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Measurement of the Equilibrium Orbit Distortion Produced by the Low-Field Dipoles in the AGS

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I. Introduction

The low-field dipoles used to correct distortions in the equilibrium orbit near injection are located in straight sections 2, 4, 6, 8, 12, 14, 16 and 18 of each superperiod of the AGS. At each location, with the exception of F12, there is a horizontal dipole which deflects the beam in the horizontal plane and a vertical dipole which deflects the beam in the vertical plane. At F12, where the AGS ring is too close to the SEB line to accommodate two dipoles, there is a horizontal diple but no vertical dipole. Thus, in the 12 superperiods of the AGS, there are 96 horizontal and 95 vertical dipoles. The integrated strengths of the horizontal and vertical dipoles are respectively[1]

$$d = 166 \text{ and } d = 158 \text{ Gauss-cm per ampere.} \qquad (1)$$

Distortions in the equilibrium orbit are corrected by eliminating those Fourier harmonics, sin $n\theta$ and cos $n\theta$, present in the dipole fields of the machine which produce the distortions. In the AGS, where the betatron tune in both planes is approximately 8.75, the n = 8 and n = 9 harmonics produce the largest distortions. To eliminate the nth harmonic, the low-field dipoles are excited with currents

$$I_{pq} = I_c \cos n\theta_{pq} + I_s \sin n\theta_{pq} = I \cos(n\theta_{pq} + \psi), \qquad (2)$$

where p = 1, 2, 3,..., 12 specifies dipoles in superperiods A, B, C, ..., L, q = 1, 2, 3, 4, 5, 6, 7, 8 specifies the dipoles in straight sections 2, 4, 6, 8, 12, 14, 16, 18, and

$$\theta_{pq} = \theta_q + (p - 1)\frac{\pi}{6} \qquad (3)$$

are the azimuthal positions of the dipoles.

In the present study, the dipoles were excited with currents given by (2) with n = 8 and n = 9, and the harmonics present in the resulting closed orbit were measured as functions of the amplitude, I, using the orbit acquisition program ORBIT. The measurements were carried out at the Booster momentum of 2.25 GeV/c to determine whether or not the present set of dipoles will be able to correct the equilibrium orbit when protons are injected into the AGS at this momentum.

II. Calculation of the Orbit Distortion Produced by the Low-Field Dipoles

The closed orbit distortion, y(s), produced by the low-field dipoles satisfies the differential equation

$$y'' + k(s)y = F(s) \qquad (4)$$

where

$$F(s) = \frac{e}{cp_o} \sum_{p=1}^{12} \sum_{q=1}^{8} d\, I_{pq}\, \delta(s - s_{pq}), \qquad (5)$$

s is the distance along the design orbit, p_o is the momentum, d is the integrated strength per ampere of each dipole, I_{pq} is given by (2), $s_{pq} = R\theta_{pq}$, and $2\pi R$ is the circumference of the design orbit. Following Courant and Snyder,[2] we find that the periodic solution of (4) is

$$y(s) = \beta^{1/2}(s)\, \eta(\phi), \qquad (6)$$

where

$$\eta = \sum_n \frac{\nu^2 f_n}{\nu^2 - n^2} e^{in\phi}, \quad \phi(s) = \int_o^s \frac{ds'}{\nu\beta(s')}, \qquad (7)$$

and

$$f_n = \frac{1}{2\pi\nu} \int_o^{2\pi R} \beta^{1/2}(s)\, F(s)\, e^{-in\phi}\, ds. \qquad (8)$$

