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RHIC Run 22 Spin Transparency Experiment: Spin Simulations in Preparation and Support

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RHIC Run 22 Spin Transparency Experiment.
Spin Simulations in Preparation and Support

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Abstract

This Tech. Note reports in detail the numerical simulations performed in preparation and in support of RHIC Run 22 spin transparency experiment in RHIC Yellow ring. Outcomes include the 9 o'clock and 3 o'clock snake current ramps.

Contents

1 Introduction

OPERA field maps of the 4 snake modules are used in these simulations. In this representation a full snake is a sequence of 4 field maps, field maps 1 and 4 computed at low field (100 A current), 2 and 3 computed at high field (322 A), as follows [1],

sign applied	field map
$\pm$	$model3a2a - x - 4\_4\_y - 4\_4\_z - 180\_180 - integral.table$
$\mp$	$model3a2a322a - x - 4\_4\_y - 4\_4\_z - 180\_180 - integral.table$
$\pm$	$model3a2a322a - x - 4\_4\_y - 4\_4\_z - 180\_180 - integral.table$
$\mp$	$model3a2a - x - 4\_4\_y - 4\_4\_z - 180\_180 - integral.table$

These OPERA maps are archived/available on C-AD computers at

$$/rap/lattice\_tools/zgoubi/RHICZgoubiModel/snakeFieldMaps/161216\_secondSet\_inclSingleHelix$$

or in zgoubi repository at

$$https://sourceforge.net/p/zgoubi/code/HEAD/tree/branches/exemples/RHIC/snakesWithFieldMaps/fieldMaps/$$

Some parameters of RHIC snakes [1]:
A snake is a series of 4 right-handed helix modules,
a helix module is 2.4 m long,
bore 10 cm,
module are spaced 0.212/0.448/0.212 m
hence an overall length of 10.472 m.

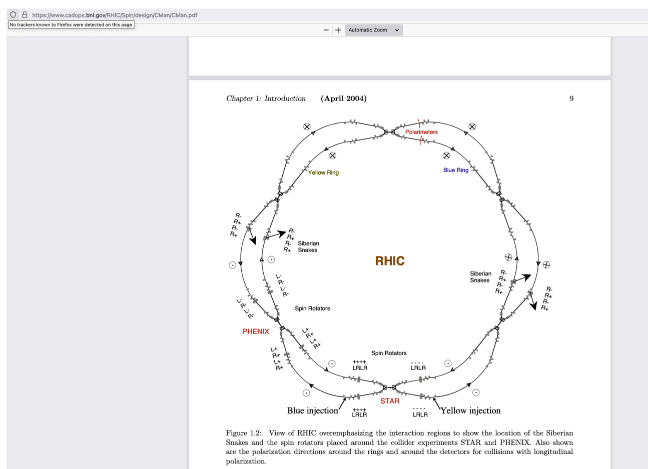


Figure 1: RHIC rings, 9 o.clock and 3 o.clock snakes in Blue and Yellow.

2 Orbits and spin across full snakes, axes at $0°$, $\pm 10°$

The left, middle and right columns in the figure below show particle and spin motion at $G\gamma = 45.5$, and field along trajectories, across 9 o'clock RHIC Yellow snake, with its axis at respectively $0°$, $+10°$ and $-10°$ to the X axis. The path is taken clockwise here (this may be different in the following, as specified in due place), hence the sequence (Fig. 1), in that order, R+R-R+R-. Orbits have been centered on the snake axis as part of the fitting procedure. A specimen data file for these simulations is given in App. A. Data files to perform these simulations can be found in zgoubi repository
https://sourceforge.net/p/zgoubi/code/HEAD/tree/branches/exemples/RHIC/spinTransparency/snakeSettings_adjustRadialPolar/

axis at 0 degree;
coil currents
164.16759 A / 220.45332 A;
Z at entrance: 0.85093724 cm

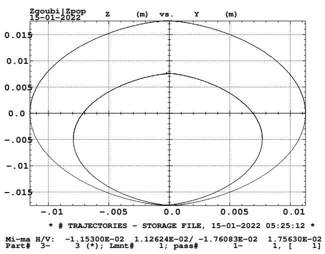


Figure 2: YZ projection of orbits.

