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Orbit Harmonics vs Tune in the Booster

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Subject Orbit Harmonics vs Tune in the Booster

Introduction

One way to test the orbit acquisition system for offset errors (absolute position) is to obtain orbit harmonics for various tunes. A measured orbit includes not only the real orbit but also distortions caused by gain and offset errors in the PUEs. Gains can be checked rather easily by comparing measured and expected difference orbits. The contribution to the 4th and 5th harmonics from offsets can be found by measuring the 4th and 5th harmonics in the orbit versus tune. An additional result from this study is an experimental measure of the quality of the harmonic analysis being used by the standard orbit correction routine.

Data

Orbits were acquired and saved for tunes over the ranges for ν_h of 4.16 to 5.31 and for ν_v of 4.07 to 5.38. The orbit dipole correctors were set to 0, there were no stopband corrections in place, and the horizontal and vertical chromaticities were set near zero. Data was taken at 40 msec t_0 on Booster User 1. Prior to collecting data it was determined that there was one bad PUE in each plane; B6 in the horizontal and B1 in the vertical. The beam momentum was basically the injection momentum, since 40 msec is near injection on the lower B-dot. The orbit files were saved in the program_data/booster_orbit.dir/data.dir area with names Mar0793_H#.##_V#.## (e.g.; Mar0793_H4.76_V4.71).

Analysis

There are four references which cover this subject rather completely (ref. 1-4). Reference 1 (Courant and Snyder) details the theory of closed orbit harmonics caused by dipole errors. Reference 2 (R. Thern, AGS Studies Report) covers this analysis as was done for the AGS orbit system. Reference 2 and 3 are Booster Technical Reports which discuss the Booster closed orbit.

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For closed orbit harmonics caused by dipole errors, one takes the equation of motion as,

$$\frac{d^2\eta}{d\varphi^2} + \nu^2 \eta = \sum_k f_k e^{ik\varphi} \tag{1}$$

where,

$$\eta = \beta^{-\frac{1}{2}} y \qquad \varphi = \int \frac{ds}{\nu\beta} \tag{2}$$

$$f_k = \frac{1}{2\pi\nu} \int_0^C \beta^{\frac{1}{2}} F(s)\, e^{-ik\varphi}\, d\varphi \tag{3}$$

The solution is,

$$\eta = \sum_k \frac{\nu^2}{\nu^2 - k^2} f_k e^{ik\varphi} \tag{4}$$

The orbit program produces harmonics such that,

$$A_k + iB_k = (A_k^0 + iB_k^0) + \frac{\nu^2}{\nu^2 - k^2}(a_k + ib_k) \tag{5}$$

where we have expressed

$$\eta = \sum_k \frac{\nu^2}{\nu^2 - k^2}(a_k Cosk\varphi + ib_k Sink\varphi) \tag{6}$$

The a_k and b_k coefficients are the integrated dipole errors in the lattice; e.g.,

$$a_k = \frac{1}{2\pi\nu} \int_0^C \beta^{\frac{1}{2}} \frac{\Delta B}{B\rho}\, Cosk\varphi\; ds \tag{7}$$

The A_k^0 and B_k^0 coefficients are the PUE offset errors.
The error of a measurement of A_k and B_k is given by,

$$\sigma^2 = \frac{1}{N-2} \sum_{data} \left(A_k^{meas.} - A_k^0 - a_k \frac{\nu^2}{\nu^2 - k^2}\right)^2 \tag{8}$$

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Analysis of Orbit Harmonics

For any given orbit the problem now is how to extract the harmonic content given, ideally, 24 equally spaced samples (PUE's) (in fact there are 3 PUE's missing in the horizontal and one missing in the vertical). Of particular interest for the Booster are the 4th and 5th harmonics. In this report three different algorithms were used to analyze the orbit harmonics. Two of these algorithms are provided by the Booster_Orbit program, the third was done by analyzing selected orbits in Mathematica, which is available on the IBM6000 ax61.

The orbit program provides an FFT analysis and a quadratic fitting to determine orbit harmonics. On Mathematica a linear fitting was used. The FFT routine makes use of the Danielson-Lanczos method of breaking up the Fourier sum into a series of even and odd terms such that the largest sum is of length N/2, then N/4, N/8, etc. This method requires that the number of samples be a power of 2 (an integer power of 2), and that the samples be evenly distributed. The quadratic fitting is basically the same as Muller's method of finding roots. This uses quadratic interpolation around three points at a time to fit the orbit data and determine the harmonics. The linear fitting used is a generalized least squares fitting of the function,

$$x = a_0 + a_3 Cos3\varphi + b_3 Sin3\varphi + a_4 Cos4\varphi + b_4 Sin4\varphi + a_5 Cos5\varphi + b_5 Sin5\varphi + a_6 Cos6\varphi + b_6 Sin6\varphi \quad (9)$$

A best fit is determined by minimizing the Chi-Square to the data.

