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Effect of emittance on performance of CeC scheme

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1 Introduction

In coherent electron cooling (CeC) the ions get cooled because their longitudinal positions with respect to a wake created by each ion in the electron bunch is correlated with an ion's energy.

A typical characteristic length of a CeC wake is just a few microns. Therefore, the coupling of a transverse and a longitudinal motion of both electrons and ions in a cooler must be taken into account.

Below we estimate an additional path-length's spread through the cooler, which particles have due to transverse emittance.

2 Model description

For a particle having an angle θ, a path-length delay dL (as compared to a reference particle) through a modulator and a kicker can be estimated from:

$$\frac{L}{L+dL} = \cos\theta \approx 1 - \frac{\theta^2}{2} \tag{1}$$

where L is the cooler's length. Figure 1 illustrates Eq. (1).

Hence, we get:

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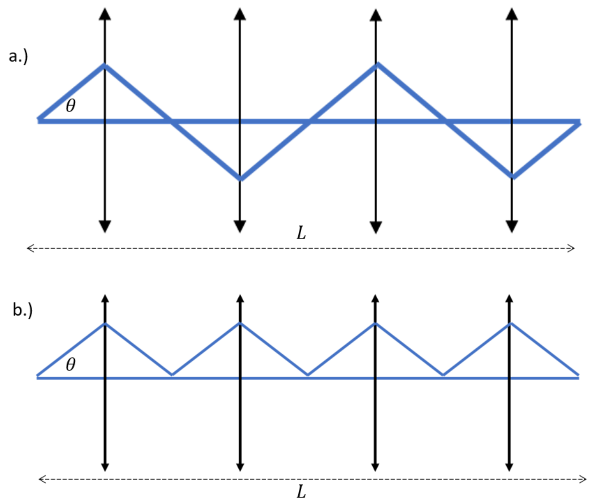

Figure 1: Paths of a test particle and a particle with $\theta \neq 0$ through the cooler for an emittance dominated case (a) and a space charge dominated case (b).

$$dL \approx \frac{L}{2}\theta^2 \tag{2}$$

The rms value for θ can be estimated as:

$$\sigma_\theta \approx \sqrt{\frac{\varepsilon}{\beta*}} \tag{3}$$

where ε is a transverse emittance of a bunch and β is the bunch's Twiss parameter.

Particles can be only delayed with respect to a test particle by the considered effect. This fact is signified by θ^2 in Eq. (2).

Let us find the first and second moments of the resulting distribution of the path-lengths. We assume that the angular spread is represented by a Gaussian distribution:

$$f_\theta = \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{-\frac{\theta}{2\sigma_\theta}} \tag{4}$$

From Eqs (2) and (4), the average path-length mismatch is:

$$\langle dL \rangle = \int_{-\infty}^{\infty} dL(\theta) f_\theta d\theta = \frac{L}{2}\sigma_\theta^2 = \frac{\varepsilon L}{2\beta*} \tag{5}$$

Similarly:

$$\langle dL^2 \rangle = \int_{-\infty}^{\infty} dL^2(\theta) f_\theta d\theta = \frac{3L^2}{4}\sigma_\theta^4 \tag{6}$$

and we obtain for the standard deviation in path-lengths distribution:

$$\sigma_L = \sqrt{\langle dL^2 \rangle - \langle dL \rangle^2} = \frac{\varepsilon L}{\sqrt{2}\beta*} \tag{7}$$

While an average path-length mismatch can be taken care of by adjusting delays between two bunches, the additional longitudinal spread will be always present and will cause a wash-out of the CeC wake.

A CeC wake (normalized by wake's amplitude) can be represented by:

$$w(z) = -\sin\left(\frac{2\pi z}{z_0}\right)\exp\left(-\frac{z^2}{\sigma_0^2}\right) \tag{8}$$

where z_0 and σ_0 are the parameters defining the wake's shape.

