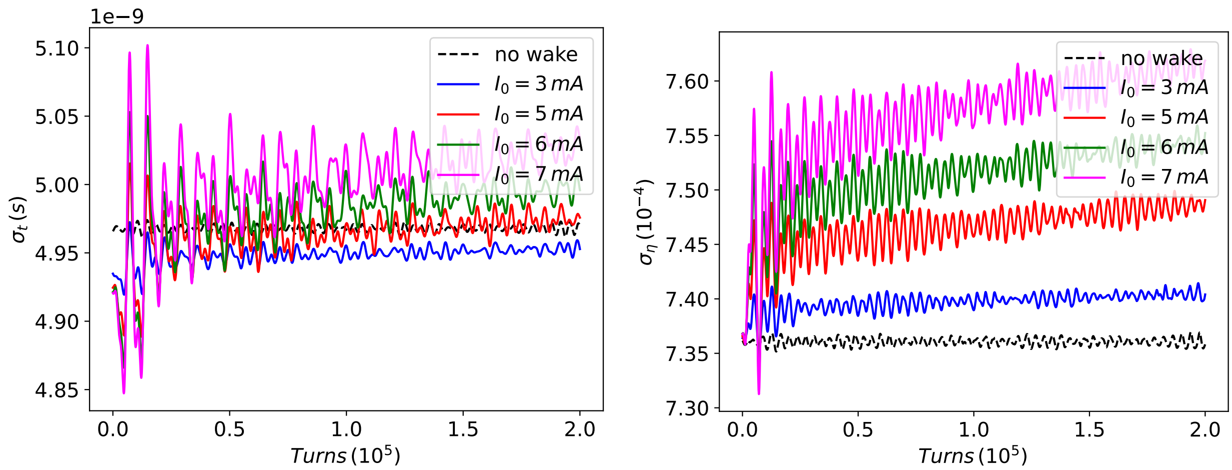

FIG. 5: ELEGANT simulations for various average current values/double RF system: the left-hand panel shows the variation of the rms bunch duration σ_t over many revolutions, while the right-hand panel plots the evolution of the rms energy spread.

are monitored in order to check for longitudinal instabilities. From the relevant results, which are presented in Figs. 4-5, one observes that ramping up the current to 7 mA leads to i) a slow, relatively modest average growth for the energy spread and bunch length over time and ii) an initial slight drop in the bunch length (driven by the space charge), eventually counterbalanced by the geometrical impedance as the current increases. Overall, one may reasonably conclude that this system configuration is longitudinally stable for average currents up to a factor of two larger than the nominal value of 3.5 mA.

IV. CONCLUSIONS

In this study, we have investigated the single-bunch longitudinal stability of the EIC proton beam at injection energy (23.8 GeV). Using a simplified model for the ring and taking into account the main sources of impedance (space charge and RF cavities), we generate the Haïssinski equilibrium profiles for various values of the average current and use them as input for parallel simulations using the ELEGANT particle tracking code. The results of these simulations show that the proton beam remains longitudinally stable for average currents of up to 7 mA.

V. ACKNOWLEDGMENTS

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