Of the three algorithms it appears the linear fit gives the best results. The FFT algorithm has some fundamental problems which cause it to not give good results. In particular the algorithm requires the number of points to be a power of 2. If all the Booster PUE's were in place there would be 24 measurements. Since bad PUE's and nonexistent PUE's can lower this number it is easy to see the criterion will not always be satisfied (it is possible to fix this by 'jamming' zero's, but this is not done). Also, the FFT requires the samples to be evenly spaced. This requirement is never satisfied. The result is that the FFT algorithm forces the samples to be even spaced, effectively shortening the Booster. The quadratic fit agrees well with the linear fit, although it appears to also not always give good results. The only problem with the quadratic fit is it must allow for complex denominators, and will become divergent for some solutions. For all the fittings there will always be observed the phenomena of 'bleedthrough', in which the strength of other harmonics is given to be larger when the tune approaches an integer (as the tune approaches 5, the calculated 4th harmonic also grows even though the 4th actually in the orbit is decreasing).

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Results

Table 1 summarizes the results of Figures 1 - 4. Figures 1 and 2 show the 4th Harmonic tune dependence for horizontal plane orbits with straight line fits. Figures 2 and 3 show the 5th Harmonic tune dependence for horizontal plane orbits with straight line fits.

Table 1: Results of Linear fits from Figures 1-4

FFT Fittings

Harm.	$A_k^0 \pm \sigma$ (mm)	$B_k^0 \pm \sigma$ (mm)	$a_k \pm \sigma$	$b_k \pm \sigma$
4th	−0.17±0.34	0.90±0.28	−0.65±0.04	0.17±0.03
5th	−1.50±0.26	−1.65±0.16	0.70±0.04	0.87±0.03

Quadratic Fittings

Harm.	$A_k^0 \pm \sigma$ (mm)	$B_k^0 \pm \sigma$ (mm)	$a_k \pm \sigma$	$b_k \pm \sigma$
4th	0.21±0.17	−0.18±0.08	−0.67±0.02	0.20±0.01
5th	0.12±0.04	−1.33±0.06	0.71±0.01	0.83±0.01

Linear Fittings

Harm.	$A_k^0 \pm \sigma$ (mm)	$B_k^0 \pm \sigma$ (mm)	$a_k \pm \sigma$	$b_k \pm \sigma$
4th	0.85±0.15	0.05±0.03	−0.75±0.02	0.01±0.003
5th	−0.02±0.09	−1.35±0.07	0.41±0.01	1.14±0.01

Table 2 summarizes the results of Figures 5 - 8. Figures 5 and 6 show the 4th Harmonic tune dependence for vertical plane orbits with straight line fits. Figures 7 and 8 show the 5th Harmonic tune dependence for vertical plane orbits with straight line fits. Linear orbit fittings were not done for the vertical data, which was much more 'well behaved'.

Table 2: Results of Linear fits from Figures 5-8

FFT Fittings

Harm.	$A_k^0 \pm \sigma$ (mm)	$B_k^0 \pm \sigma$ (mm)	$a_k \pm \sigma$	$b_k \pm \sigma$
4th	−0.32±0.04	0.22±0.08	0.24±0.001	−0.18±0.003
5th	−0.08±0.02	−0.58±0.02	0.29±0.001	0.39±0.001

Quadratic Fittings

Harm.	$A_k^0 \pm \sigma$ (mm)	$B_k^0 \pm \sigma$ (mm)	$a_k \pm \sigma$	$b_k \pm \sigma$
4th	−0.27±0.03	−0.14±0.05	0.21±0.001	−0.22±0.002
5th	−0.14±0.03	−0.26±0.03	0.34±0.002	0.31±0.003

Conclusions

The 4th and 5th harmonic content in the offsets errors of the Booster PUEs are generally very small, the largest being slightly over 1 mm in the horizontal plane and a few tenths of a mm in the vertical plane. From this we conclude that the offsets are small. The data is surprisingly good, enough so that the differences in different kinds of fittings become significant. It appears that a simple linear fitting gives the best results, in that the deviation from linearity caused by 'bleedthrough' is smallest. The fact that the vertical data is so much better than the horizontal is probably due to the fact that the vertical orbit is not sensitive to changes in momentum. An improvement in this experiment can be made by keeping the RF frequency at the time of the measurements fixed, so that changes in the momentum versus tune due to errors in the radial loop PUEs become minimized. This might also allow a more well behaved 6th harmonic, which reflects the Booster dispersion function.

The quality of the harmonic analysis is of concern since the result of this analysis is used by the orbit correction program (using low field dipoles to minimize the harmonics in the orbit). Since the effect of the 'bleedthrough' can impose an error to the true harmonic content, using a routine which minimizes this effect should give the most reliable results. We therefore recommend that the linear fitting be added to the application programs.

Acknowledgements

Discussions with L. Ahrens and T. Roser were very helpful. It was Tom Roser who suggested trying the linear fitting algorithm.

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References

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2. Orbits at Various Tunes, R. Thern et al; AGS Studies Report no. 235, 10/19/87.