For a quick estimates of severity of the washout one can compare σ_L to wake's width σ_0. If σ_L and σ_0 are comparable (are of the same order of magnitude), then the emittance-driven dilution of a wake must be modeled more accurately.

A wake diluted due to the path-length's spread can be estimated as:

$$\begin{aligned} w_d(z) &= \int_{-\infty}^{\infty} w(z + dL(\theta)) f_\theta d\theta = \\ &= -\frac{1}{\sqrt{2\pi}\sigma_\theta}\int_{-\infty}^{\infty} \sin\left(\frac{2\pi(z+L\theta^2/2)}{z_0}\right)\exp\left(-\frac{(z+L\theta^2/2)^2}{\sigma_0^2} - \frac{\theta^2}{2\sigma_\theta^2}\right) d\theta \end{aligned} \tag{9}$$

When applying Eq. (9) to the case of electrons' path-length spread we are assuming the following equivalency: $\frac{d}{dz}\left[\int f_\rho(z+\zeta(\theta)) f_\theta d\theta\right] = \int \frac{df_\rho(z+\zeta(\theta))}{dz} f_\theta d\theta$, where f_ρ is electrons' density distribution producing a wake ($w = df_\rho/dz$). Application of Eq. (9) to the case of ions' path-length spread is clearly justified by treating such a spread as an additional noise.

3 Results for CeC experiment

Parameters of CeC experiment are listed in Table 1

Table 1: Parameters of CeC experiment

	2021	2023
cooler length [m]	12	
electrons geometric emittance (ε_e) [m·rad]	$10 \cdot 10^{-8}$	$5.3 \cdot 10^{-8}$
electrons minimal β-function $\beta{*}_e$ [m]	0.1	0.19
ions geometric emittance (ε_i) [m·rad]	$9 \cdot 10^{-8}$	
ions minimal β-function $\beta{*}_i$ [m]	5	
CeC wake width σ_0 [μm]	6	
CeC wake wavelength z_0 [μm]	19	

Substituting experimental parameters into Eq. (7) we see that while for gold ions σ_L is just 0.2 μm, for electrons $\sigma_L = 8.5$ μm for 2021 parameters and $\sigma_L = 2.4$ μm for 2023 parameters. Such spreads in the pathlengths are comparable to the wake's width and can cause a substantial wake dilution.

Figure 2 shows the expected wash-out of the CeC wake for 2021 and 2023 parameters. The considered effect is significant for both sets of parameters.

One interesting feature of the diluted wake is its asymmetry. This is exactly the wake's asymmetry that we observe in the 3D simulations of the CeC process.

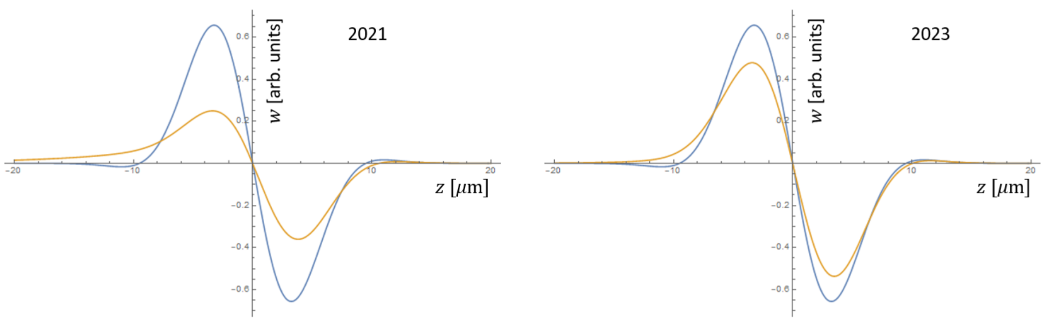

Figure 2: An ideal normalized CeC wake (blue) and a wake diluted by the emittance-driven path-length spread of electrons (orange). The offset of the diluted wake was removed to allow better comparison to the ideal wake.