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EIC'S HSR ROTATOR AND SNAKE SWAP STUDY

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Rotator and Snake Configuration at IP6

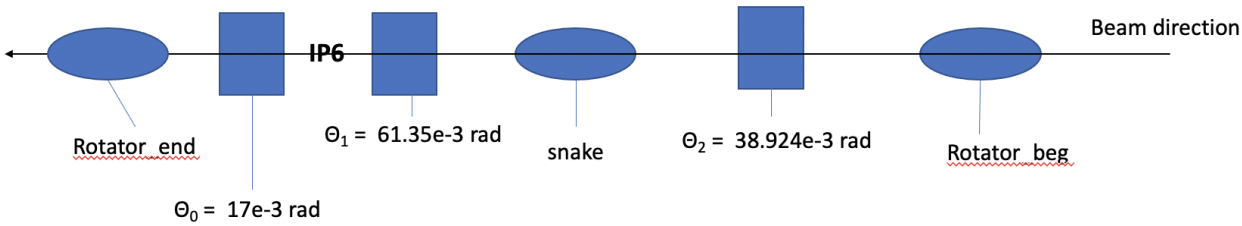

Figure 1: Proposed layout of IP6 with rotators and snake

Abstract

The Electron-Ion Collider's Hadron Storage Ring (HSR) will use a pair of spin rotators to achieve longitudinal polarization at IP6. Additionally there are to be six snakes located at azimuthal angles of 60 degrees from each other. Due to space constraints in the whole lattice, in order to achieve a 60 degree separation between the snakes, we are forced to place a snake near IP6 where the rotator normally would be. We explore if a powering scheme exists which would recover a longitudinal polarization at IP6 and the same spin rotation of a normal orthogonal snake at collision energies.

INTRODUCTION

The design of the collision optics for the EIC is particularly challenging due to the fact that beams with dramatically different rigidities will be collided. Further space around the HSR ring is limited due to the need for cooling and the legacy of the RHIC lattice. This situation forces the place of a snake just before IP6 to achieve the needed 60 degree orbital separation between it and the nearest next snake. This situation displaces the spin rotator as illustrated in Fig. 1. Using this configuration we can accelerate using it as part of a six snake set. However, when we reach top energy to obtain longitudinal polarization while maintaining the function of the snake we propose tuning the snake and two rotators such that the spin Transport Matrix through the whole line at exit of rotator want to be same as if we had an ideal snake and the rotators were off. We call this matrix the STM_0 matrix. As well at IP6 the polarization should be longitudinal.

SYPHERS AND COURANT'S HELICAL DIPOLE MODEL

Using a reduced model of a helical dipole developed by M. Syphers and E. Courant [1] the 3x3 spin transport matrix for a helical dipole is approximated using the following expressions:

$$M_h = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\mu & -\cos\phi\sin\mu & \sin\phi\sin\mu \\ \cos\phi\sin\mu & \cos^2\phi\cos\mu + \sin^2\phi & \cos\phi\sin\phi(1-\cos\mu) \\ -\sin\phi\sin\mu & \cos\phi\sin\phi(1-\cos\mu) & \sin^2\phi\cos\mu + \cos^2\phi \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

Here α is the angle the field makes from the vertical (y-axis) toward the horizontal (x-axis) at the entrance/exit of the magnet. For snakes α should be zero. For a rotator, α should be 90 degrees. The other parameters are given as follows:

$$\begin{aligned} \mu &= -2\pi\sqrt{1+(\frac{\kappa}{k})^2} \\ \phi &= \arctan(\kappa/k) \\ \kappa &= (1+G\gamma)B_0/(B\rho) \\ k &= \pm\frac{2\pi}{L} \end{aligned} \quad (2)$$

Here L is the length of the helix and the sign (±) indicates a right-handed helix (+) or left handed helix (-). A full snake is made up of four helices all right handed. The outer helices are powered with equal but opposite polarity as are in the

inner. A full snake transport matrix is thus given:

$$M(B_1, B_2)_{sn} = M_{hs}(-B_1)M_{hs}(-B_2)M_{hs}(B_2)M_{hs}(B_1) \tag{3}$$

Here hs subscript indicate that these are snake-like helices with $\alpha = 0$ and $k = +2\pi/L$. For the spin rotators both right handed and left handed helices are used with $\alpha = \pi/2$ we use the subscript hrl and hll to indicate right handed and left handed helices respectively. In this case the outer and inner two helices are power equally but using opposite right-handed and left-handed helices in the following manner.