3. Computer Study of Harmonic Orbit Correction In The AGS Booster, A.U. Luccio; ADD Booster Technical Note no. 131, 10/3/88.

4. Closed Orbit Analysis For The AGS Booster, J. Milutinovic and A.G. Ruggiero; ADD Booster Technical Note no. 107, 2/1/88.

5. Numerical Recipes in C, W.H. Press, B.P. Flannery, S.A. Teukolsky, W.T. Vetterling; Cambridge Univ. Press, 1990.

6. Booster Proton Book III for Startup FY 93, pp. 1-5; Logbook for data taken in this report.

Table 3: Horizontal Orbit Harmonic Data

FFT Fittings

nu{h}	Cos3	Sin3	Cos4	Sin4	Cos5	Sin5	Cos6	Sin6
4.16	1.1233	0.726	-9.067	3.3445	-3.497	-3.898	0.6783	0.4688
4.26	0.7956	0.7392	-5.39	2.1928	-3.388	-3.946	0.7842	0.4188
4.36	0.7693	0.4783	-3.98	1.7313	-3.555	-4.291	0.9173	0.2473
4.46	0.7406	0.4102	-3.269	1.5534	-3.986	-4.9	0.9586	0.1502
4.56	0.5288	0.6375	-2.9	1.5268	-4.828	-5.906	0.8323	0.1218
4.66	0.4933	0.5873	-2.755	1.6238	-6.031	-7.371	0.8523	0.0358
4.76	0.3273	0.8	-2.89	1.9363	-8.436	-10.17	0.6368	-0.04
4.86	-0.093	1.4482	-3.573	2.8312	-14.28	-16.93	-0.173	-0.098
5.21	0.5868	-0.19	1.7458	-0.353	13.099	11.163	1.6448	1.807
5.24	1.359	-0.446	0.9054	-0.239	9.4652	8.497	3.8138	1.0174
5.31	1.974	0.5325	0.781	0.358	9.347	7.5745	5.1145	1.115

Quadratic Fittings

nu{h}	Cos3	Sin3	Cos4	Sin4	Cos5	Sin5	Cos6	Sin6
4.16	1.7303	-0.463	-8.804	2.594	-1.665	-3.299	0.082	-0.679
4.26	1.4154	-0.465	-5.248	1.5044	-2.026	-3.519	0.193	-0.693
4.36	1.2705	-1.626	-3.85	1.0348	-2.37	-3.91	0.3755	-0.641
4.46	1.1934	-0.491	-3.108	0.806	-2.86	-4.511	0.439	-0.632
4.56	1.0825	-0.432	-2.663	0.677	-3.672	-5.46	0.2843	-0.762
4.66	1.0228	-0.428	-2.397	0.6258	-4.788	-6.836	0.311	-0.768
4.76	0.9508	-0.36	-2.28	0.643	-6.961	-9.389	0.0753	-0.904
4.86	0.8404	-0.193	-2.351	0.8186	-12.19	-15.59	-0.788	-1.269
5.21	1.2182	-1.568	0.2342	0.6386	10.952	9.8828	0.8304	0.1196
5.24	1.4326	-0.947	-0.402	0.3888	8.916	7.5974	3.0156	0.3266
5.31	1.316	0.2475	-0.223	0.6715	7.908	6.6705	4.209	0.657

Linear Fittings

nu{h}	Cos3	Sin3	Cos4	Sin4	Cos5	Sin5	Cos6	Sin6
4.16	1.995	-0.282	-9.202	0.193	-0.814	-3.903	0.406	-0.862
4.26								
4.36	1.5	-0.347	-3.7	0.106	-1.365	-4.896	0.689	-0.682
4.46								
4.56	1.319	-0.324	-2.371	0.062	-2.12	-7.068	0.695	-0.851
4.66								
4.76	1.197	-0.339	-1.758	0.08	-3.97	-12.3	0.473	-1.198
4.86	1.161	-0.317	-1.612	0.116	-6.966	-20.81	-0.187	-2.053
5.21	2.116	-0.512	-1.585	0.444	8.237	14.822	0.32	1.043
5.24	1.533	-0.313	-0.791	1.006	5.951	11.343	2.657	1.973
5.31	1.288	0.99	-1.39	1.021	5.606	10.582	3.989	3.239

