

Electron beam emittance at operational intensity in fourth-generation synchrotron light sources

V. Smaluk

February 2024

Photon Sciences

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC), Basic Energy Sciences (BES)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

NSLS-II TECHNICAL NOTE BROOKHAVEN NATIONAL LABORATORY	NUMBER NSLSII-ASD-TN-403
AUTHORS: Smaluk, V. and Shaftan, T.	DATE 02/09/2024
<i>Electron beam emittance at operational intensity in fourth-generation synchrotron light sources</i>	

Electron beam emittance at operational intensity in fourth-generation synchrotron light sources

Victor Smaluk* and Timur Shaftan
Brookhaven National Laboratory, Upton, NY 11973, USA
(Dated: February 9, 2024)

For synchrotron light sources, the brightness of user X-ray beams is primarily determined by the electron beam emittance and energy spread at operational intensity. A common feature of fourth-generation synchrotrons is the short length of electron bunches combined with a very small transverse beam size. Consequently, the particle density is much higher than in machines of previous generations, leading to strong collective effects that significantly increase the emittance and limit the achievable brightness at operational beam intensity. In this article, we summarize our studies of the emittance scaled with the beam energy and intensity, taking into account the effects of intrabeam scattering, beam-impedance interaction, and bunch lengthening provided by higher-harmonic RF systems, to identify optimal combinations of machine and beam parameters.

1. INTRODUCTION

Over the past three decades, there has been a remarkable evolution in synchrotron light sources, transforming the landscape of scientific research and technological advancements. The ultimate brightness of light sources is the key to advancing to a smaller scale, faster response, and higher rate of data measurement and processing. The evolution of synchrotron light sources in past decades follows a path of continued increased brightness of photon beams. The development of third- and fourth-generation synchrotrons has enabled scientists to explore more complex and dynamic systems at unprecedented resolutions, from materials science and chemistry to biology and medicine. Looking ahead, ongoing advancements in accelerator technology and facility upgrades continue to shape the next generation of synchrotron light sources, promising even greater scientific insights and technological innovations.

Since brightness is one of the main figures of merit for synchrotron light sources, all projects of future synchrotrons consider an increase of brightness by a few orders of magnitude compared to the present standards. Brightness is a function of the electron beam energy, intensity, emittance, energy spread, and choice of light-generating insertion devices (wigglers and undulators). Reducing the emittance is a straightforward and efficient way to increase the brightness.

2. EVOLUTION OF BEAM EMITTANCE

The natural emittance of an electron beam in a storage ring is determined by a balance of the radiation damping and quantum excitation. For a planar ring without vertical bending, this relation is expressed as a ratio of

radiation integrals [1] in the following equation

$$\varepsilon_{x0} = C_q \gamma^2 \frac{I_5}{I_2 - I_4}. \quad (1)$$

Here ε_{x0} is the emittance; γ is the Lorentz factor; $C_q = \frac{55}{32\sqrt{3}} \frac{\hbar c}{E_e} \simeq 3.83 \cdot 10^{-13}$ m; the radiation integrals are

$$I_2 = \int \frac{1}{\rho^2} ds, \quad I_4 = \int \frac{\eta_x}{\rho^3} (1 + 2\rho^2 K_1) ds, \quad I_5 = \int \frac{\mathcal{H}}{|\rho|^3} ds; \quad (2)$$

$$\mathcal{H} = \beta_x \eta_x'^2 + 2\alpha_x \eta_x \eta_x' + \gamma_x \eta_x^2; \quad (3)$$

β_x is the amplitude function of betatron oscillation (beta function), $\alpha_x \equiv -\beta_x'/2$, $\gamma_x \equiv \frac{1+\alpha_x^2}{\beta_x}$; η_x and η_x' is the dispersion function and its derivative, respectively; ρ is the local bending radius; $K_1 = \frac{1}{B\rho} \frac{\partial B_y}{\partial x}$ is the normalized quadrupole strength.

The emittance can be represented in a simple way as

$$\varepsilon_{x0} = F \frac{E^2}{J_x N_B^3}, \quad (4)$$

where F is some function of the magnet lattice, E is the electron energy, J_x is the horizontal damping partition, and N_B is the number of bending (dipole) magnets in the ring.