Putting (5) into (8), we obtain

$$f_n = \frac{1}{2\pi\nu} \frac{ed}{cp_o} \sum_{pq} \beta_{pq}^{1/2} I_{pq} e^{-in\phi_{pq}} \tag{9}$$

where

$$\phi_{pq} = \phi(s_{pq}) = \phi_q + (p - 1) \frac{\pi}{6} , \phi_q = \phi(R\theta_q), \tag{10}$$

and

$$\beta_{pq} = \beta(s_{pq}) = \beta(s_q) = \beta_q. \tag{11}$$

Then using (2-3) and (10-11) in (9) and summing over p, we find

$$f_n = \frac{1}{2\pi\nu} \frac{ed}{cp_o} \frac{I}{2} 12 \sum_{q=1}^{8} \beta_q^{1/2} (A_q + SB_q) \tag{12}$$

where

$$A_q = e^{i[n(\theta_q - \phi_q) + \psi]}, B_q = e^{-i[n(\theta_q + \phi_q) + \psi]},$$

$$S = \frac{1}{12} \sum_{p=1}^{12} e^{-i \frac{n\pi}{3} (p - 1)}. \tag{13}$$

Now, for $n \neq 0, \pm 6, \pm 12, \pm 18, \ldots$, we have $S = 0$ and therefore

$$f_n = \frac{1}{2\pi\nu} \frac{ed}{cp_o} \frac{I}{2} 12 e^{i\psi} \sum_q \beta_q^{1/2} e^{in(\theta_q - \phi_q)}. \tag{14}$$

Defining

$$S_n = \sum_q \beta_q^{1/2} e^{in(\theta_q - \phi_q)} \tag{15}$$

and evaluating this sum for n = 8, 9 using the BEAM program, we find

$$S_8 = S_9 = \beta_{AV}^{1/2}(8.23 \pm 0.5\, i) \simeq 8\, \beta_{AV}^{1/2}, \tag{16}$$

in which $\beta_{AV} = R/\nu$, and the upper and lower signs refer to the horizontal and vertical planes. Thus, we have to a good approximation

$$f_8 = f_9 = \frac{I}{2} \frac{ed}{cp_o} \beta_{AV}^{3/2} \left(\frac{96}{2\pi R}\right) e^{i\psi}. \tag{17}$$

Using

$d = 160$ Gauss-cm per amp

$\frac{cp}{e} = 3.335641 \times 10^6$ Gauss-cm per GeV

$p_o = 2.25$ GeV/c

$\beta_{AV} = R/\nu = 1480$ cm

$R = 12845$ cm

we find

$$\boxed{\beta_{AV}^{1/2} f_8 = \beta_{AV}^{1/2} f_9 = ICe^{i\psi}, \quad C = 0.028 \text{ cm per amp.}} \tag{18}$$

This is the result which is to be compared with our measurements. (Note that if we excite a cosine harmonic with the dipoles then $\psi = 0$, and if we excite a sine harmonic then $\psi = -\pi/2$.)

III. Measurement of the Orbit Distortion Produced by the Low-Field Dipoles

The equilibrium orbit (E.O.) acquisition program ORBIT,[3] measures the position, y(s), of the E.O. at the PUEs in straight sections 2, 4, 8, 12, 14, 18 of each superperiod and after interpolating between these measured positions, it calculates and displays the harmonic amplitudes a_n, b_n in the Fourier expansion,

$$y(s) = \frac{a_o}{2} + \sum_{n=1}^{\infty} \{a_n \cos \frac{ns}{R} + b_n \sin \frac{ns}{R}\}. \qquad (19)$$

In the present study, the low-field dipoles were excited with harmonics n = 8, 9 and the resulting amplitudes a_8, b_8, a_9, b_9 were obtained from the ORBIT program. The data acquired are summarized in Table I. The measurements were made at the Booster momentum of 2.25 GeV/c where the tunes were found to be

$$\nu_H = 8.652, \quad \nu_V = 8.754. \qquad (20)$$

Before each ORBIT measurement, the Linac pulse width was adjusted so that the beam intensity at the time of the measurement was 7 to 8 x 10^{12} ppp. This was done to insure that all measurements were made at the same intensity, thereby eliminating any systematic variations of the measurements due to variations in intensity.

