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# Normalized Emittance Measurements Using a Current Transformer Signal of Spiraling Beam

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# DISCLAIMER

| AGS Complex Machine Studies<br>(AGS Studies Report No. 314)<br>Normalized Emittance Measurements Using a Current Transformer Signal of Spiraling Beam |
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| **Study Period:** April 1, 1994 |
| **Participants:** K. Zeno |
| **Reported by:** K. Zeno |
| **Machine:** Booster |
| **Beam:** Proton |
| **Tools:** Circulating Current Transformer, LeCroy Scope, Spreadsheet, MUX/Xbar, Spiral Program |
| **Aim:** To test the Spiral program |

# I. Introduction

An application is being developed using Labview data acquisition software. The application's aim is to extract information about the horizontal particle distribution and emittance of the circulating beam. The measurement uses a signal from a circulating beam current transformer as the beam spirals and is lost on an aperture. This condition exists when there is no RF accelerating voltage and the main magnet field is changing. In general, the measurement is taken with an increasing magnet field (the beam is lost on inside apertures). The analysis is based on the premise that the horizontal phase space distribution is related to the form of the spiraling current transformer signal.

This analysis is performed as if the revolution frequency of the circulating beam is infinite and all the beam scrapes off from the circulating beam at one azimuthal location. If this were the case, and it is approximated as being so, betatron oscillations would cause the beam to be scraped off in circular rings as viewed in normalized phase space. The time at which no circulating beam remains would be where the horizontal center of charge of the beam finally scrapes the inside aperture (neglecting the effects of the momentum spread). The rate at which the circulating beam moves into the aperture is determined by the dB/dt and B of the main magnet (assumed constant), and the dispersion (D) at the azimuthal location where the scraping occurs. Let $t' = (t_o - t_s)$, where $t_o$ is the time where the last part of the beam scrapes, and $t_s$ is the time when the ring in question is scraping. Then,

$$\frac{dx}{dt'} = D \cdot \frac{dB/dt}{B} \Rightarrow \frac{d}{dt'}\left(\sqrt{\beta}\, r\right) \Rightarrow \frac{dr}{dt'} = \frac{D}{\sqrt{\beta}} \cdot \frac{dB/dt}{B},$$

x represents the distance from the center of charge to the aperture, r represents the radius from the center of charge of the beam to the circular ring which is scraping (in normalized phase space), and β is the value of the horizontal Beta function at the scraping point. In a time interval Δt', the amount of radius change is given by,

$$\Delta r = \int_{\Delta t'} \frac{dr}{dt'} dt' = \frac{dr}{dt'} \int_{\Delta t'} dt' \doteq \frac{dr}{dt'} \cdot \Delta t',$$

where dr/dt' is assumed to be constant. So, for a constant interval Δt', the radius change is also a constant. It is also assumed that the particle distribution of the circulating beam is unaffected by the spiraling and scraping processes, and does not change over the measurement interval.

The distance (in normalized phase space) from the center of the distribution to the ring that is scraping can be calculated in terms of t',

$$r(t') = \int_{t'=0}^{t'=t_0-t_s} \frac{dr}{dt'} dt' = \frac{dr}{dt'} \cdot \left((t_0 - t_s) - 0\right) = \frac{dr}{dt'} \cdot t' = \frac{D}{B\sqrt{\beta}} \frac{dB}{dt} \cdot t'.$$

The area of the circular ring (dε) at a given time t', can be determined by differentiating the relation $r = \sqrt{\varepsilon}$,

$$r = \sqrt{\varepsilon} \Rightarrow r^2 = \varepsilon \Rightarrow d\varepsilon = 2r\,dr$$

where πε is the area enclosed in phase space by the ring. The average radial charge density in the horizontal plane of the beam in this ring can be determined if the rate at which circulating beam is lost at the time $t_s$, the time $t_s$, and the time $t_o$ are known. A spiraling current transformer signal contains this information for the entire set of rings which compose the beam distribution in phase space. The average radial density of a ring of area Δε and width Δr at a time t' is,

$$\rho(\varepsilon(t')) = \frac{\Delta Q(t')}{\Delta\varepsilon(t')} = \frac{\Delta Q(t')}{\Delta t'} \cdot \frac{\Delta t'}{\Delta\varepsilon(t')} = \frac{\Delta Q(t')}{\Delta t'} \cdot \frac{\Delta t'}{2r(t')\,\Delta r} = \frac{\Delta Q(t')}{\Delta t'} \cdot \frac{\Delta t'}{2r(t')\frac{dr}{dt'}\Delta t'}$$

$$= \frac{\Delta Q(t')}{\Delta t'} \cdot \frac{1}{2r(t')\left(\frac{D}{\sqrt{\beta}} \frac{dB/dt}{B}\right)} = \frac{\Delta Q(t')}{\Delta t'} \cdot \frac{\sqrt{\beta}\,B}{2D \cdot \frac{dB}{dt}} \cdot \frac{\sqrt{\beta}B}{t'\,D(dB/dt)}$$

$$= \frac{\Delta Q(t')}{\Delta t'} \cdot \frac{\beta\,B^2}{2D^2 \cdot \left(\frac{dB}{dt}\right)^2 \cdot t'}$$