Since the emittance is inversely proportional to the cube of the number of bending magnets, increase of their number is a most efficient way to design a low-emittance lattice. As a result, we see transition from Double-Bend and Triple-Bend Achromat lattices previously used to build 3rd-generation light sources worldwide, to Multi-Bend Achromat, which is the basic lattice option for new and upgrade project of light sources. The fast development of advanced Multi-Bend Achromat (MBA) technology resulted in a new generation of light source facilities. Recently, three new low-emittance synchrotrons based on MBA have been commissioned: MAX-IV (Sweden, 2016) [2], ESRF-EBS (France, 2020) [3], and SIRIUS (Brazil, 2020) [4]. APS-U [6] (USA) and HEPS [5]

* smaluk@bnl.gov

(China) will enter the commissioning stage soon, and many projects of new or upgraded facilities are being developed worldwide: ALS-U [7] (USA), Elettra-2 [8] (Italy), Diamond-II [9] (UK), Soleil-2 [10] (France), PETRA IV [11] (Germany), CLS-2 [12] (Canada), Korea-4GSR [13], and others. The beam emittance is continuously reduced in few decades of synchrotron development, as illustrated by Figure 1.

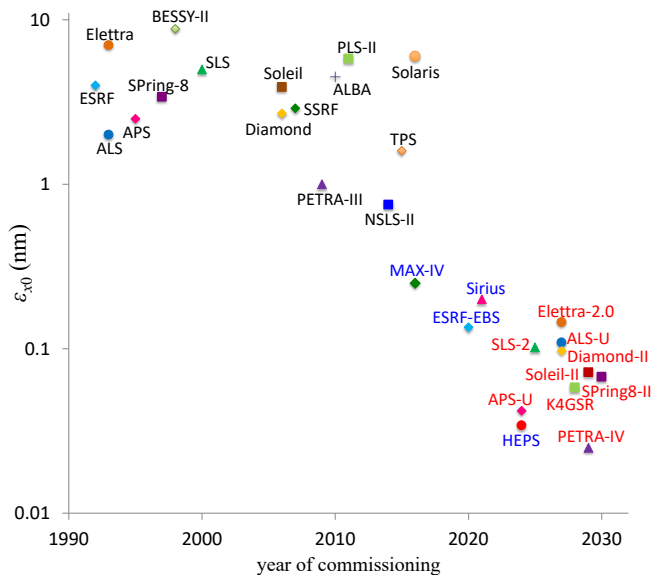


FIG. 1. Evolution of the electron beam emittance in synchrotron light sources.

A new approach of low-emittance lattice design alternative to MBA has been recently proposed at NSLS-II, Brookhaven National Laboratory: use of a new lattice element “Complex Bend” replacing regular dipole magnets [14–17]. The main advantage of the Complex Bend design is to enable many more dense dipoles integrated into the same element. Since the emittance scales inversely as the cube of the number of dipoles, this opens a possibility of achieving gains in emittance. For example, replacement of the dipole magnets with Complex Bends in the NSLS-II DBA lattice keeping the layout of matching quadrupole triplets and straight sections unchanged, results in the emittance reduction by a factor of 30 [18]. A more advanced lattice for the NSLS-II upgrade assuming the replacement of the whole ring provides the emittance of 24 pm [19].

There is a recent trend in magnet design towards combined magnets with field profiles tailored to the lattice requirements. In near future, we expect a transition to the permanent-magnet bending/focusing elements providing high quadrupole gradients, saving space, and reducing total power consumption. The use of superconducting high-field high-gradient magnets looks also promising for future projects.

3. ENERGY AND INTENSITY CONSTRAINTS

The major practical limit of the beam energy E and intensity results from the synchrotron radiation power

$$P_{\text{rad}} = I_b U_0 = I_b \frac{C_\gamma}{2\pi} E^4 I_2, \quad (5)$$

where U_0 is the energy loss per turn, $C_\gamma = 8.846 \cdot 10^{-5} \text{ m/GeV}^3$, and the 2-nd radiation integral is $I_2 = \oint \frac{ds}{\rho}$, ρ is the local radius of electron trajectory curvature.

