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# STABLE SPIN DIRECTION MEASUREMENTS AT RHIC WITH POLARIZED PROTON BEAMS 

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#### Abstract

We describe methods for measuring the three-dimensional stable spin vector at the STAR detector (using STAR local polarimetry) and at the proton-Carbon coulomb nuclear interaction polarimeter at the 12 o'clock RHIC interaction region (hereafter the pC polarimeter). These polarimeters can only provide information about the two transverse components of the stable spin direction. If a known, local spin rotation can be generated at the location of the polarimeter, then the longitudinal component can be calculated by comparing the transverse components before and after the rotation. At STAR the stable spin direction can be rotated using the helical dipole spin rotators. At the pC polarimeter, a local horizontal orbital angle is introduced to rotate the stable spin direction. The stable spin direction at the hydrogen jet polarimeter is determined by transporting the spin vector at the pC polarimeter target to the location of the jet using a Zgoubi model. We describe the measurement and analysis methods used and present results of measurements made during RHIC Run 22.


## INTRODUCTION

For a proton launched in a synchrotron ring, we may define a stable spin direction. If the proton is launched with its spin aligned (or anti-aligned) with this direction, the proton's spin direction will repeat turn-to-turn. Protons launched with other orientations will precess about the stable spin direction turn after turn [1]. The stable spin direction is a function of the 6 D phase space coordinates, so full experimental characterization is in general difficult. When we refer here to "the" stable spin direction of the beam, we mean the average over the phase space of the entire beam, sampled for many turns (of order minutes).

Polarization is preserved during acceleration in RHIC via use of a pair of helical dipole snakes in each ring [2]. These magnets, located azimuthally opposite each other in the ring, each rotate the spin vector a full $180^{\circ}$ about an axis in the horizontal plane (from up to down or vice versa) in a single pass through the magnet. This fixes the spin tune at $1 / 2$ and the stable spin direction along the vertical axis, independent of the proton's energy. This allows avoidance of both imperfection and intrinsic depolarizing resonances [1]. However, in the presence of imperfection resonances (due to quadrupole misalignment or residual orbit error, for example), the stable spin direction can be moved away from the vertical even with the snakes. This "spin tilt" from machine errors has been studied at RHIC extensively [3,4]. These deviations from an ideal lattice make it necessary to measure the stable spin direction in order to properly cali-
brate polarization measurements and determine their impact on the physics process at the point of collision.

Additionally, in Run 22, two coils in one of the snakes in the Blue ring failed and in order to continue operation, the remaining two modules in that snake were reconfigured as a partial snake (one that rotates the spin vector by an angle less than $180^{\circ}$ ). This configuration breaks the symmetry of the two-full-snake configuration which results in an additional contribution to the spin tilt, since the partial snake no longer guarantees energy independent vertical stable spin direction. The Yellow ring was unaffected and still had two nominally full snakes.

It should be noted also that the store energy for Run 22 was lowered for both beams from Lorentz factor $\gamma=271.635$ to $\gamma=270.938$ relative to previous RHIC runs. This change was to rotate the non-vertical stable spin direction as much as possible into the transverse plane at STAR and the pC polarimeters. All measurements described here were made during Run 22 at $\gamma=270.938$. Details of this partial snake setup during Run 22 can be found in Refs. [5, 6]. These deviations (snake configuration and energy change) should be born in mind if comparisons of these measurements are made to other runs.

## SPIN DIRECTION AT STAR

The STAR local polarimetry reports information about the transverse components of the stable spin direction as a transverse asymmetry $\left(A_{n}\right)$ and an angle, $\theta$ of that asymmetry axis relative to the vertical axis. The stable spin direction lies along this assymmetry axis. The coordinate system is shown in Fig.1. The coordinate axes $\vec{x}, \vec{y}, \vec{z}$ form a right-handed coordinate system and point radially outward, vertically upward and longitudinally in the Blue beam direction respectively. The same coordinate system is used for both Blue and Yellow beams, which means the Yellow beam travels in the negative $\vec{z}$ direction. We call the stable spin direction vector at the interaction point $\vec{S}$ which is normalized to have magnitude one. The vector $\vec{\rho}$ is the projection of $\vec{S}$ onto the transverse $(\vec{x}-\vec{y})$ plane with an amplitude $\rho$ and an angle $\theta$ with respect to the vertical axis. This is the same as the $\theta$ reported by the polarimetry. A rotation angle is positive if it is counterclockwise when viewed with the axis of rotation pointed toward the observer.


Figure 1: Coordinate system for measurements at the STAR interaction point.