Table 4: Vertical Orbit Harmonic Data

FFT Fittings

nu{h}	Cos3	Sin3	Cos4	Sin4	Cos5	Sin5	Cos6	Sin6
4.07	1.7007	0.1833	6.592	-4.962	-0.719	-1.5	2.722	0.711
4.12	1.565	0.2163	3.775	-2.894	-0.721	-1.469	2.094	0.626
4.23	1.466	0.219	1.973	-1.386	-0.787	-1.569	1.623	0.483
4.33	1.3127	0.2293	1.2693	-0.881	-0.961	-1.702	1.4837	0.5333
4.43	1.1475	0.248	0.9695	-0.731	-1.122	-1.979	1.475	0.441
4.53	1.1008	0.2198	0.7428	-0.594	-1.424	-2.36	1.4425	0.4493
4.63	0.8887	0.262	0.5783	-0.593	-1.793	-2.926	1.5027	0.3947
4.73	0.6273	0.3257	0.4387	-0.686	-2.497	-3.876	1.581	0.3007
4.88	-0.275	0.5798	0.104	-1.201	-5.851	-8.339	1.7858	-0.073
4.92	-1.092	0.862	-0.094	-1.69	-8.903	-12.15	1.907	-0.403
5.13	2.488	-0.391	0.773	0.645	5.242	7.074	1.788	1.503
5.2	1.928	-0.302	0.606	0.098	3.988	4.77	2.159	1.374
5.38	1.423	-0.108	0.428	-0.152	2.052	2.11	2.679	1.418

Quadratic Fittings

nu{h}	Cos3	Sin3	Cos4	Sin4	Cos5	Sin5	Cos6	Sin6
4.07	2.3457	-0.065	5.806	-6.438	-1.005	-0.614	0.7043	0.239
4.12	2.01	0.0223	3.3213	-3.962	-0.958	-0.778	0.6977	0.2263
4.23	1.749	0.048	1.746	-2.169	-1.005	-0.991	0.68	0.168
4.33	1.5647	0.062	1.1327	-1.544	-1.18	-1.138	0.7157	0.2523
4.43	1.4075	0.0745	0.8595	-1.304	-1.365	-1.375	0.79	0.1975
4.53	1.3653	0.0198	0.6643	-1.114	-1.713	-1.694	0.8308	0.2493
4.63	1.2123	0.0313	0.51	-1.041	-2.152	-2.147	0.9463	0.2533
4.73	1.068	0.0397	0.3693	-1.066	-2.982	-2.896	1.082	0.2577
4.88	0.7198	0.0208	-0.007	-1.395	-6.949	-6.382	1.515	0.3332
4.92	0.404	0.073	-0.262	-1.756	-10.52	-9.327	1.831	0.397
5.13	1.538	-0.012	0.843	0.096	6.375	5.651	0.783	0.223
5.2	1.286	-0.055	0.662	-0.34	4.79	3.818	1.158	0.277
5.38	1.09	-0.03	0.458	-0.421	2.511	1.742	1.801	0.483

Figure 1: Cos 4th Harmonic Tune Dependence
Horizontal Data: Linear Fits for each of the data fittings

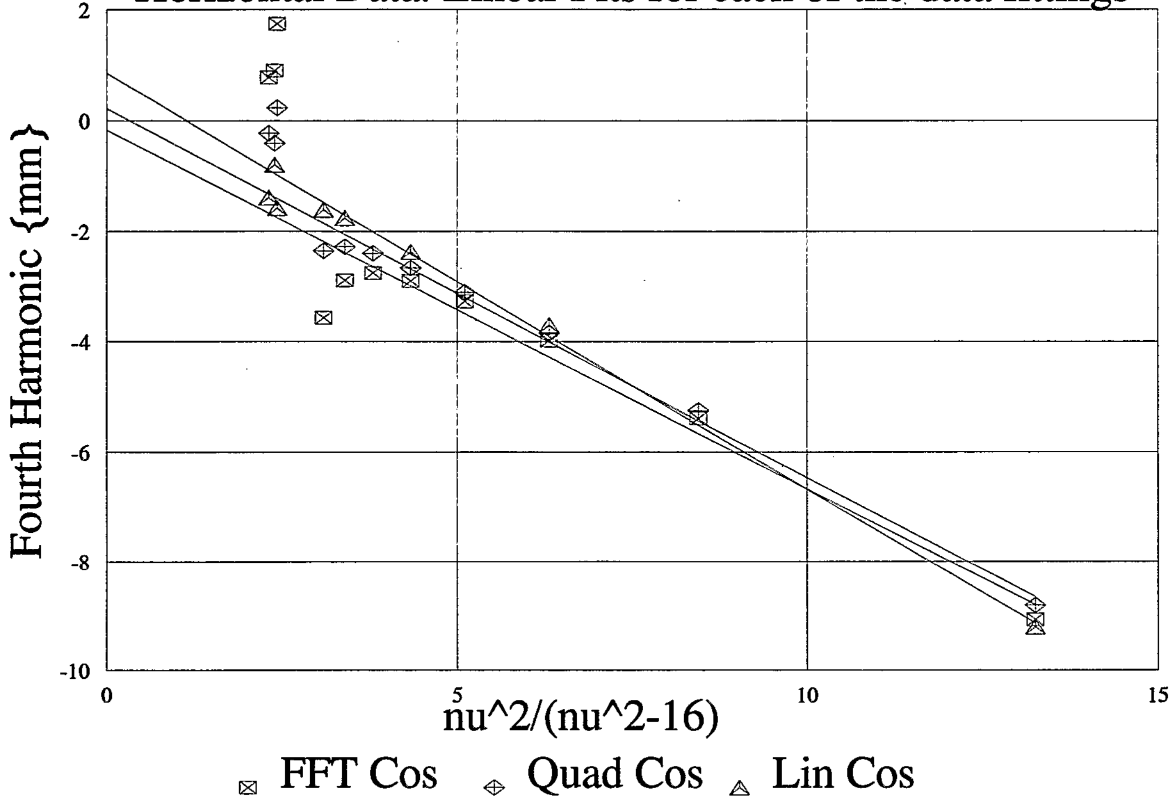

Figure 2: Sin 4th Harmonic Tune Dependence
Horizontal Data: Linear fits for each of the data fittings.