In modern synchrotrons, the total radiation energy is dominated by the light-generating insertion devices (wigglers and undulators), whose contribution is usually a few times higher than the contribution of dipole magnets. The radiation power rapidly increases with the beam energy, as E^4 , and the beam power loss is compensated for by complex and expensive RF systems, which contribute a significant part to the total cost of the construction and operation, due to high power consumption.

Another important factor that could limit the beam intensity is the beam-induced heating of vacuum chambers, which is directly proportional to the longitudinal impedance (mainly the resistive-wall one) and to the square of the beam current. Strong focusing magnets and high-brightness insertion devices require low-aperture vacuum chambers. Since the longitudinal impedance is inversely proportional to the vacuum chamber size, small vacuum chambers and short bunch lengths lead to higher beam-induced power. Moreover, the transverse impedance is inversely proportional to the cube of the vacuum chamber size. The beam interaction with the impedance can also lead to beam instabilities, further limiting the maximum stable beam current. The interaction with the residual gas in vacuum chambers can excite the instabilities too, and this problem is also more severe for the modern synchrotrons because the vacuum chambers are smaller and the pumping is more difficult.

One more intensity-limiting effect is the shorter Touschek lifetime inversely proportional to the beam current. Lower emittance needs stronger beam focusing resulting in high natural chromaticity, and strong sextupoles are needed to compensate for it. So beam dynamics becomes very nonlinear with smaller momentum acceptance leading to the degradation of the Touschek lifetime. While the DBA and TBA lattices of 3rd-generation synchrotrons typically operate with a beam current up to 500 mA and a beam lifetime exceeding 10 hours, it is about an order of magnitude lower for most of the new MBA-based machines.

For the above-mentioned reasons, higher-energy synchrotrons operate with lower beam intensity, as illustrated in Figure 2. The dashed line represents an empirical limit curve for the beam intensity.

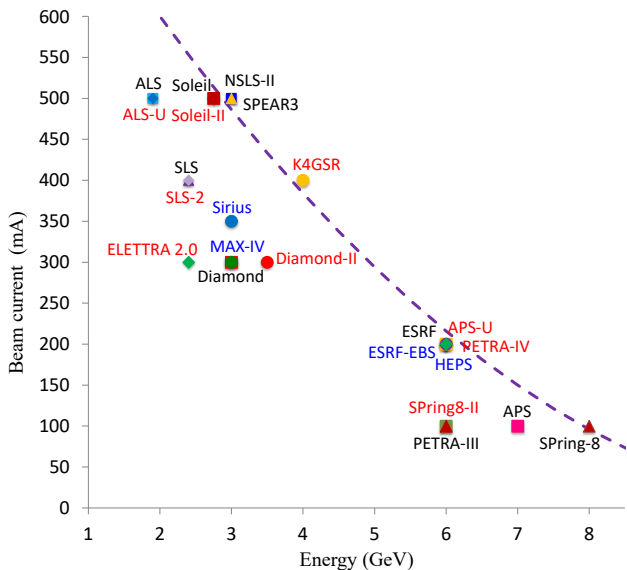


FIG. 2. Operation beam current of synchrotrons vs beam energy.

4. INTENSITY-DEPENDENT EFFECTS

Ideally, the photon flux and brightness should be directly proportional to the electron beam current. However, the collective effects of beam dynamics at operational beam intensity play a crucial role in determining the practically achievable performance of light sources. In modern low-emittance rings, electron beams are small in all three dimensions: a small momentum compaction results in a short bunch length, while a low emittance determines small transverse sizes.

Figure 3 demonstrates the bunch volume

$$V_b = \sqrt{4\pi^3} \sigma_x \sigma_y \sigma_z \quad (6)$$

as a function of the emittance for a set of synchrotrons worldwide, in operation or under development. Here σ_z is the r.m.s. bunch length, $\sigma_{x,y}$ is the horizontal/vertical beam size. Note that the bunch volume was calculated at zero beam intensity and without higher-harmonic cavities widely used to increase the bunch length for mitigation of collective effects, so the bunch length σ_z is completely determined by the lattice and RF parameters:

$$\sigma_z = \sigma_\delta \sqrt{\frac{\lambda_{\text{RF}} R_{\text{aver}} \alpha_c E}{\sqrt{e^2 V_{\text{RF}}^2 - U_0^2}}}, \quad (7)$$

where σ_δ is the relative energy spread, λ_{RF} is the RF wavelength, V_{RF} is the RF voltage, R_{aver} is the average ring radius, α_c is the momentum compaction factor.