The components of $\vec{S}$ and $\vec{\rho}$ and the measured asymmetry, $A_{N}$, are therefore related by

$$
\begin{align*}
S_{x} & =\rho \sin \theta \\
S_{y} & =\rho \cos \theta \\
\rho & =\sqrt{S_{x}^{2}+S_{y}^{2}}=\sqrt{1-S_{z}^{2}}  \tag{1}\\
\rho & =\frac{A_{N}}{A_{\max }}
\end{align*}
$$

The total asymmetry $A_{\text {max }}$ is not directly measureable. It is the asymmetry that would be measured by transverse scattering if the stable spin direction were entirely in the transverse plane.

Inferring $A_{\text {max }}$, or equivalently $S_{z}$, requires measuring the purely transverse quantities $A_{n}$ and $\theta$ in at least two different orientations. A large local rotation of the stable spin direction at the STAR interaction point is possible with the helical dipole spin rotators on either side of the interaction region (IR). A schematic of the STAR IR with its spin rotators is shown in Fig.2. A design particle exiting the RHIC arc will see spin rotations first from the rotator itself, which rotates the spin about an axis in the horizontal plane and then a rotation about the vertical due to the DX and D0 dipole magnets. The rotators and dipoles on the outgoing side of the IR serve to return the spin direction to vertical before entry into the outgoing arc [7].

If we call the spin vector at the upstream face of the incoming rotator $S_{a r c}$, then the initial state of the spin direction at the IP with rotators off, $S_{i}$, and the final state with rotators on, $S_{f}$, are given respectively by

$$
\begin{align*}
S_{i} & =R_{d} \cdot S_{a r c}  \tag{2}\\
S_{f} & =R_{d} R_{r} \cdot S_{a r c}
\end{align*}
$$

where $R_{d}$ and $R_{r}$ are the rotation matrices representing the rotations from the DX/D0 system and the spin rotator. Subscripts $i$ and $f$ denote quantities at the IP with the rotators before and after powering the rotators, respectively. Under the design assumption that the outgoing rotator cancels the effect of the incoming rotator $S_{a r c}$ must be the same whether


Figure 2: Schematic view of the STAR interaction region.
the rotators are on or off. This cancellation is verified by measuring the polarization with the pC polarimeters which are outside the STAR IR and observing no change in the stable spin direction at that location. In that case we can directly relate the spin directions as measured at the IP in the two cases with

$$
\begin{align*}
& S_{f}=R_{d} R_{r} R_{d}^{-1} \cdot S_{i}  \tag{3}\\
& S_{f}=R \cdot S_{i}
\end{align*}
$$

where we have defined the transport matrix $R=R_{d} R_{r} R_{d}^{-1}$. Expressing Eq. 3 explicitly in terms of the quantities defined in Eq. 1 we have

$$
\begin{align*}
-\rho \sin \theta_{f} & =-R_{11} \rho_{i} \sin \theta_{i}+R_{12} \rho_{i} \cos \theta_{i}+R_{13} S_{z i} \\
\rho \cos \theta_{f} & =-R_{21} \rho_{i} \sin \theta_{i}+R_{22} \rho_{i} \cos \theta_{i}+R_{23} S_{z i}  \tag{4}\\
S_{z f} & =-R_{31} \rho_{i} \sin \theta_{i}+R_{32} \rho_{i} \cos \theta_{i}+R_{33} S_{z i}
\end{align*}
$$

The first two lines of Eq. 4 together with the condition $S_{z i}=$ $\sqrt{1-\rho_{i}^{2}}$ can be used to solve for the unknown quantities in terms of the known and measured quantities.

Defining a few convenient constants that depend only on the known matrix elements $R_{i j}$ and the measured angles $\theta_{i}, \theta_{f}$ we have

$$
\begin{align*}
A_{1} & =\frac{R_{11} \sin \theta_{i}-R_{12} \cos \theta_{i}}{\sin \theta_{f}} \\
A_{2} & =-\frac{R_{13}}{\sin \theta_{f}} \\
A_{3} & =\frac{-R_{21} \sin \theta_{i}+R_{22} \cos \theta_{i}}{\cos \theta_{f}}  \tag{5}\\
A_{4} & =\frac{R_{23}}{\cos \theta_{f}} \\
B & =\frac{A_{3}-A_{1}}{A_{2}-A_{4}}
\end{align*}
$$

we then have

$$
\begin{align*}
\rho_{i} & =\sqrt{\frac{1}{1+B^{2}}} \\
S_{z i} & =\sqrt{\frac{B^{2}}{1+B^{2}}}  \tag{6}\\
S_{x i} & =\rho_{i} \sin \theta_{i} \\
S_{y i} & =\rho_{i} \cos \theta_{i} \\
A_{\max } & =\frac{A_{n}}{\rho_{o}}
\end{align*}
$$

which gives the full 3D spin vector in terms of measured quantities and known machine parameters.