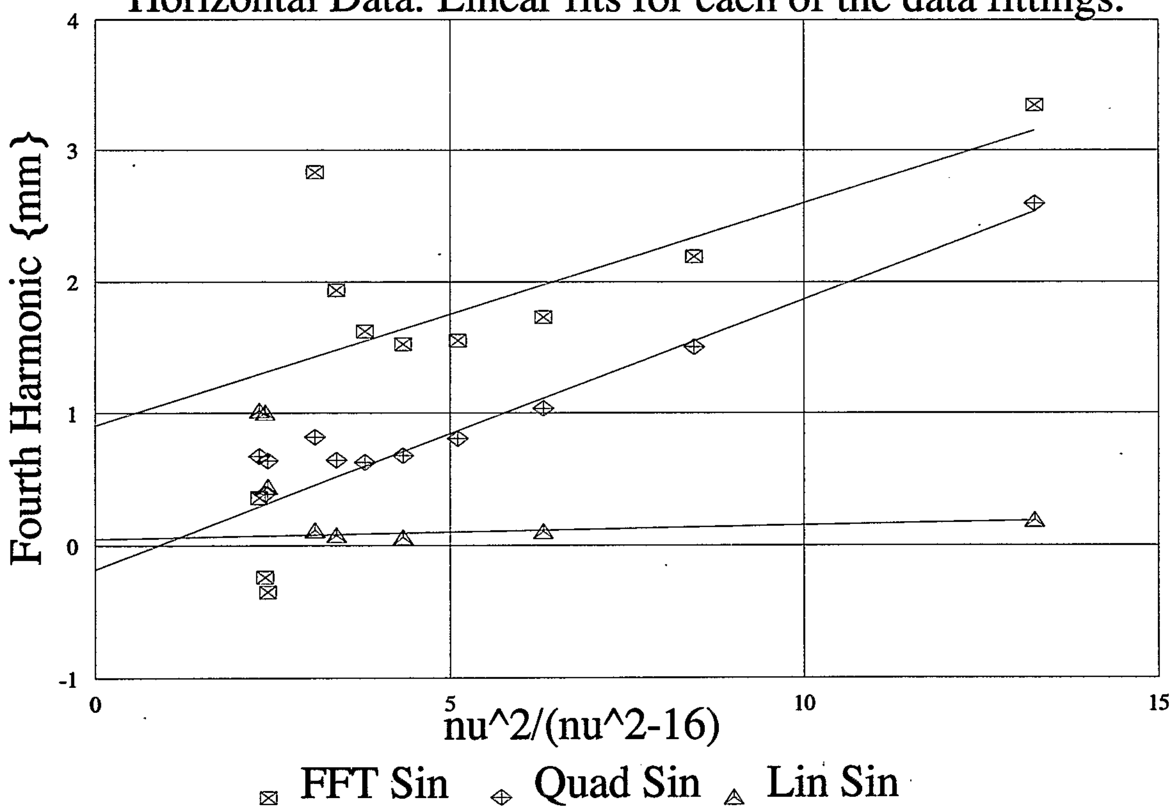

Figure 3: Cos 5th Harmonic Tune Dependence
Horizontal Data: Linear fits for each of the data fittings.