A trend of a significant reduction of the bunch volume in modern low-emittance synchrotrons is clearly seen. As a result, the particle density within the bunch is considerably high, amplifying the collective effects.

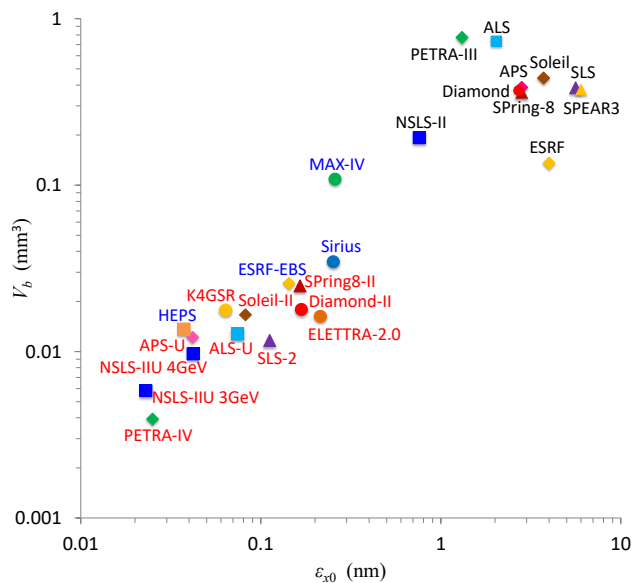


FIG. 3. Bunch volume in synchrotrons vs beam emittance.

Intra-beam scattering (IBS) is one of the adverse effects that can impact beam quality and impose limitations on the ultimate performance of low- and medium-energy synchrotrons [20, 21]. IBS is a small-angle scattering that does not cause particle loss but results in a substantial intensity-dependent increase in emittance, energy spread, and bunch length. The theory of IBS has been well-developed for quite some time [22–25] and has been implemented into particle tracking codes [26].

We employed the high-energy approximation of the IBS theory [27] to examine the combined effect of IBS and the bunch lengthening resulting from the longitudinal impedance and higher-harmonic cavities. The equilibrium emittance $\varepsilon_{x,y}$ and relative energy spread σ_δ are expressed as follows:

$$\varepsilon_{x,y} = \frac{\varepsilon_{x0,y0}}{1 - \tau_{x,y}/T_{x,y}}, \quad \sigma_\delta^2 = \frac{\sigma_{p0}^2}{1 - \tau_p/T_p}, \quad (8)$$

where $\varepsilon_{x0,y0}$ and σ_{p0} are the emittance and energy spread at zero beam current; τ_x , τ_y , and τ_p are the radiation damping times; T_x , T_y , and T_p are the IBS growth times:

$$\frac{1}{T_p} \simeq \frac{r_0^2 c N}{32 \gamma^3 \varepsilon_x \varepsilon_y \sigma_z \sigma_\delta^2} \left(\frac{\varepsilon_x \varepsilon_y}{\langle \beta_x \rangle \langle \beta_y \rangle} \right)^{1/4} \ln \frac{\langle \sigma_y \rangle \gamma^2 \varepsilon_x}{r_0 \langle \beta_x \rangle}, \quad (9)$$

$$\frac{1}{T_{x,y}} \simeq \frac{\sigma_\delta^2 \langle \mathcal{H}_{x,y} \rangle}{\varepsilon_{x,y}} \frac{1}{T_p}, \quad (10)$$

r_0 is classical electron radius, $\mathcal{H}_{x,y}$ is a function determined by the lattice (3).

Since the IBS strongly depends on the beam energy, its effect is not so significant for high-energy rings.

In a practical range of the beam energy and current, we analyzed the impact of intensity-dependent effects on

the light source performance using a hybrid 7BA lattice, based on the ESRF-EBS design. This lattice design represents a typical example adopted for future synchrotron light sources. The lattice was scaled to the size of NSLS-II and optimized to achieve minimum emittance, decent dynamic aperture, and beam lifetime [28].