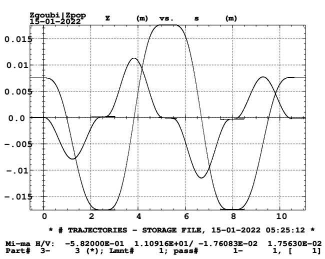


Figure 3: Y(s) and Z(s) orbits.

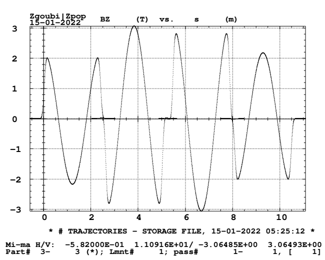


Figure 4: $B_Z(s)$ along along orbits.

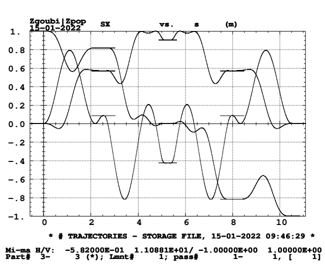


Figure 5: $S_X(s)$, $S_Y(s)$ and $S_Z(s)$. $S_Z(s=0)=1$.

axis at 10 degree;
coils currents
184.08957 A / 199.07343 A;
Z at entrance: 1.3120770 cm

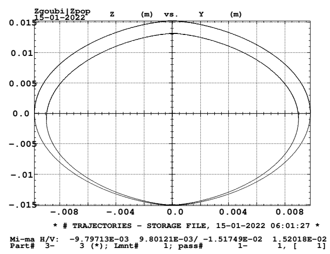


Figure 6: YZ projection of orbits.

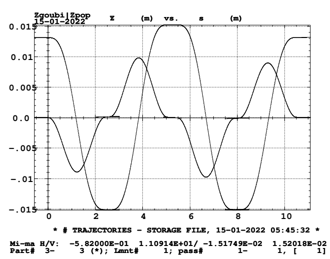


Figure 7: Y(s) and Z(s) orbits.

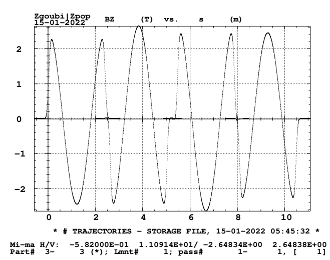


Figure 8: $B_Z(s)$ along along orbits.

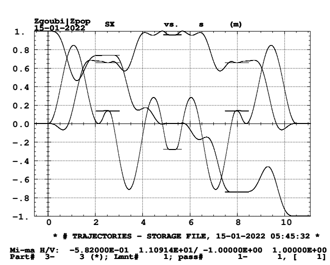


Figure 9: $S_X(s)$, $S_Y(s)$ and $S_Z(s)$. $S_Z(s=0)=1$.

axis at -10 degree;
coil currents
146.03718 A / 241.80863 A;
Z at entrance: 0.42093414 cm

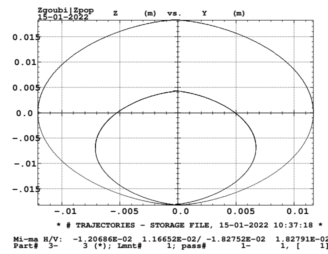


Figure 10: YZ projection of orbits.

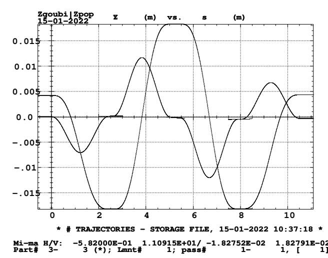


Figure 11: Y(s) and Z(s) orbits.

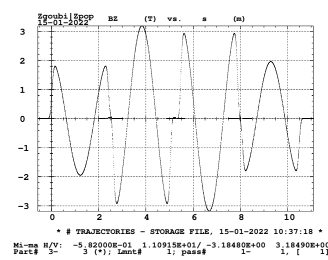


Figure 12: $B_Z(s)$ along along orbits.

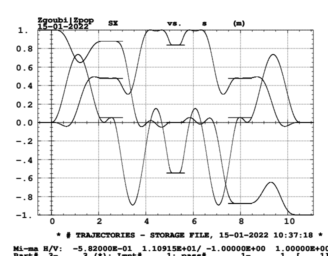


Figure 13: $S_X(s)$, $S_Y(s)$ and $S_Z(s)$. $S_Z(s=0)=1$.

