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Compound refractive lenses for X-rays

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I. INTRODUCTION

Compund refractive lenses are primarily used for focusing and transport of X-rays and for forming a stable optical cavity and mode matching to maximize FEL gain [1]. In this report, we cover focusing behavior and effects of curvature of compound refractive lenses on X-ray beam propagation. Our analysis is limited to X-ray beams with transverse Gaussian profile and is thus purely analytical in nature.

The transmission amplitude of a radiation beam upon passing through a thin refractive lens is given by [2, 3]

$$\mathrm{T}(x, y) = \exp\left[-ik\int_0^d (1 - n(x, y, z))\, dz\right], \tag{1}$$

where d is the thickness of the lens, and $n(x, y, z)$ is its complex refractive index and $k = 2\pi/\lambda$ is the wave number of radiation with wavelength λ. Here x and y refers to the transverse coordinates and z is the longitudinal coordinate for beam propagation. For X-rays, the index of refraction takes the form $n(x, y, z) = 1 - \delta(x, y, z) + i\beta(x, y, z)$, with δ and β at any point in space given by following expressions [2, 4]

$$\delta = N_A/(2\pi) r_0 \lambda^2 \rho (Z + f')/A, \tag{2a}$$

$$\beta = \frac{\mu\lambda}{4\pi}. \tag{2b}$$

N_A is Avogadro's number, r_0 is the classical electron radius, λ is X-ray wavelength, ρ is the mass density of the lens material, Z is the mass number, A is the atomic mass, f' is the dispersion correction, and μ is the attenuation coefficient of the X-ray in the lens material. The typical values of δ and β are on the order of 10^{-6} or less for X-rays in the range of 5-100 KeV (see [2, 4] for data). For a radiation beam with electric field amplitude $\mathcal{E}_{\text{in}}(x, y)$ passing through this lens, we get

$$\begin{aligned}\mathcal{E}_{\text{out}}(x, y) &= \mathrm{T}(x, y)\mathcal{E}_{\text{in}}(x, y) \\ &= \exp\left[-\int_0^d (\mu/2) dz\right] e^{-ik\epsilon}\mathcal{E}_{\text{in}}(x, y),\end{aligned} \tag{3}$$

where we substituted $\mu = 2k\beta$ and $\epsilon = \int_0^d \delta(x, y) dz$. In other words, the real component of the refractive index induces phase shift whereas the imaginary component brings about attenuation in the radiation field amplitude.

II. FOCUSING

A converging/diverging lens can be obtained with bi-concave/bi-convex surface due to the contribution of the real component of the refractive index. If R is the radius of the curvature of a bi-concave surface, its focal length for the X-ray given by the Lensmaker's equation becomes $f = R/(2\delta)$ to the lowest order [2]. If N thin elements are stacked together, the resulting focal length becomes

$$f = \frac{R}{2\delta N}. \tag{4}$$

Beryllium (Be) lenses with focal lengths on the order of tens of meters can be designed with radius of curvatures of few hundred micrometers as shown in Fig. 1 for 10.3 KeV X-rays. Tighter focusing with smaller focal lengths can be achieved by stacking several single lens together as is evident from equation (4) and Fig. 1. We ignored the contribution from dispersion correction (i. e. $f' \approx 0$) and used $\delta_{Be} = 3.20465 \times 10^{-6}$ to obtain the results of Fig. 1. The demonstration of using compound refractive lens for focusing X-rays was first reported in Ref. [5] with drilled holes in Aluminium (Al) block for 14 KeV X-ray beam. Parabolic lenses made from Be are preferred choice for X-rays in the 5-25 KeV because they are free from spherical aberration and thus are ideally suitable for imaging; in addition, Be has lower attenuation coefficients compared to higher Z materials like Al [2].

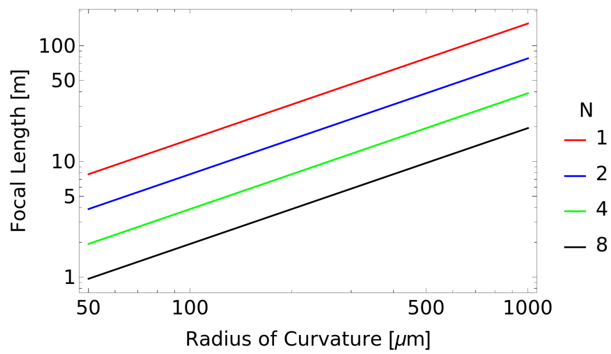

FIG. 1. Focal lengths obtained by stacking bi-concave Beryllium surfaces with various radius of curvatures for X-ray with wavelength of 1.20 Å (10.3 KeV) based on equation (4).

III. CURVATURE AND ATTENUATION

A monoenergetic pencil beam with initial intensity $I_0 \propto |\mathcal{E}_{\rm in}|^2$ traversing through a matter of small thickness z gets reduced by the linear attenuation factor μ of the matter based on equation (3). In this case, the transmitted intensity is given by Beer-Lambert law as

$$\frac{dI}{dz} = -\mu I \Rightarrow I(z) = I_0 e^{-\mu z}, \tag{5}$$

where the attenuation factor is determined by scattering cross-section of X-ray in the given lens material and depends on the X-ray wavelength and material (see equation (2b)). It also defines the mean free path of the X-ray in the material, the distance by which the incident intensity reduces by a factor of e. For a 10.3 KeV X-ray in Be, $\mu = 1.114\,\text{cm}^{-1}$, which means the mean free path of 10.3 KeV in Be is ∼0.9 cm.