We calculated the emittance as a function of the beam current and energy for this 7BA lattice, considering both IBS and impedance effects. The bunch lengthening caused by the beam interaction with the longitudinal impedance was computed using the modified Zotter equation [29, 30]:

$$\left(\frac{\sigma_t}{\sigma_{t0}}\right)^3 - \frac{\sigma_t}{\sigma_{t0}} = \frac{I_b \alpha_c}{4\sqrt{\pi} \nu_s^2 \omega_0^3 \sigma_{t0}^3 E/e} \text{Im}\left(\frac{Z_{\parallel}}{n}\right)_{\text{eff}}, \quad (11)$$

where $\nu_s = \omega_s/\omega_0$ is the synchrotron tune, σ_{t0} is the bunch length at zero intensity, $(\text{Im}Z_{\parallel}/n)_{\text{eff}}$ is the effective normalized longitudinal impedance.

The RF voltage was scaled with the energy to keep the RF acceptance of 5%. The effect of higher-harmonic cavities was simply modeled by multiplying the zero-intensity bunch length σ_{t0} by a moderate factor of 3. In this calculation, we assumed the "bare" lattice without wigglers and undulators, 100% coupling, $\varepsilon_y = \varepsilon_x = \varepsilon_{x0}/2$, and an inductive longitudinal impedance $\text{Im}(Z_{\parallel}/n)_{\text{eff}} = 0.5 \Omega$. Figure 4 shows the emittance ε_{x0} affected by these collective effects, as a function of the energy E and average beam current I_{aver} uniformly distributed in 1000 bunches.

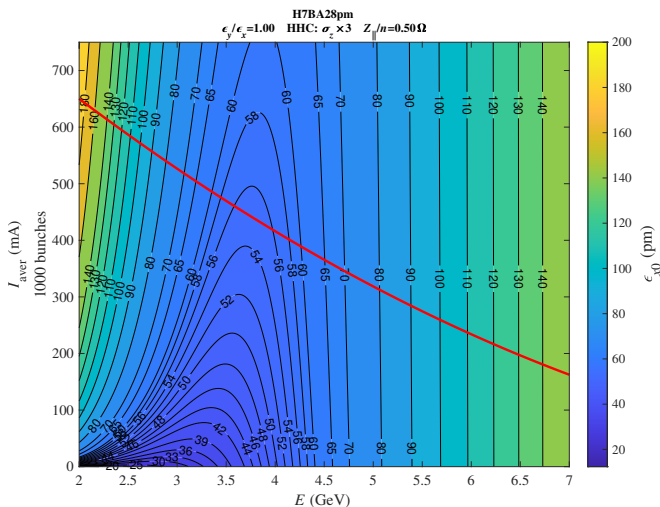


FIG. 4. Combined effect of IBS, impedance, and higher-harmonic cavities on the beam emittance.

We found that the emittance at the operational beam intensity reaches a minimum in the energy range of 3.5-4 GeV. This is due to the strong emittance blow-up caused by IBS at lower energies, while the E^2 factor in Equation (4) leads to an increase in emittance at higher energies. So there is an optimal energy with the smallest emittance at the operational beam intensity for a particular low-emittance lattice.

For example, we compared the emittances of four lattices, ESRF-EBS [3], APS-U [6], NSLS-IIU 7BA [28], and NSLS-IIU CBA [31], assuming the empirical energy-dependent limit of the operational beam current presented as the dashed line in Figure 2. The "bare" lattices without wigglers and undulators were used, and a 100% coupling was assumed. The result presented in Figure 5 shows that the energy of the minimum operational emittance depends on the lattice design.

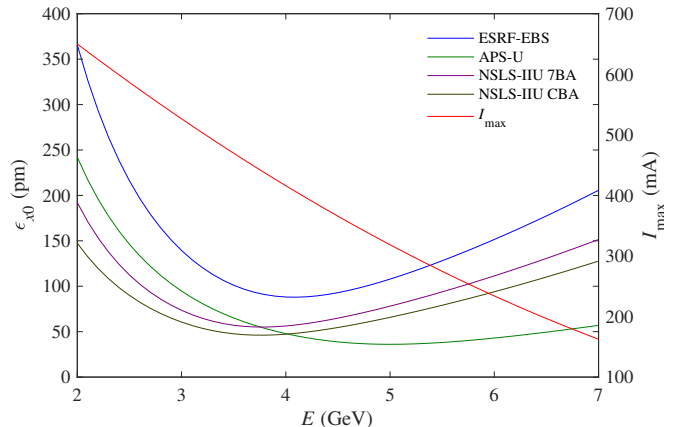


FIG. 5. Minimum emittance along the the energy-dependent operational beam current.