For reference, regular 9 o'clock snake, axis at -45 degree at 255, 100 and 23 GeV:

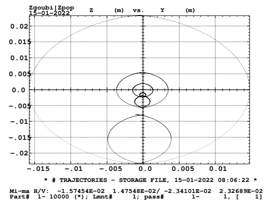


Figure 14: YZ projection of orbits.

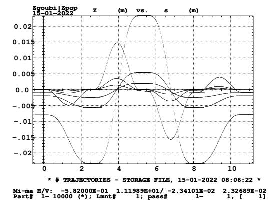


Figure 15: Y(s) and Z(s) orbits.

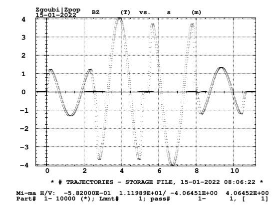


Figure 16: $B_Z(s)$ - vertical field along orbits.

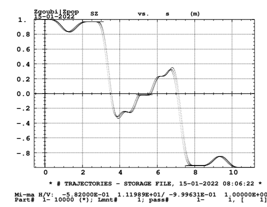


Figure 17: $S_Z(s)$.

2.1 Snake spin matrices, time of flight

In the three columns below, trajectory length (s) and time of flight (TOF) are over an optical axis straight extent of

$$10.472000\,\mathrm{m}.$$

which is the length of the snake from entrance to exit magnetic face [1, Fig. 5].

9 o'clock snake
axis at 0 degree
Injection

```
Spin transfer matrix, momentum group # 1 :

 1.00000        3.133707E-06   -8.179472E-05
 3.133669E-06   -1.00000        -4.657628E-07
-8.179472E-05    4.655065E-07    -1.00000

Determinant =       1.0000000000
Trace =      -1.0000000000;
spin precession =     179.9999732999 deg
Precession axis :  ( 1.0000, -0.0000, -0.0000)
     ->  angle to X axis :    -0.0001 deg
```

s = 1.047347E+01 m;
TOF = 3.49628868E-02 μs.

9 o'clock snake
axis at 10 degree
Injection

```
Spin transfer matrix, momentum group # 1 :

 0.939691        0.342026       -1.033279E-04
 0.342026       -0.939691       -1.833964E-05
-1.033689E-04   -1.810721E-05    -1.00000

Determinant =       1.0000000000
Trace =      -1.0000000000;
spin precession =     179.9998995155 deg
Precession axis :  ( 0.9848,  0.1736, -0.0001)
     ->  angle to X axis : 10.0000 deg
```

s = 1.047344E+01 m
TOF = 3.49627763E-02 μs.

9 o'clock snake
axis at -10 degree
Injection

```
Spin transfer matrix, momentum group # 1 :

 0.939703       -0.341991       -4.463609E-05
-0.341991       -0.939703        7.654557E-06
-4.456247E-05    8.072114E-06    -1.00000

Determinant =       1.0000000000
Trace =      -1.0000000000;
spin precession =     179.9998957697 deg
Precession axis :  ( 0.9848, -0.1736, -0.0000)
     ->  angle to X axis :  -10.0000 deg
```

s = 1.047354E+01 m
TOF = 3.49631135E-02 μs.

For comparison, regular 9 o'clock snake, axis at -45 degree

255 GeV:

```
Spin transfer matrix, momentum group # 1 :

 3.214335E-02   -0.999077       -2.848681E-02
-0.999072       -3.129976E-02   -2.958057E-02
 2.866164E-02    2.941120E-02   -0.999156

Determinant =       1.0000000000
Trace =      -0.9983127948;
spin precession =     177.6463758024 deg
Precession axis :  ( 0.7182, -0.6958,  0.0001)
   ->  angle to X axis :     -44.0907 deg
```

s = 1.047202E+01 m;
TOF = 3.49311247E-02 μs;

100 GeV:

```
Spin transfer matrix, momentum group # 2 :

 3.180516E-02   -0.999085       -2.859672E-02
-0.999080       -3.095522E-02   -2.968848E-02
 2.877609E-02    2.951465E-02   -0.999150

Determinant =       1.0000000000
Trace =      -0.9983001153;
spin precession =     177.6375472367 deg
Precession axis :  ( 0.7181, -0.6959,  0.0001)
    ->  angle to X axis:  -44.1005 deg
```

s = 1.047211E+01 m;
TOF = 3.49327140E-02 μs;