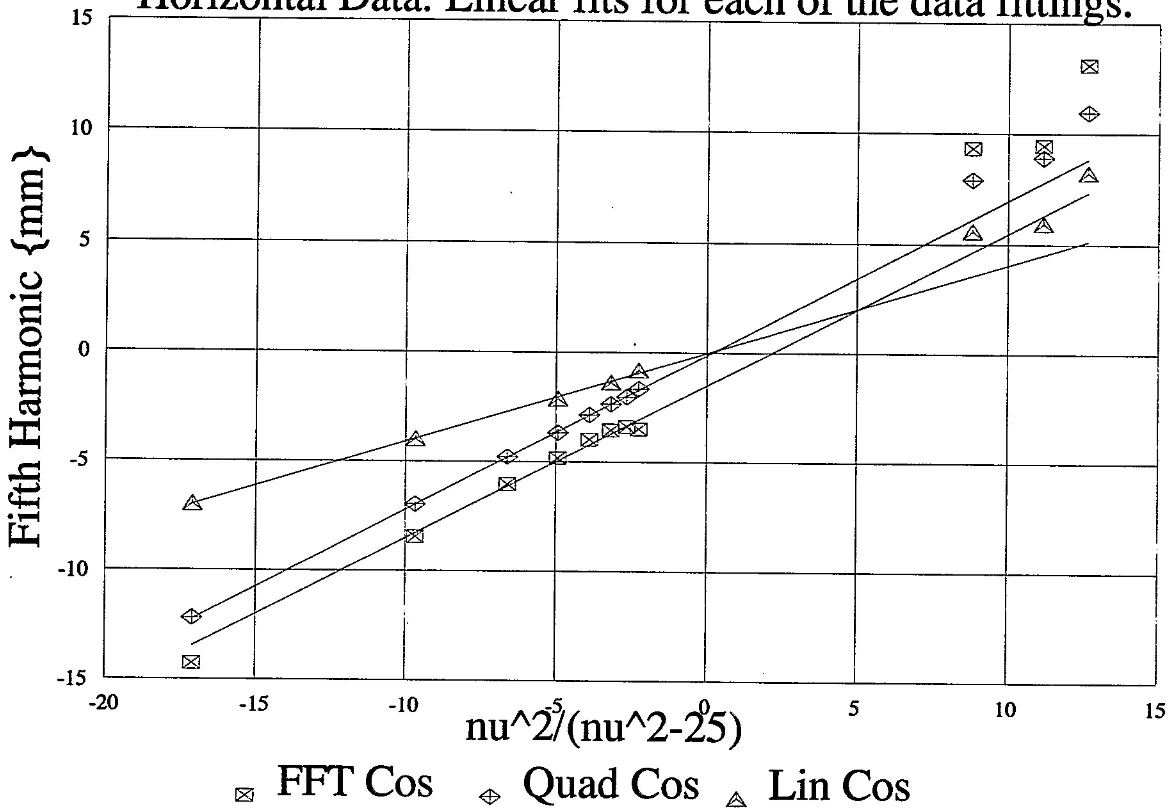

Figure 4: Sin 5th Harmonic Tune Dependence
Horizontal Data: Linear fits for each of the data fittings.

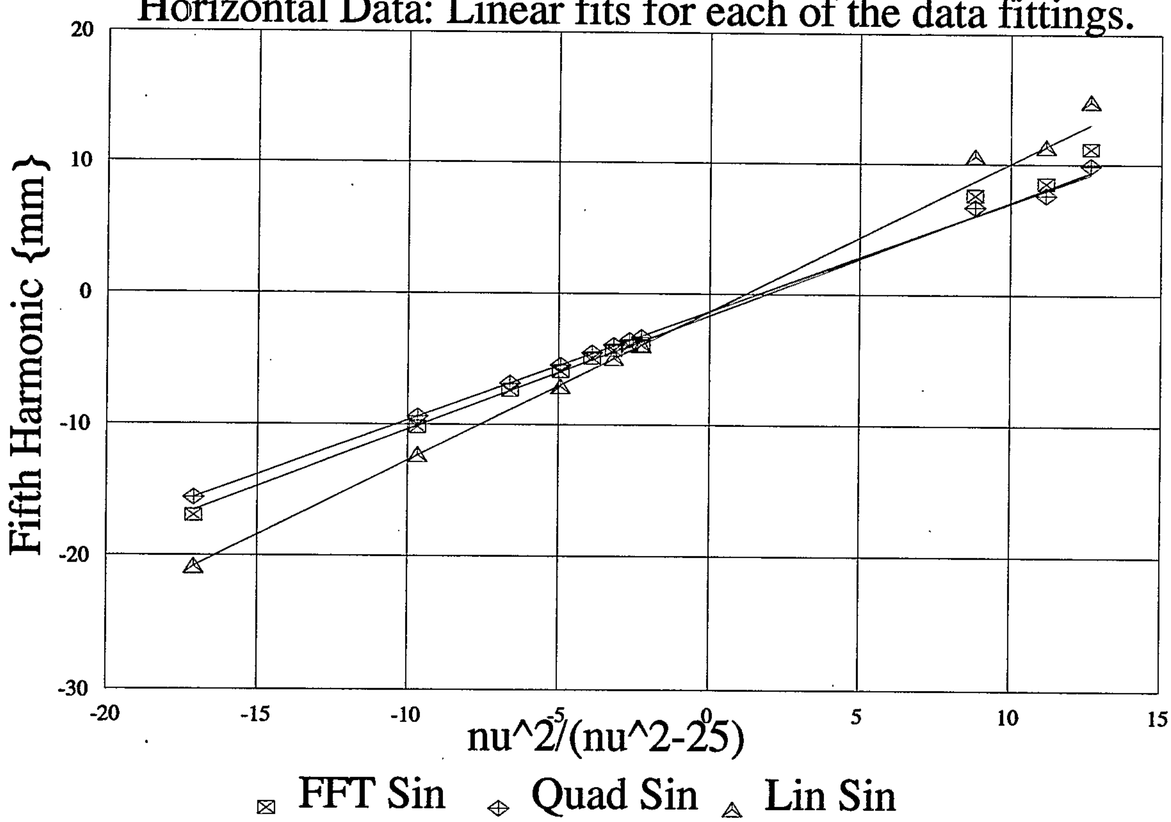

Figure 5: Cos 4th Harmonic Tune Dependences
Vertical Data: Linear Fits for each of the data fittings