Figures 1-6 are plots of the data from Table I. From the lines fitted to the data, we obtain in the horizontal plane

$$\begin{aligned} a_9 &= -0.71\ (I - I_o) - 0.064\ (cm) \\ b_9 &= -0.68\ (I - I_o) + 0.037\ (cm) \\ a_8 &= 0.43\ (I - I_o) + 0.236\ (cm) \\ b_8 &= 0.42\ (I - I_o) - 0.030\ (cm) \end{aligned} \qquad (21)$$

and in the vertical plane

$$\begin{aligned} a_8 &= 0.27\ (I - I_o) + 0.093\ (cm) \\ b_8 &= 0.27\ (I - I_o) - 0.002\ (cm) \end{aligned} \qquad (22)$$

where $I = I_o + \Delta I$ is the amplitude, in amps, with which the dipole harmonic is excited. Here we see, along with I_o from Table I, that in the horizontal plane the currents, I, required to make a_9, b_9, a_8, and b_8 zero are respectively -45, -126, -887, and 259 mA. In the vertical plane, the currents required to make a_8 and b_8 zero are respectively -390 and 16 mA. Since the power supplies for each dipole can provide currents of ±2 amps, we conclude that the present orbit correction system will be adequate at the Booster momentum of 2.25 GeV/c.

Now, if we define

$$F_n = \frac{1}{2}(a_n - ib_n) \tag{23}$$

then (19) becomes

$$y(s) = \sum_{n=-\infty}^{n=+\infty} F_n e^{ins/R}. \tag{24}$$

Comparing (24) and (6-7), we obtain

$$\beta_{AV}^{1/2} f_n \simeq \frac{\nu^2 - n^2}{\nu^2} F_n = \frac{\nu^2 - n^2}{2\nu^2} (a_n - ib_n). \tag{25}$$

Thus using the measured tunes (20) and harmonic amplitudes (21-22) in (25), we find in the horizontal plane,

$$\begin{aligned}
\frac{\nu^2 - 9^2}{2\nu^2} a_9 &= 0.029 (I - I_o) + 0.003 \text{ (cm)} \\
\frac{\nu^2 - 9^2}{2\nu^2} b_9 &= 0.028 (I - I_o) - 0.002 \text{ (cm)} \\
\frac{\nu^2 - 8^2}{2\nu^2} a_8 &= 0.031 (I - I_o) + 0.017 \text{ (cm)} \\
\frac{\nu^2 - 8^2}{2\nu^2} b_8 &= 0.030 (I - I_o) - 0.002 \text{ (cm)}
\end{aligned} \tag{26}$$

which are in good agreement with (18), and in the vertical plane,

$$\frac{\nu^2 - 8^2}{2\nu^2}\ a_8 = 0.022\ (I - I_o) + 0.008\ (cm)$$

$$\frac{\nu^2 - 8^2}{2\nu^2}\ b_8 = 0.022\ (I - I_o)\quad (cm) \tag{27}$$

which are in fair agreement with (18).

TABLE I

DIPOLE HARMONIC EXCITED	AMPLITUDE (mA) I_o	+ ΔI	MEASURED HARMONIC AMPLITUDES a_9(cm)	b_9(cm)	a_8(cm)	b_8(cm)
Horz Cos 9	45	-200	0.08	0.02	0.23	-0.02
	45	-100	0.01	0.02	0.24	-0.03
	45	0	-0.07	0.02	0.23	-0.03
	45	+100	-0.14	0.04	0.24	-0.03
	45	+200	-0.20	0.03	0.23	-0.04
Horz Sin 9	-180	-200	-0.07	0.18	0.24	-0.02
	-180	0	-0.07	0.02	0.23	-0.03
	-180	+200	-0.06	-0.09	0.24	-0.02
Horz Cos 8	-337	-500	-0.06	0.04	0.02	-0.03
	-337	-250	-0.07	0.02	0.13	-0.02
	-337	0	-0.07	0.02	0.23	-0.03
	-337	+250	-0.07	0.02	0.35	-0.03
	-337	+500	-0.07	0.01	0.45	-0.02
Horz Sin 8	188	-450	-0.08	0.02	0.23	-0.22
	188	0	-0.07	0.02	0.23	-0.03
	188	+450	-0.06	0.03	0.23	0.16
Vert Cos 8	- 50	-600	-0.03	0.01	-0.07	0.00
	- 50	-300	-0.03	0.01	0.01	0.00
	- 50	0	-0.04	0.02	0.10	0.00
	- 50	+300	-0.03	0.03	0.17	0.00
Vert Sin 8	9	-400	-0.04	0.01	0.10	-0.11
	9	0	-0.04	0.02	0.10	-0.00
	9	+200	-0.03	0.01	0.10	+0.05