Injection:

```
Spin transfer matrix, momentum group # 3 :

 2.452685E-02   -0.999220       -3.094373E-02
-0.999210       -2.353451E-02   -3.203606E-02
 3.128283E-02    3.170501E-02   -0.999008

 Determinant =       1.0000000000
 Trace =      -0.9980152480;
 spin precession  =     177.4472300590 deg
 Precession axis :  ( 0.7156, -0.6986,  0.0001)
     ->  angle to X axis :   -44.3112 deg
```

s = 1.047402E+01 m;
TOF = 3.49647201E-02 μs

2.2 APEX data

Working hypotheses, from Vahid:

> So this is what I have for the Experiment:
> First we set Yellow 9 oclock snake to :
>
> ```
> Iout  Iin   MU  PHI
>
> 184.0 197.0 179.975 10.18
> ```
>
> And Yellow 3 oclock snake to:
>
> ```
> Iout  Iin   MU  PHI
>
> 164.0 219.0 179.668 0.026
> ```
>
> With 9 oclock fixed we perform the first ramp on Yellow Snake at 3 oclock using:
>
> ```
> Ramp1
> Iout  Iin   MU  PHI
> 164.0 219.0 179.668 0.026
> 163.0 218.0 178.941 179.985
> 162.0 217.0 177.549 179.945
> 161.0 215.0 175.459 0.115
> 160.0 214.0 174.057 0.072
> 159.0 213.0 172.653 0.029
> 158.0 212.0 171.248 179.986
> 157.0 211.0 169.841 179.943
> 156.0 209.0 167.75 0.109
> 155.0 208.0 166.339 0.064
> 154.0 207.0 164.926 0.02
> ```
>
> Next we ramp only the 9 oclock snake using:
>
> ```
> Ramp2
> Iout  Iin   MU  PHI
> 184.0 197.0 179.975 10.18
> 181.0 198.0 178.553 9.101
> 178.0 199.0 177.045 8.027
> 175.0 200.0 175.533 6.97
> 172.0 201.0 174.019 5.929
> 169.0 202.0 172.438 4.923
> 166.0 203.0 170.933 3.914
> 163.0 204.0 169.433 2.921
> 160.0 205.0 167.905 1.933
> 157.0 206.0 166.411 0.969
> 154.0 207.0 164.926 0.02
> ```
>
> Since we have been ramping these snakes a bit lately I dont think we need a special ramp. But we need to involve Vincent or Chuyu and give him and ops a heads up. What is missing how fast you need to ramp this to get good data from the polarimeter.

Simulated snake data:

Beam goes CCW in Yellow, so the snake module current signs are (Fig. 1), 9oclock: - + - +, 3oclock: + - + -; this determines the sign of the vertical orbit along the snake.

9 o'clock Snake:

```
'SCALING'
1  2
TOSCA    snk1LowB
-1
184.47838    !  low-field coils current (A)
1
TOSCA    snk1HighB
-1
198.70954    !  high-field coils current (A)
1
```

```
                    Spin transfer matrix, momentum group # 1 :

         0.937525       -0.347918       -1.392460E-04
        -0.347918       -0.937525       -4.184721E-04
         1.504733E-05    4.407743E-04    -1.00000

Determinant =       1.0000000000
Trace = -0.9999998095;   spin precession acos((trace-1)/2) =     179.9749904825 deg
Precession axis: (0.9843, -0.1767, -0.0000)
             -> angle to (X,Y) plane, to X axis : -0.0018, -10.1800 deg
Spin precession/2pi (or Qs, fractional) :    4.9993E-01
```

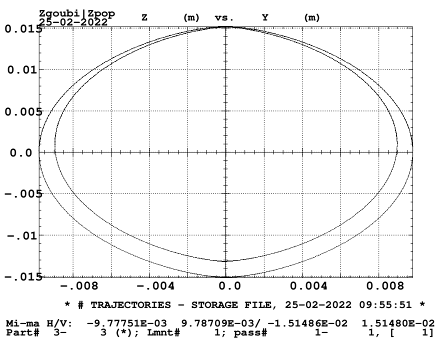


Figure 18: YZ projection of orbits.

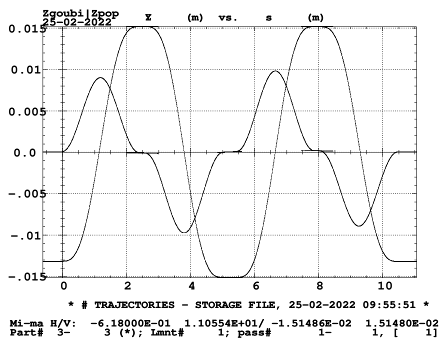


Figure 19: Y(s) and Z(s) orbits.