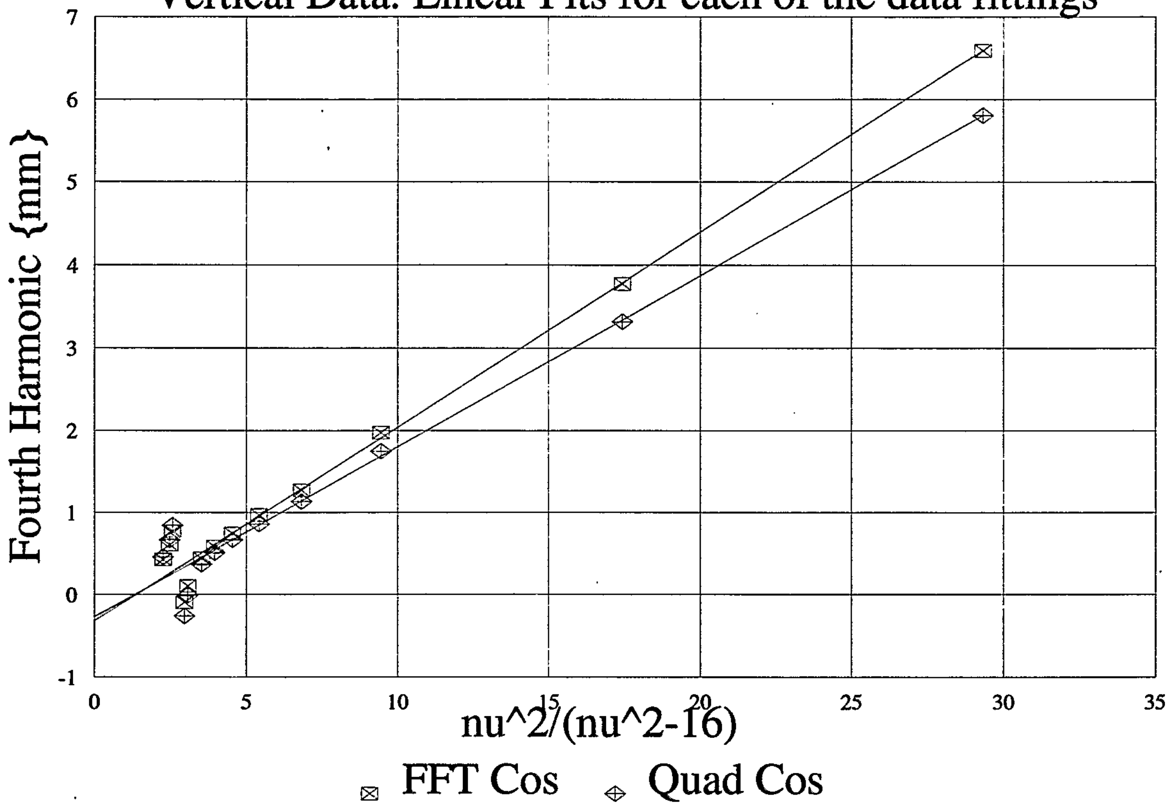

Figure 6: Sin 4th Harmonic Tune Dependence
Vertical Data: Linear fits for each of the data fittings.