SLOPE= -0.000710 +/- 0.000032
INTERCEPT= -0.064000 +/- 0.004472

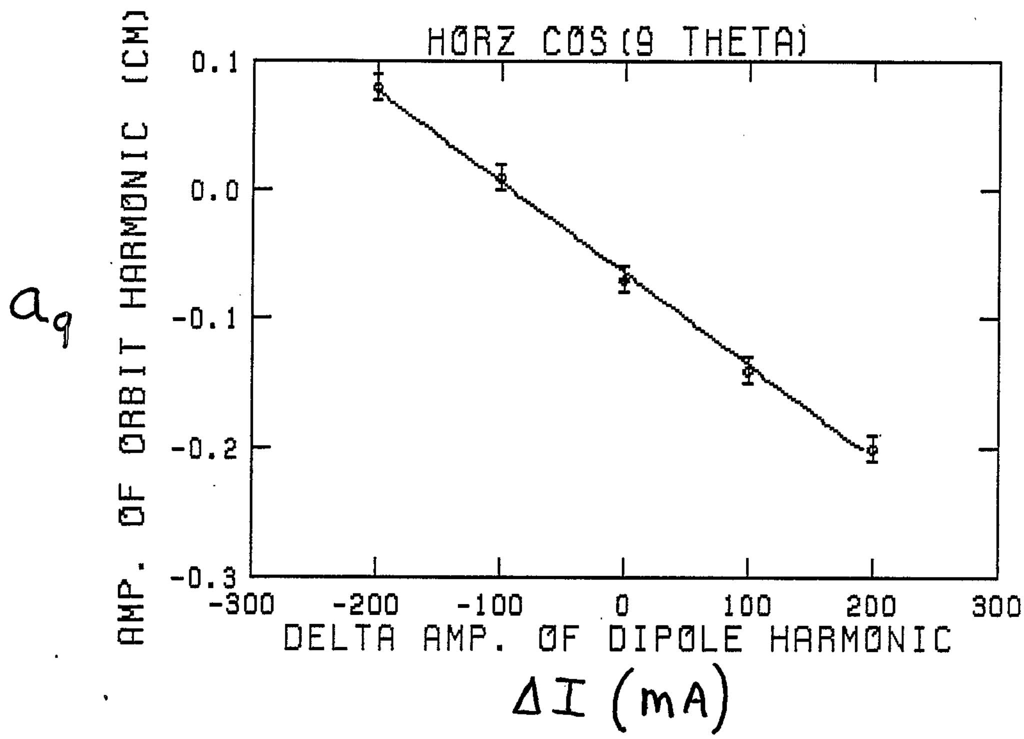

Fig. 1

SLOPE= -0.000675 +/- 0.000035
INTERCEPT= 0.036667 +/- 0.005774

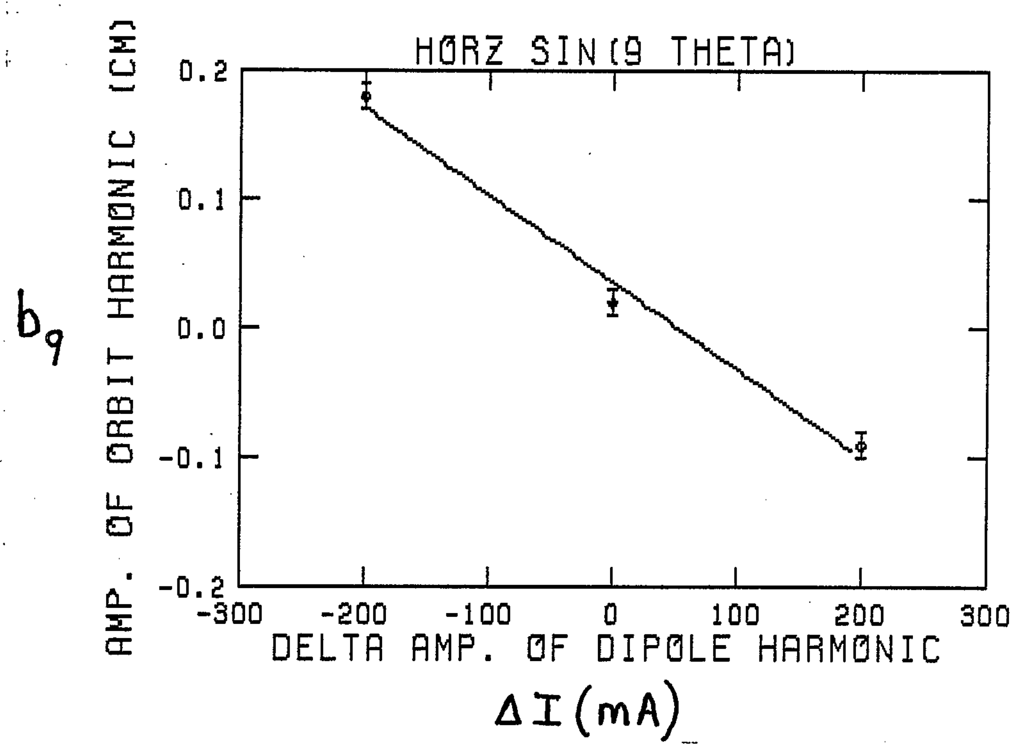