$$M(B_1, B_2)_{rot} = M_{hrr}(B_1)M_{hrl}(B_2)M_{hrr}(B_2)M_{hrl}(B_1) \tag{4}$$

FITTING REDUCED MODEL

We built a python code to implement the transport line shown in fig. 1 using rotation matrices to model the dipoles as follows:

$$R_y(G\gamma\theta) = \begin{pmatrix} \cos G\gamma\theta & 0 & \sin G\gamma\theta \\ 0 & 1 & 0 \\ -\sin G\gamma\theta & 0 & \cos G\gamma\theta \end{pmatrix} \tag{5}$$

We obtain the transport matrix for the whole line and to IP6:

$$\begin{aligned} STM_W &= M(B_1, B_2)_{rot}R_y(G\gamma\theta_0)R_y(G\gamma\theta_1)M(B_3, B_4)_{sn}R_y(G\gamma\theta_2)M(B_5, B_6)_{rot} \\ STM_{ip6} &= R_y(G\gamma\theta_1)M(B_3, B_4)_{sn}R_y(G\gamma\theta_2)M(B_5, B_6)_{rot} \\ \theta_0 &= 17.0 \ \ mrad \\ \theta_1 &= 61.35 \ \ mrad \\ \theta_2 &= 38.924 \ \ mrad \end{aligned} \tag{6}$$

For an ideal snake the transport matrix is given by:

$$Mideal_{sn} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{7}$$

Thus our ideal STM_0 matrix with only idea snakes and dipoles is:

$$STM_0 = R_y(G\gamma\theta_0)R_y(G\gamma\theta_1)Mideal_{sn}R_y(G\gamma\theta_2) \tag{8}$$

At 275 GeV STM_0 matrix becomes:

$$\begin{pmatrix} 0.95643894 & 0 & -0.29193243 \\ 0 & -1 & 0 \\ -0.29193243 & 0 & -0.95643894 \end{pmatrix} \tag{9}$$

Using $\vec{b} = (B_1, B_2, B_3, B_4, B_5, B_6) = (2.816, 3.107, -0.41332, -1.725, 2.6339, 2.552)$ (T) we can recover the STM_0 and at IP6 a vertical spin vector (0,1,0) becomes (-0.00787843, 0.01496793, 0.99985694). At 100 GeV the STM_0 becomes:

$$\begin{pmatrix} 0.94902352 & 0. & 0.31520527 \\ 0. & -1. & 0. \\ 0.31520527 & 0. & -0.94902352 \end{pmatrix} \tag{10}$$

This is recovered using $\vec{b} = (-1.7177, -3.0292, -3.933, -3.643, , 1.67338, 3.0326)$ and yielding a spin vector at IP6 of (0.00858315 0.02527488 0.99964369). For 41 GeV the STM_0 matrix is:

$$\begin{pmatrix} 0.05291137 & 0. & -0.99859921 \\ 0. & -1. & 0. \\ -0.99859921 & 0. & -0.05291137 \end{pmatrix} \tag{11}$$

This is recovered using $\vec{b} = (-1.70116, -3.56313, 2.58481, -4.0, -3.355, -2.798)$ and yielding a spin vector at IP6 of (0.01825893 0.02319105 0.9995643). Finally for He3 at 183.4 GeV the STM_0 becomes:

$$\begin{pmatrix} 0.97436538 & 0. & -0.22497133 \\ 0. & -1. & 0. \\ -0.22497133 & 0. & -0.97436538 \end{pmatrix} \tag{12}$$

This is recovered using $\vec{b} = (-2.68880, 1.3333794, 0.85326, -2.140, 1.02, 2.878)$ and yielding a spin vector at IP6 of (8.49386689e-17 -2.39672806e-16 -1.00000000e+00).