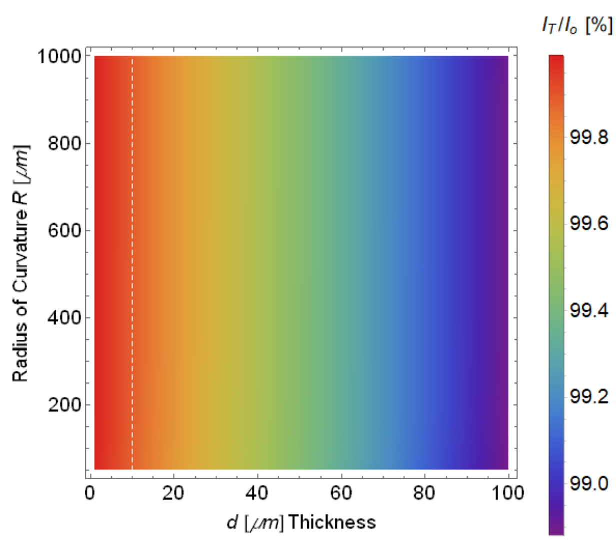

FIG. 2. Ratio of transmitted intensity of an incident Gaussian beam with rms beam sizes of $\sigma_x = 46.8\,\mu\text{m}$ and $\sigma_y = 40.2\,\mu\text{m}$ in a Be lens plotted various radius of curvatures and lens thickness for X-ray with wavelength of 1.20 Å (10.3 KeV). The lens aperture size is $2\sqrt{2\pi}$ times the rms beam size in each dimension.

Typical X-ray beams from undulators and FELs incident on the lens surface will have non-negligible transverse size. In fact, the design transverse beam profile is typically Gaussian in nature. For a lens with parabolic curvature in transverse plane, the thickness of the lens increases away from the lens center according to the parabolic profile as follows

$$z(x,y) = d + \frac{x^2}{R_x} + \frac{y^2}{R_y}. \tag{6}$$

Here d is the lens thickness at the center and R_x and R_y are radii of curvature in x and y direction respectively. At lens position, the transverse intensity profile of the X-ray beam is given by

$$I(x,y) = \frac{I_0}{2\pi\sigma_x\sigma_y}\, e^{-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}}, \tag{7}$$

where σ_x and σ_y are the corresponding rms sizes of the X-ray beam in x and y dimensions. The overall transmitted intensity of this Gaussian beam upon passing through a lens can be obtained by using equation (5) and integrating over the transverse profile after substituting z with equation (6). The resulting expression for transmittivity is

$$\begin{aligned} \mathcal{T} &= I_T/I_0 \\ &= e^{-\mu d}\sqrt{\frac{R_x}{R_x + 2\mu\sigma_x^2}}\sqrt{\frac{R_y}{R_y + 2\mu\sigma_y^2}} \\ &\quad \times \operatorname{erf}\left[a\sqrt{\frac{1}{2\sigma_x^2} + \frac{\mu}{R_x}}\,\right] \operatorname{erf}\left[b\sqrt{\frac{1}{2\sigma_y^2} + \frac{\mu}{R_y}}\,\right], \end{aligned} \tag{8}$$

where a and b are effective aperture radii of the lens in x and y dimensions respectively. For $a \gg \sigma_x$, $\operatorname{erf}\left[a\sqrt{\frac{1}{2\sigma_x^2} + \frac{\mu}{2f_x}}\right] \approx 1$. The effect of finite aperture size of the lens is thus included via the error functions expressions in second line of equation (8). Again using Be as a lens material candidate for 10.3 KeV X-ray beam, Fig. 2 shows the contour plot of the percent ratio of transmitted to incident intensity over various lens thickness and radii of curvature where we used $R_x = R_y = R$ for simplicity. The lens aperture in each direction is assumed to be $2\sqrt{2\pi}$ times the rms beam sizes. The X-ray beam has $\sigma_x = 46.8\,\mu\text{m}$ and $\sigma_y = 40.2\,\mu\text{m}$ when it strikes the lens. This corresponds to a X-ray beam source located 30 m away from the lens with rms waist sizes of $\sigma_x = 6.184\,\mu\text{m}$ and $\sigma_y = 7.25\,\mu\text{m}$ and corresponding Rayleigh ranges of 4.0 m and 5.5 m in x and y respectively. While the transmitted intensity is $\geq 99\,\%$ for $50 \leq R(\mu m) \leq 1000$ and $1 \leq d(\mu m) < 100$, the finite aperture effects become prominent if $r_{x,y} \leq 3\sigma_{x,y}$, where $r_x = a$ and $r_y = b$. For instance, when $r_{x,y} = 3\sigma_{x,y}$, the transmittivity drops by additional $\sim 1.2\%$ over the specified range. Even with finite aperture effects, highly transmittive lens with Be are feasible to operate in the range of 5−25 keV by choosing smaller thickness. Highly transmittive lenses can be designed with $10 \leq d(\mu m) < 30$ [6] restricting power loss to $\sim 0.12\%$ for $d = 10\,\mu\text{m}$ for a single lens as indicated by the white dashed line in Fig. 2. The thin-lens approximation is sufficient for focal lengths in the range of 10-50 m. However, it may not be adequate for thicker lenses or compound refractive lenses with imperfections. Recently, Celestre *et al.* [7] applied multi-slicing approach to model and simulate the compound refractive lenses with phase imperfection effects.

ACKNOWLEDGMENTS

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