Fig. 2

SLOPE= 0.000432 +/- 0.000013
INTERCEPT= 0.236000 +/- 0.004472

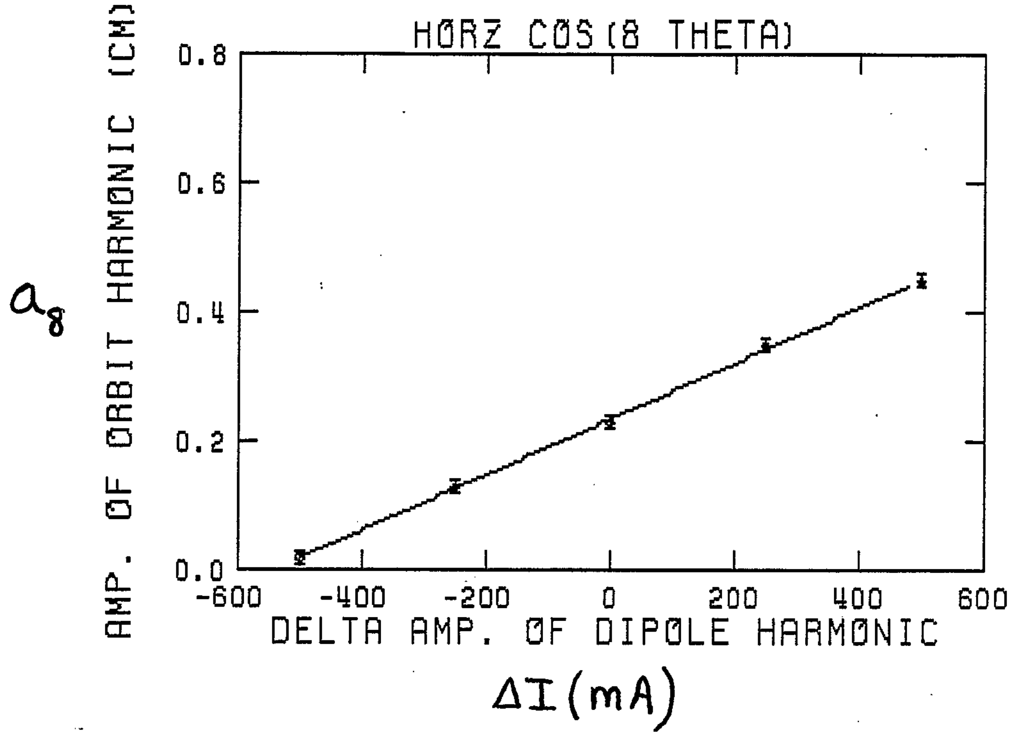

Fig. 3

SLOPE= 0.000422 +/- 0.000016
INTERCEPT= -0.030000 +/- 0.005774

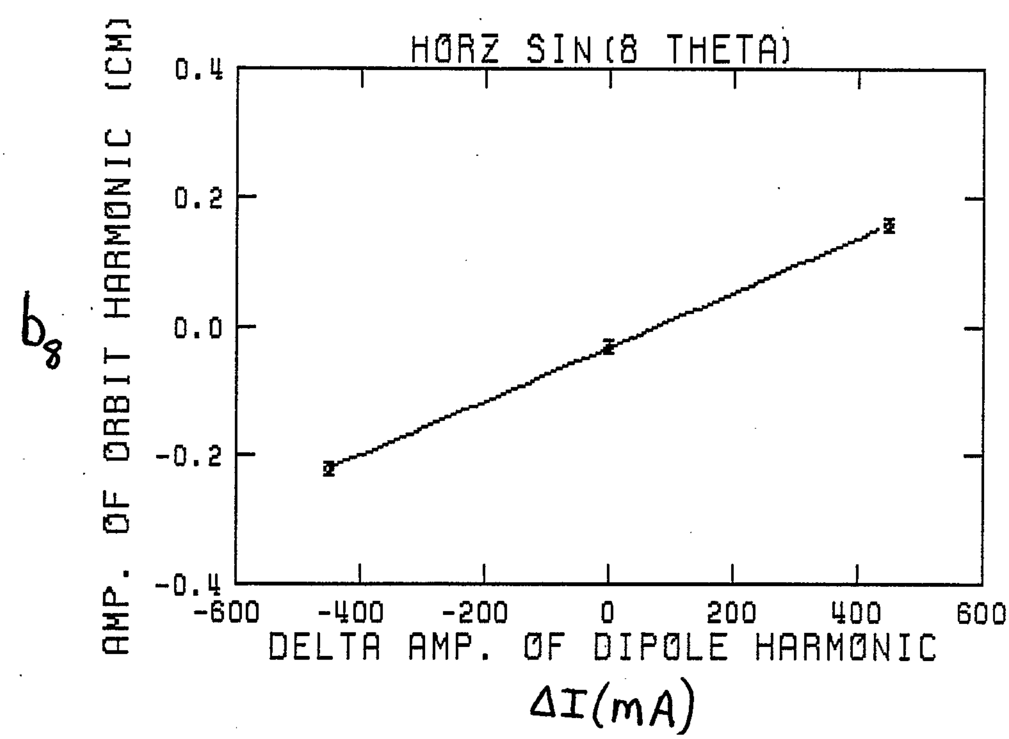