$\frac{\Delta Q(t')}{\Delta t'} = -\frac{\Delta Q(t)}{\Delta t}$, which can be obtained from the current transformer. So, the entire average radial phase space charge density in the horizontal plane can be reconstructed. The Spiral program takes the digitized current transformer signal, together with β and D at the scraping point, and B and dB/dt of the main magnet as inputs. It analyzes this data to reconstruct the distribution. It also calculates the 95% emittance from this data.

The program uses a nonlinear fitting routine to find the time at which the current transformer trace no longer has any beam induced signal on it. This is taken as the point at which the beam at the center of the distribution scrapes on the aperture. The routine assumes there is some beam at the center of the distribution, but that the charge density near the center is essentially constant. For a constant charge density (ρ) the current transformer signal should be parabolic since,

$$Q_{enclosed}(\varepsilon_s) = \pi \int_0^{\varepsilon_s} \rho(\varepsilon)\, d\varepsilon = \pi\rho \int_0^{\varepsilon_s} d\varepsilon = \pi\rho\varepsilon_s$$
$$= \pi\rho\left(t' \cdot \frac{D}{\sqrt{\beta}B}\frac{dB}{dt}\right)^2 = \frac{\pi\rho}{\beta}\left(\frac{D}{B}\frac{dB}{dt}\right)^2 \cdot t'^2$$

Where $\pi\varepsilon_s$ is the area enclosed and $Q_{enclosed}(\varepsilon_s)$ is the amount of charge enclosed in that area. The amount of charge enclosed (and hence the amplitude of the current transformer trace) is a quadratic function of t'. The program uses the values for the intensity obtained from the current transformer as a function of x, where x is essentially the time from the end of the trace transformed to distance by the scale factor, $D \cdot \frac{dB/dt}{B}$. The program fits a range of this data from about 10% of the maximum intensity to the end of the trace. It fits it to the following function to find the value of x at the center of the distribution ($x_o$),

$$V(x) = \begin{cases} ax^2 + V_o, & x > x_o \\ V_o, & x < x_o \end{cases}.$$

Where V(x) is the current transformer voltage and is proportional to the intensity, and $V_o$ is the offset of the signal. Once the value of $x_o$ is found (and hence $t_o = x_o B / D \cdot (dB/dt)$ ), the distance from the center of the distribution at which the circulating beam intensity is 95% of its maximum value can be found. This is given by,

$$r_{95\%} = \frac{D(dB/dt)}{B\sqrt{\beta}}(t_o - t_{95\%}),$$

where $t_{95\%}$ is the time at which the intensity is 95% of its maximum value. The 95% emittance can then be calculated using the relation, $\varepsilon_{95\%} = r_{95\%}^2$. This is the value output by the program. $x_o$ is also used in calculating the density distribution since $t' = (t_o - t_s)$, and $t_o$ is obtained from $x_o$. Hence, testing the program's ability to find the 95% emittance tests this critical component of the program.

The density distribution calculation becomes extremely sensitive to errors in $x_o$ near the beam's center since the value of t' in the denominator of the equation for $\rho(\varepsilon(t'))$

approaches 0. The emittance measurement is not as sensitive to this. However, accurate emittance measurements made from the spiraling current transformer signal by the method described above would indicate that there is some understanding of the way in which the beam is lost. This in turn would support the density distribution measurement since it is based on the same reasoning. The aim of this study is to see if the 95% emittance obtained by this method and then normalized by momentum is nearly constant throughout the acceleration cycle with a low intensity beam. This is the behavior one would expect from the normalized emittance.