Note the bunch volume in the denominator of the IBS growth rate (9) is also relevant for the Touschek lifetime and plays its role in the reduction of the beam lifetime in 4th-generation synchrotrons alongside with the smaller momentum acceptance.

There are several ways to mitigate the challenges caused by collective effects. Lattice optimization can help to increase natural bunch length. To reduce the peak current of the beam, bunch lengthening is essential and can be achieved by implementing advanced schemes of higher-harmonic cavities. Increasing number of bunches is also helpful to reduce the peak current and beam-induced heating of the vacuum chamber. To reduce the impedance, larger vacuum chamber should be used where possible and smooth transitions must be implemented. Minimization of the impedance needs to be a part of the vacuum chamber design from the very beginning of a project.

5. CONCLUSION

Next-generation synchrotrons have a common feature of short electron bunches and a small transverse beam size, resulting in a significant reduction in bunch volume, higher particle density, and stronger collective effects. Since the brightness of user X-ray beams at operational intensity is predominantly determined by the electron beam emittance, we studied the intensity-dependent emittance scaled with beam energy, considering the ef-

fects of intrabeam scattering, beam-impedance interaction, and bunch lengthening by higher-harmonic RF cavities, to identify optimal combinations of machine and beam parameters. We found the emittance at operational beam intensity reaches a minimum at specific energy due to the interplay of the quadratic increase of zero-intensity emittance and IBS-induced blow-up at lower energies. This optimal energy point with the smallest operational

emittance is determined by a specific lattice design.

ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy under Contract No. DE-SC0012704 and by Brookhaven National Laboratory Directed Research and Development Program, Project No. 20-041.