Fig. 4

SLOPE= 0.000270 +/- 0.000015
INTERCEPT= 0.093000 +/- 0.005477

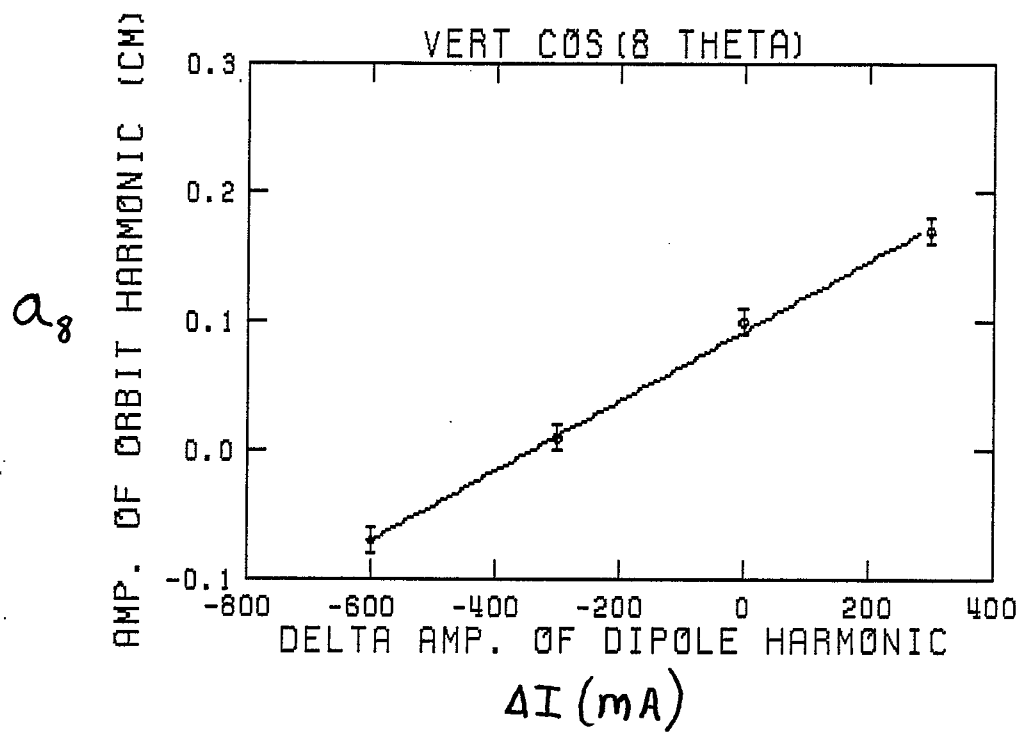

Fig. 5

SLOPE= 0.000268 +/- 0.000023
INTERCEPT= -0.002143 +/- 0.005976

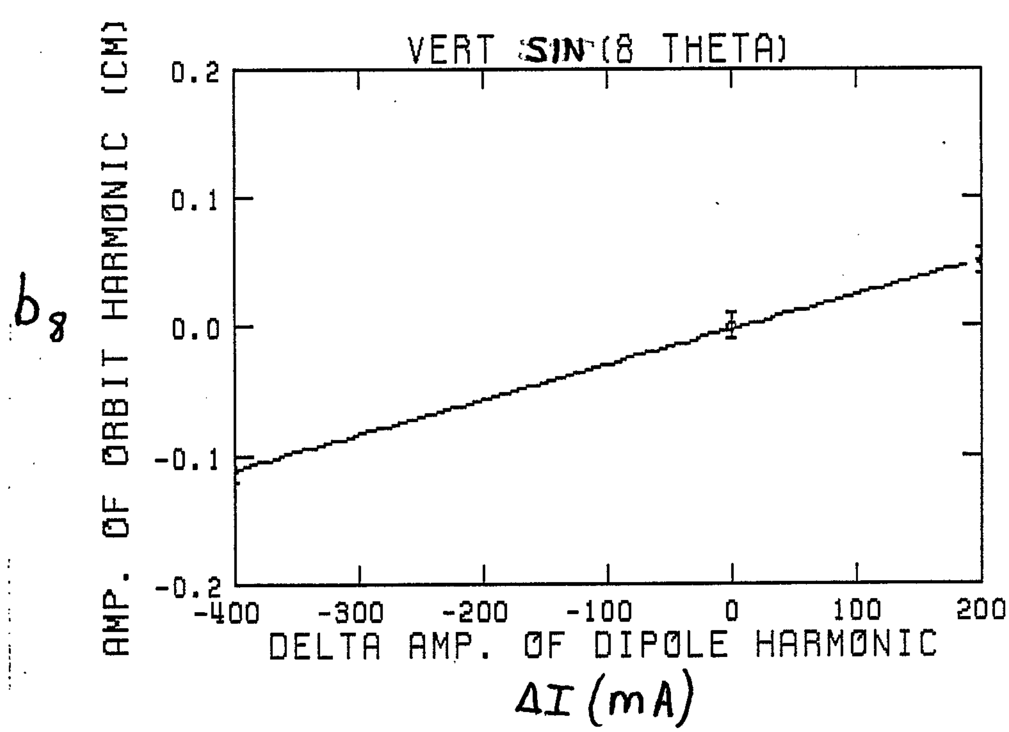

Fig. 6

References

1. C.J. Gardner, AGS/AD/Op. Note No. 17, February 4, 1988.

2. E.D. Courant and H.S. Snyder, Annals of Physics: 3, 1-48 (1958).

3. L.A. Ahrens, AGS/AD/Op. Note No. 2, March 4, 1985.