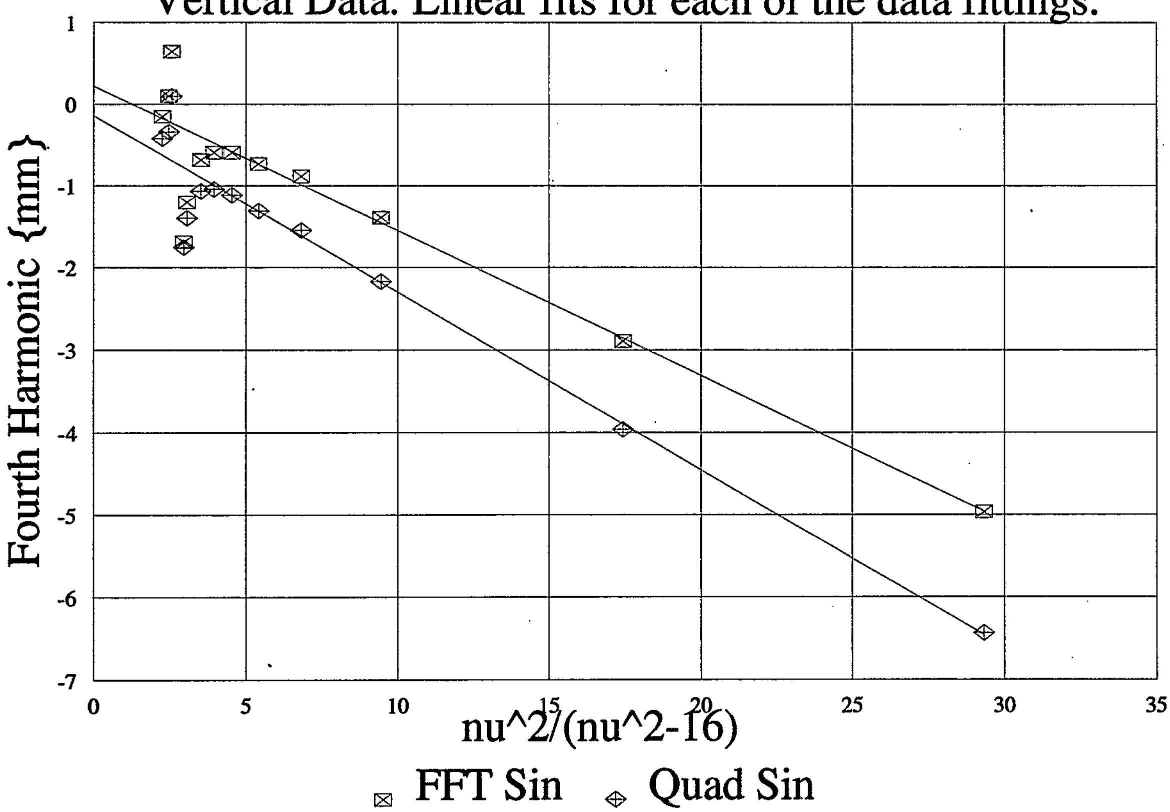

Figure 7: Cos 5th Harmonic Tune Dependence
Vertical Data: Linear fits for each of the data fittings.

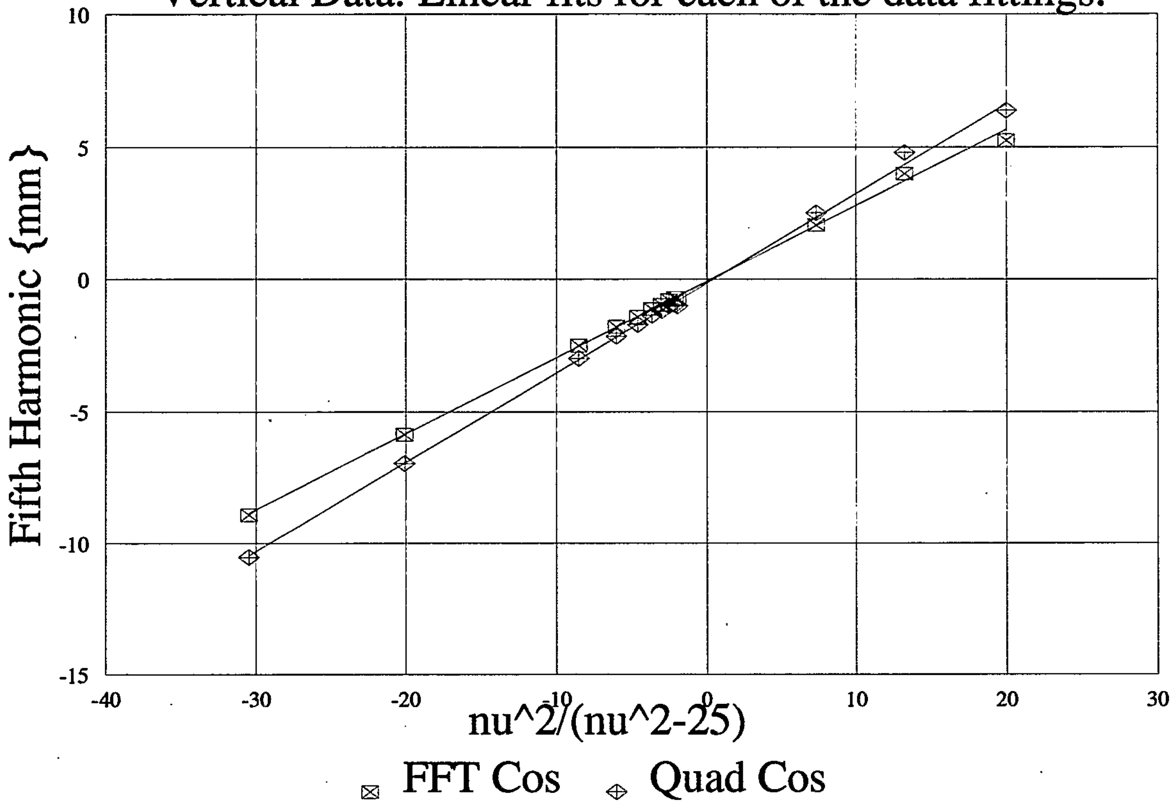

Figure 8: Sin 5th Harmonic Tune Dependence
Vertical Data: Linear fits for each of the data fittings.