-
- [1] R.H. Helm, M.J. Lee, P.L. Morton, M. Sands, “Evaluation of Synchrotron Radiation Integrals”, in Proc. of PAC-1973, San Francisco, pp. 900-901.
- [2] P. Tavares, E. Al-Dmour, A. Andersson, F. Cullinan, B. Jensen, D. Olsson, D.K. Olsson, M. Sjöström, H. Tarawneh, S. Thorina, A. Vorozhtsov, “Commissioning and first-year operational results of the MAX IV 3 GeV ring”, *J. Synchrotron Rad.* 25 (2018) 1291–1316.
- [3] P. Raimondi, et al., “The Extremely Brilliant Source storage ring of the European Synchrotron Radiation Facility”, *Commun. Phys.* 6, 82 (2023).
- [4] L. Lin, “Sirius Accelerators Overview”, in Proc. of EIC Workshop, 2020.
- [5] Y. Jiao, et al., “The HEPS project”, *J. Synchrotron Rad.* 25 (2018) 1611–1618.
- [6] M. Borland, et al., “The Upgrade of the Advanced Photon Source”, in Proc. of IPAC-2018, Vancouver, THXGBD1.
- [7] C. Steier, et al., “Status of the Conceptual Design of ALS-U”, in Proc. of IPAC-2017, Copenhagen, WEPAB104.
- [8] E. Karantzoulis, “Elettra 2.0 - The diffraction limited successor of Elettra”, *Nuclear Inst. and Methods in Physics Research, A* 880 (2018) 158-165.
- [9] Diamond-II: Conceptual Design Report, 2019.
- [10] A. Loulergue, “Baseline Lattice for the Upgrade of SOLEIL”, in Proc. of FLS-2018, Shanghai, MOP2WB03.
- [11] C.G. Schroer, H.C. Wille, O.H. Seeck, et al., “The synchrotron radiation source PETRA III and its future ultra-low-emittance upgrade PETRA IV”, *Eur. Phys. J. Plus* 137, 1312 (2022).
- [12] L.O. Dallin, “Design Considerations for an Ultralow Emittance Storage Ring for the Canadian Light Source”, in Proc. of IPAC-2018, Vancouver, TUPMF038.
- [13] S. Shin, “Lattice Design and Beam Dynamics Studies for Fourth-Generation Storage Ring”, 23rd International Conference on Accelerators and Beam Utilization, Daejeon, 2019.
- [14] T. Shaftan, V. Smaluk, G. Wang, “A Concept of the Complex Bend”, BNL-211211-2019-TECH, Upton, 2018.
- [15] G. Wang, T. Shaftan, V. Smaluk, N. Mezentssev, S. Sharma, O. Chubar, Y. Hidaka, C. Spataro, “Complex bend: Strong-focusing magnet for low-emittance synchrotrons”, *Phys. Rev. Accel. Beams* 21 (2018) 100703.
- [16] T. Shaftan, G. Wang, V. Smaluk, Y. Hidaka, O. Chubar, T. Tanabe, J. Choi, “Complex Bend II”, BNL-211223-2019-TECH, Upton, 2018.
- [17] G. Wang, T. Shaftan, V. Smaluk, Y. Hidaka, O. Chubar, T. Tanabe, J. Choi, S. Sharma, C. Spataro, N. Mesentsev, “Complex bend. II. A new optics solution”, *Phys. Rev. Accel. Beams* 22 (2019) 110703.
- [18] V. Smaluk, T. Shaftan, “Realizing low-emittance lattice solutions with Complex Bends”, *J. Phys.: Conf. Ser.* 1350 (2019) 012044.
- [19] F. Plassard, G. Wang, T. Shaftan, V. Smaluk, Y. Li, Y. Hidaka, “Simultaneous correction of high order geometrical driving terms with octupoles in synchrotron light sources”, *Phys. Rev. Acc. Beams* 24 (2021) 114801.
- [20] C. Steier, J. Byrd, H. Nishimura, D. Robin, S. De Santis, F. Sannibale, C. Sun, M. Venturini, W. Wan, “Physics Design Progress Towards a Diffraction Limited Upgrade of the ALS”, in Proc. of IPAC-2016, Busan, WEPOW049.
- [21] A. Blednykh, B. Bacha, G. Bassi, V. Smaluk, T. Shaftan, M. Borland, R. Lindberg, “Combined Effect of IBS and Impedance on the Longitudinal Beam Dynamics”, in Proc. of IPAC-2021, Campinas, THPAB240.
- [22] A. Piwinski, “Intra-beam Scattering”, in Proc. of 9th Int. Conf. on High Energy Accelerators, Stanford, 1974, p. 405.
- [23] J.D. Bjorken, S.K. Mtingwa, “Intrabeam Scattering”, *Particle Accelerators Vol. 13* (1983) pp. 115-143.
- [24] K. Bane, “A Simplified Model of Intrabeam Scattering”, in Proc. of EPAC-2002, Paris, WEPRI120.
- [25] K. Kubo, S.K. Mtingwa, A. Wolski, “Intrabeam scattering formulas for high energy beams”, *Phys. Rev. ST Accel. Beams* 8 (2005) 081001.
- [26] M. Borland, “elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation”, Advanced Photon Source LS-287, September 2000.
- [27] K. Bane, H. Hayano, K. Kubo, T. Naito, T. Okugi, J. Urakawa, “Intrabeam Scattering Analysis of ATF Beam Measurements”, SL AC-PUB-8875 (2001).
- [28] Y. Li, K. Hwang, C. Mitchell, R. Rainer, R. Ryne, V. Smaluk, “Design of double- and multi-bend achromat lattices with large dynamic aperture and approximate invariants”, *Phys. Rev. Accel. Beams* 24 (2021) 124001.
- [29] B. Zotter, “Potential-well bunch lengthening”, CERN SPS/81-14 (DI). Geneva, Switzerland, 1981.
- [30] D. Zhou, G. Mitsuka, T. Ishibashi, K. Bane, “Potential-well Bunch Lengthening in Electron Storage Rings”, arXiv:2309.00808.
- [31] M. Song, T. Shaftan, “Design study of a low emittance complex bend achromat lattice”, arXiv:2310.20010v2 (2023).