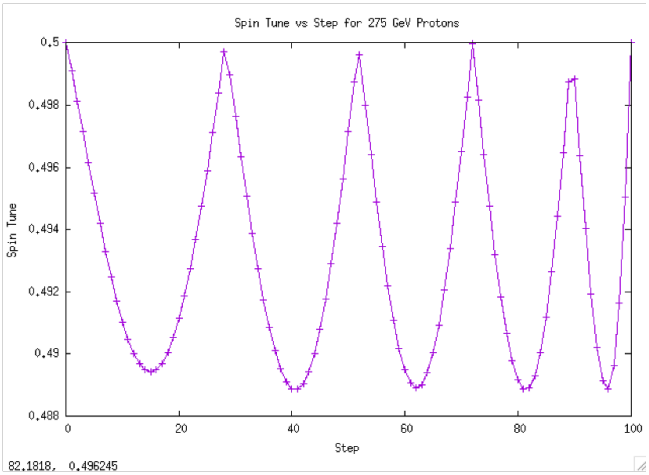

Figure 2: Rotator Ramp for 275 GeV

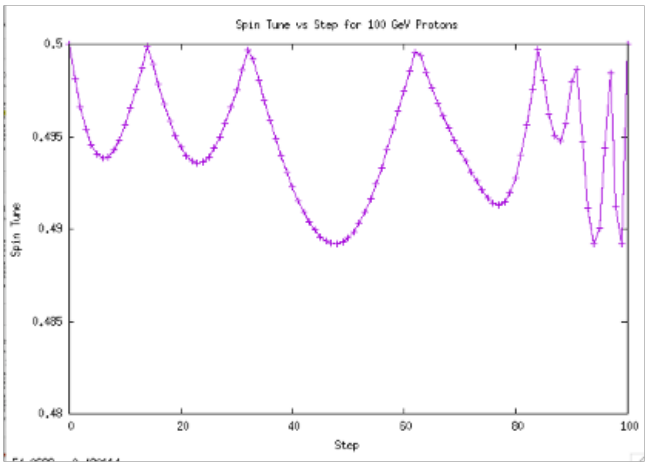

Figure 3: Rotator Ramp for 100 GeV

RAMPING INTO STORE CONFIGURATION

After accelerating to store energy the currents in the rotators and snakes will need to be ramped to these field settings calculated in the previous section. However one must do this in such a way that our spin tune is not perturbed too far away from 0.5 while keeping our fields below the 4 Tesla limit. To accomplish this we modeling the field ramp for each of the elements of the b vector using a third order polynomial,

$$B = m_0 + m_1 t + m_2 t^2 + m_3 t^3 \tag{13}$$

For the rotator currents B_1, B_2, B_5, B_6 the initial values are zero since they are off during the acceleration ramp so $m_0 = 0$ while for the snakes m_0 is just the snake settings at the end of the acceleration ramp. We chose to vary the m_1 and m_2 coefficient while fixing the m_3 based on the final field setting B and the chosen m_1 and m_2 values for the rotators as follows:

$$m_3 = B/step^3 - m_1/step^2 - m_2/step \tag{14}$$

and for snakes:

$$m_3 = (B - B_i)/step^3 - m_1/step^2 - m_2/step \tag{15}$$

Here the variable $step$ is the total number of time steps for the rotator ramp. At each time step the one turn spin transport matrix was calculated for whole lattice assuming it was made up of just dipoles R_y, rotators and snakes. Using this a spin tune is calculated. An optimized path for the spin tune using these polynomials is shown in the case of protons at 275, 100 and 41 GeV in Fig. 1-4 In Fig. 5-7 the field trajectories are plotted for each case,