## 2. Method

The horizontal unnormalized emittance was measured on ppm user 3 at regular intervals throughout the acceleration cycle. The normalized circulating current transformer (BXI.CIRC_XFMR_NORM on Xbar) was used with BMD.CIRCXF_GN=AAA and CIRCXF_CNTL=NEG. With a 1 MΩ termination this corresponds to $10^{11}$charges/V. The signal was acquired by the LeCroy 9414 scope at MCR4. Measurements were taken at 5ms intervals starting at 15 ms from T0, and ending at 65 ms from T0. The RF was turned off at these times using the early turn off SLD on spreadsheet (BRF.EARLY_OFF2.RT), given in microseconds from T0. At each measurement a Xbar generic gauss trigger was viewed on the scope together with the current transformer. Its value was changed until it occured within about 1 ms of the point at which the beam intensity began to drop (from the spiraling/scraping process). This was the value input into the spiral program for the magnetic field (in gauss clock counts). The scalers at MCR4 were set up to view the gauss clock counts over a millisecond interval starting when the RF was turned off. This was the value of dB/dt input into the program (in gauss clock counts per millisecond).

The values of the Dispersion and $\beta_x$ used were arbitrary (D=2.77 m, $\beta_x$=8.10 m). The beam dump bump was off during the measurements. If the scraping occurs at the same azimuthal location, and the values of D and $\beta_x$ are constant across the acceleration cycle, then the normalization of the emittance data should be unaffected, except for a scale factor. This is what was assumed.

The intensity throughout the acceleration cycle was approximately 3.5 x $10^{11}$ charges. The first measurement (at 15 ms) was after any loss associated with injection or capture. There was no extraction equipment firing on user 3. One measurement was made at each field value. Each measurement was made on a different pulse over a period of about an hour.

Since the momentum is proportional to the field, the normalized emittance was obtained from the following,

$$p[GeV/c] = \gamma m \beta c = 0.2997925\, B\rho[Tm] = (2.997925x10^{-5})\cdot(13.88m)\cdot B[g] = (4.161x10^{-4})\cdot B[g]$$

$$\beta\gamma = \frac{4.161x10^{-4}\, m}{(0.93826\, GeV/c)}\cdot B[g] = (4.435x10^{-4}\, mc/GeV)\cdot B[g]$$

where ρ is the Booster's bending radius.

## 3. Results and Conclusions

Table I gives the results of the measurements. The results show that the normalized emittance is relatively constant, increasing by 26% from 15 ms to 65 ms. Alot of this increase occurs early in the cycle (from 15 to 20 ms). In this 5 ms interval the normalized emittance increases by 11%. If there were significant emittance growth one might expect it to occur early in the cycle where the momentum is relatively low.

| Time (ms) | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Field (gauss) | 1590 | 1790 | 2080 | 2430 | 2830 | 3270 | 3700 | 4130 | 4550 | 5000 | 5430 |
| dB/dt (g/ms) | 34.9 | 49.2 | 65.0 | 75.9 | 83.0 | 85.2 | 87.0 | 88.7 | 84.5 | 85.6 | 85.6 |
| $\varepsilon_{95\%}$ | 203 | 202 | 180 | 160 | 132 | 111 | 100 | 100 | 84 | 79 | 75 |
| $\beta\gamma$ | 0.71 | 0.79 | 0.92 | 1.08 | 1.26 | 1.45 | 1.64 | 1.83 | 2.02 | 2.22 | 2.41 |
| norm. $\varepsilon_{95\%}$ | 144 | 160 | 166 | 173 | 166 | 161 | 164 | 183 | 170 | 175 | 181 |

Table I: Emittance measurements and related data. "Time" is the time from T0 that the RF was shut off; "Field" and "dB/dt" are the values put into the Spiral program; "$\varepsilon_{95\%}$"is the emittance measurement from the Spiral program, "norm. $\varepsilon_{95\%}$" is the emittance multiplied by $\beta\gamma$ (the normalized 95% emittance). The units of emittance are $\pi$ mm-mrad.

In figure 1 the emittance data is graphed versus time. IPM data from 8 March 1994 (Ahrens and Thern, Booster Studies Book IV of 1994, pg. 20) with similar intensity on user 1 shows a 1 $\sigma$ norm. emittance which varies within the range of $6\pi$-$8\pi$ mm-mrad through the cycle (see figure 2). In one set of data it increases slowly from a 'faint' minimum at about 20-25 ms of $\sim 6.5\pi$ mm-mrad to a maximum at extraction momentum of $\sim 8\pi$ mm-mrad.