## Run 22 Measurement Results

Since the STAR request for Run 22 physics operation consisted entirely of transverse (vertical) spin orientation, the spin rotators were not powered for typical physics fills. They were used only for dedicated measurements of the stable spin direction. During these dedicated fills (fill 33001 for the Blue measurement and fill 33025 for the Yellow measurement), the rotators were powered such that they would rotate the nominally vertical stable spin direction in the arc to the longitudinal direction at the STAR interaction point. The rotator outer coils were powered to 272 A , and the inner coil currents to 247 A (in both rings, on both sides of the IR). Spin direction measurements made with the rotators powered establish the value of $\phi_{f}$. In order to reduce statistical uncertainty of the final result, the value of $\phi_{o}$ used in the calculation of the stable spin direction is the average of measurements made during the five RHIC physics fills before and five fills after the dedicated rotator fill. The results of the measurements are reported in Table 1.

The values for the transport matrix $R_{i j}$ are produced by a Zgoubi model of RHIC. Information about Zgoubi and the particular model used to generate these matrix coefficients is given in Refs. [8,9].

The Blue stable spin direction for normal physics fills has a non-vertical component that is forward (relative to the Blue beam direction) and radially inward (North). The Yellow stable spin direction has a non-vertical component that is almost entirely radially outward (South), with a much smaller longitudinal component in the backward direction (relative to the Yellow beam direction).

## SPIN DIRECTION MEASUREMENTS AT THE PROTON-CARBON POLARIMETERS

The pC polarimeters are located between the Q 3 and Q 4 on the 1 o'clock side of RHIC IR 12. Like the STAR local polarimetry, the pC polarimeter can only measure the two components of the spin direction transverse to the beam direction [10]. Since this IR does not have spin rotators, the rotation of the longitudinal component of the stable spin direction into the transverse plane is accomplished instead by the introduction of a local horizontal orbit angle at the target


Figure 3: Coordinate system for measurements at the p-C and hydrogen jet polarimeters.
location. For a given orbital angle change $\Delta \theta$, the stable spin precesses by $G \gamma \Delta \theta$, where $G$ is the anomalous magnetic ratio and $\gamma$ the Lorentz factor. The machine aperture allowed a maximum angular range of $\pm 350 \mu \mathrm{rad}$, which produced $\pm 9^{\circ}$ of spin precession (this was also very close to the power supply limits of some of the local dipole corrector magnets).

The coordinate systems at the pC polarimeters are shown in Fig. 3. Note that unlike in the common pipe at the STAR detector, the coordinate systems are different for the Blue and Yellow rings. In both case the ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) triad is (radial, vertical,longitudinal beam direction), but in the Blue ring the positive $x$ direction is radially outward, where in the Yellow ring it is radially inward. This also means that a small positive rotation angle about the longitudinal axis is inward in the Blue ring and outward in the Yellow ring. A positive horizontal orbit angle is one in which the position at Q4 is radially outward relative to the position at Q3 (this is true in both the Blue and Yellow coordinate systems).
For a simple precession about the vertical axis, the radial component of the stable spin direction $S_{x}$ has the form

$$
\begin{align*}
S_{x} & =A \sin \left(\Delta \phi+\phi_{o}\right) \\
A & =\sin \psi \tag{7}
\end{align*}
$$

The angles $\psi$ and $\phi$ are spherical coordinates denoting the angles with respect to the vertical and longitudinal axes, respectively, $\Delta \phi$ is the precession angle about the vertical caused by the horizontal orbit angle change, and $\psi$ and $\phi_{o}$ are the unknown orientation angles of the spin vector without the additional precession and $S_{x}$ is the radial component of the spin direction measured by the polarimeter. Equation 7 can be fit to measurements of $S_{x}$ as a function of $\Delta \phi$ to infer values for $\psi$ and $\phi_{o}$ which together fully determine the spin orientation.

The measurement results from RHIC Run 22 are shown together with the sinusoidal fits in Fig 4. These measurements were taken during three separate RHIC fills - 33242, 33276 , and 33307 . The results of the fit are summarized in Table 2.