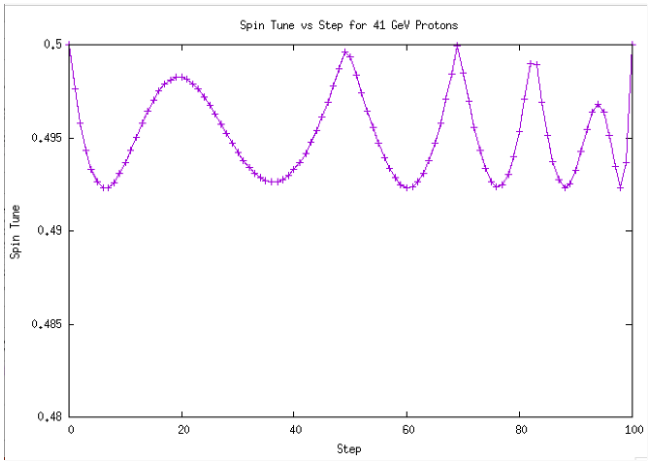

Figure 4: Rotator Ramp for 41 GeV

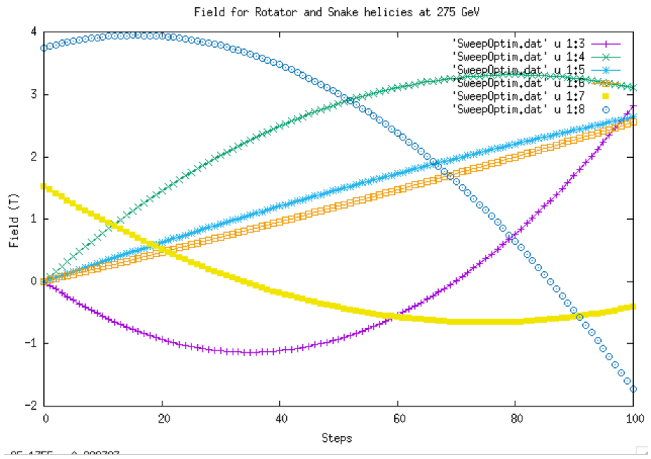

Figure 5: Rotator Ramp for 275 GeV, field trajectory

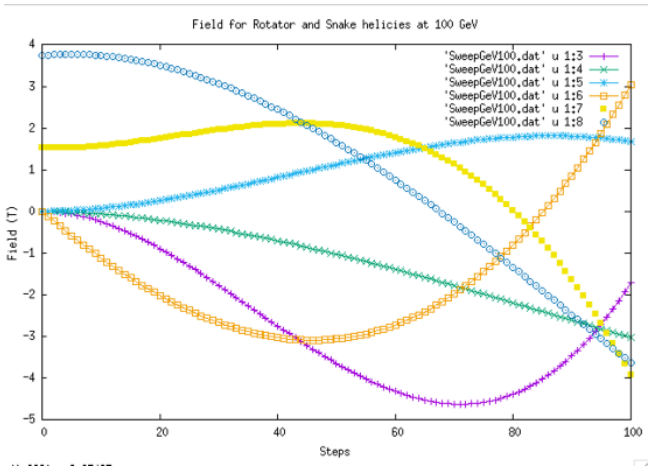

Figure 6: Rotator Ramp for 100 GeV, field trajectory

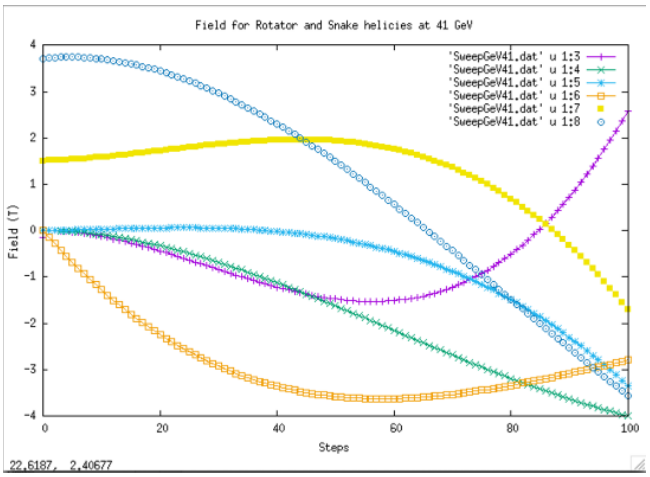

Figure 7: Rotator Ramp for 41 GeV, field trajectory

USING ZGOUBI MODEL

Next we tried the fit using the spin orbit tracking code Zgoubi using the exact field maps and the current to field transfer functions in each rotator and snake. We found the following current settings for 275 GeV [-231.80, 350.9094, 123.3368, 283.1899, -198.963, -191.876] (Amps), for 100 GeV [-151.058, -218.025, 49.44, 356.247, -278.1867, -369.806] (Amps) and for 41 GeV [275.604, 355.5366, 265.939, 132.680, -103.173, -346.504] (Amps) However these were very challenging to obtain and the python optimization algorithm would often get stuck in a local minimum.

CONCLUSION

We did find possible solutions for all the given energies and species, however in several cases the fields go as high as 4 T and during the rotator ramp they would need to swing from positive to negative fields and for some magnets they would need to reach fields nearing 5 T. For this reason as well as due to the difficulty tuning this rotator/snake set up I wouldn't advise using this approach. I anticipate it will be very hard to tune and the ability to achieve some other polarization orientation at the IP will be very limited.

ACKNOWLEDGMENTS

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REFERENCES

[1] M. J. Syphers "Spin Motion through Helical Dipole Magnets", BNL Tech Note, BNL-103646-2014-TECH, (1996)