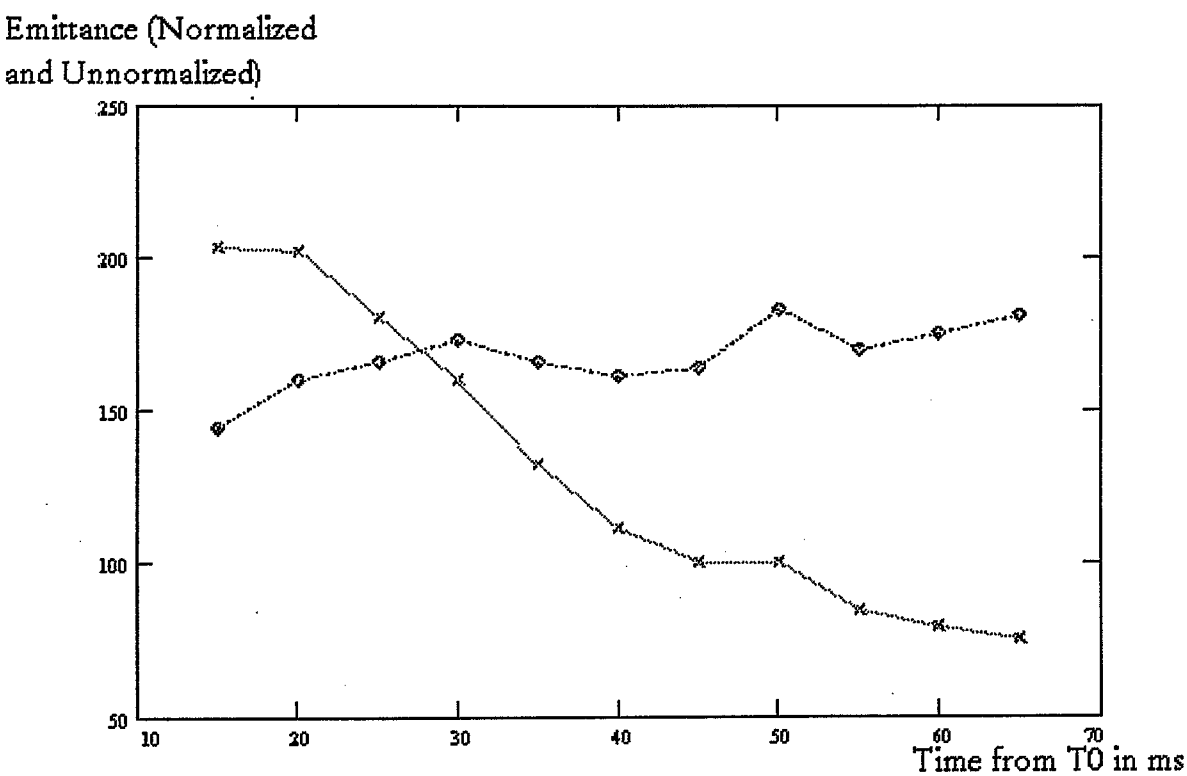


Figure 1: Normalized (diamonds) and unnormalized (x's) emittances versus time from T0. Units of emittance are $\pi$ mm-mrad.

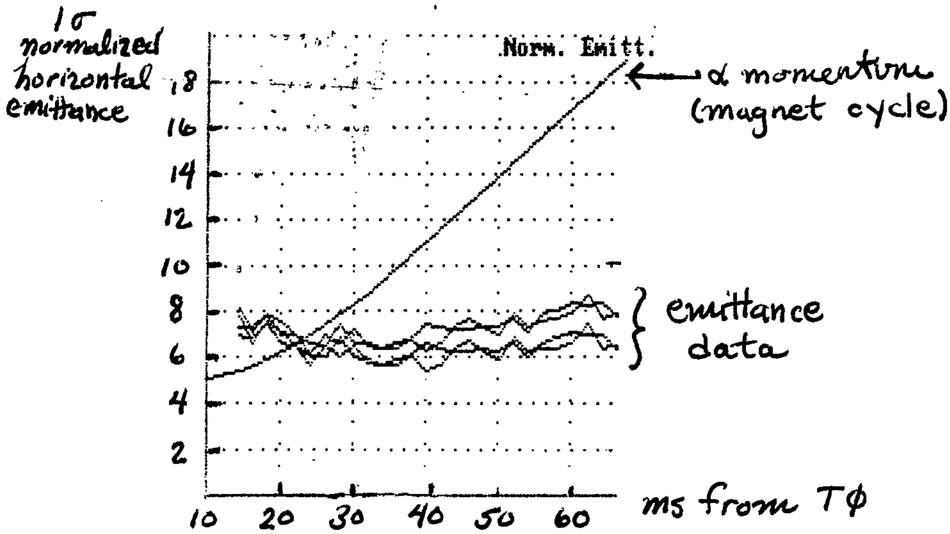


Figure 2: IPM 1 σ normalized emittance data from 8 March 1994 at 4 x $10^{11}$ charges/cycle on cycle 2 of user 1. Taken from Booster Studies Book IV, pg 20. Horizontal axis is time from T0, each grid is 10 ms. The vertical axis is 1 σ normalized emittance in π mm-mrad, ranging from 0 to 20.