In both rings the deviation away from the vertical is directed mainly in the longitudinal direction (Blue tipped

Table 1: Stable spin direction measurements at the STAR interaction point.

| Ring | $\phi_{o}(\mathrm{deg})$ | $\phi_{f}(\mathrm{deg})$ | $A_{o}$ | $A_{f}$ | $A_{\max }$ | $S_{x}$ | $S_{y}$ | $S_{z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Blue | $-2.275 \pm 0.87$ | $-178.16 \pm 1.49$ | $3.588 \pm 0.046$ | $0.88 \pm 0.029$ | $4.370 \pm 0.053$ | $-0.040 \pm 0.015$ | $0.998 \pm 0.002$ | $0.042 \pm 0.031$ |
| Yellow | $8.692 \pm 0.692$ | $111.596 \pm 2.326$ | $3.431 \pm 0.05$ | $0.653 \pm 0.023$ | $3.431 \pm 0.045$ | $0.151 \pm 0.011$ | $0.989 \pm 0.002$ | $0.003 \pm 0.006$ |

Table 2: Stable spin direction measurements at the p-C polarimeters in Run 22.

| Ring | $\psi(\mathrm{deg})$ | $\phi(\mathrm{deg})$ | $S_{x}$ | $S_{y}$ | $S_{z}$ | $\chi^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Blue | $7.82 \pm 3.45$ | $177.12 \pm 3.54$ | $0.007 \pm 0.008$ | $0.991 \pm 0.008$ | $-0.136 \pm 0.060$ | 1.79 |
| Yellow | $7.40 \pm 2.98$ | $-2.56 \pm 3.64$ | $-0.006 \pm 0.007$ | $0.992 \pm 0.007$ | $0.129 \pm 0.052$ | 1.14 |



Figure 4: p-Carbon polarimeter measurements of the radial component of the stable spin direction in the Blue (left) and Yellow (right) rings as a function of the spin precession angle about the vertical induced by an applied horizontal orbital angle. Each point represents the average of several polarization measurements. The dashed line indicates the sinusoidal fit.

Table 3: Stable spin direction measurements at the H-Jet polarimeter in Run 22.

| Ring | $S_{x}$ | $S_{y}$ | $S_{z}$ |
| :---: | :---: | :---: | :---: |
| Blue | $-0.130 \pm 0.06$ | $0.991 \pm 0.008$ | $0.029 \pm 0.012$ |
| Yellow | $0.015 \pm 0.012$ | $0.992 \pm 0.007$ | $0.125 \pm 0.051$ |

slightly forward, Yellow backward, relative to their respective beam directions).

## Transport of spin vector to hydrogen jet polarime-

 terThe polarized hydrogen jet polarimeter (H-Jet) is located at the interaction point in the center of the common pipe at 12 o'clock in the RHIC ring. The H-Jet is the only absolute polarimeter in the RHIC rings and is used to calibrate the relative polarization measurements made with the pC polarimeters [11]. Since the H-Jet detectors are sensitive only to the vertical component of the stable spin direction, it is important for the overall calibration of the polarization measurements to determine how much of the stable spin vector lies along the vertical. The event rate from the hydrogen jet is much lower than that of the pC polarimeters. A measurement with statistical precision of 2-3\% requires about 8 hours, which makes extensive orientation scans impractical. Instead, the spin orientation measured at the pC polarimeter is transported to the location of the H-Jet using a transport matrix generated hy zgoubi. The Zgoubi matrix includes the effects of the horizontal dipoles between the
pC and H -Jet locations as well as the vertical bumps used to separate the Blue and Yellow beams in the common section at the H-Jet [9]. Here the coordinate system used at the H-Jet for the Blue and Yellow beams are the same as those used at the pC polarimeters.

## CONCLUSION

Knowing the orientation of the stable spin direction is important for understanding the performance of a polarized beam collider. The stable spin direction affects both the physics processes at the point of collision and the calibration of the polarization measurements, especially when not all components are directly measurable by the available polarimetry. Methods have been developed and implemented to measure the full three-dimensional stable spin direction with polarimeters which can only measure the two transverse components by making use of localized spin rotations. The extra spin rotations were achieved using either spin rotators or a local horizontal orbit bump. These methods require propagating measured spin vector components from point to point in the accelerator, which is realized here by using the ray-tracing code Zgoubi. Methods for measuring the stable spin direction at both the STAR IP and the pC polarimeters were derived and results of such measurements from RHIC Run 22 were presented. The stable spin direction at the hydrogen jet was inferred from the pC measurement by propagating the spin vector using a Zgoubi accelerator model from the pC location to the H -Jet location.

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