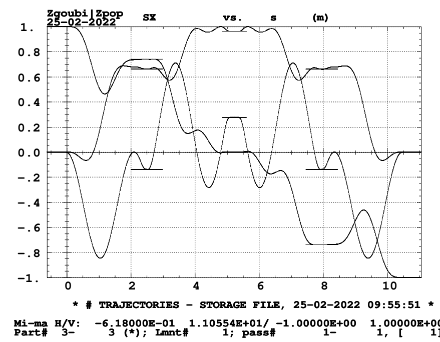


Figure 20: Spin components.

3 o'clock Snake:

```
'SCALING'
1  2
TOSCA    snk2LowB
-1
164.43807    !  low-field coils current (A)
1
TOSCA    snk2HighB
-1
220.65730    !  high-field coils current (A)
1
```

```
                    Spin transfer matrix, momentum group # 1 :

         1.00000         9.053152E-04   -7.919528E-05
         9.048411E-04   -0.999983       -5.793846E-03
        -8.443917E-05    5.793772E-03   -0.999983

Determinant =       1.0000000000
Trace = -0.9999664315;   spin precession acos((trace-1)/2) =     179.6680372920 deg
Precession axis: (1.0000,  0.0005, -0.0000)
             -> angle to (X,Y) plane, to X axis : -0.0023,   0.0259 deg
     Spin precession/2pi (or Qs, fractional) :    4.9908E-01
```

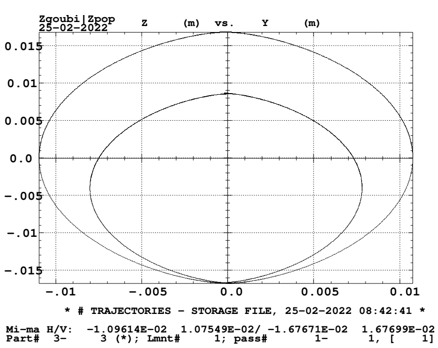


Figure 21: YZ projection of orbits.

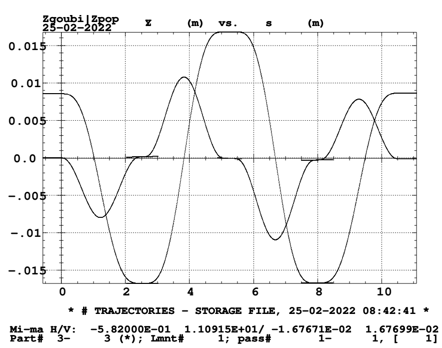


Figure 22: Y(s) and Z(s) orbits.

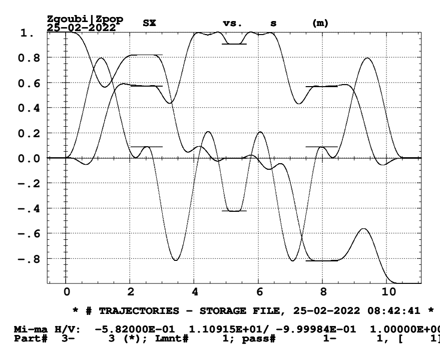


Figure 23: Spin components.

2.3 Coil current scan

2.3.1 Vertical to radial polarization

Goal angles are in the table below. Coil currents for that are determined.
Snakes are considered CCW, 3OClock series is R+R-R+R-, all right-helicity, 9OClock series is R-R+R-R+. In particular, coil polarities for this spin transparency experiment are the same as in regular snake operation, see Sec. 3.1 for details.
All files for that simulation can be found in the dedicated RHIC repository

https://sourceforge.net/p/zgoubi/code/HEAD/tree/branches/exemples/RHIC/spinTransparency/snakeSettings_adjustRadialPolar/

	9 o' snake, CCW				3 o' snake, CCW			
Step #	μ	ϕ	$I_{\rm out}$	$I_{\rm in}$	μ	ϕ	$I_{\rm out}$	$I_{\rm in}$
0	180	170	184.08966	199.07334	180	0	164.16759	220.45335
1	180	170	184.08967	199.07334	177	0	166.17474	222.79739
2	180	170	184.08966	199.07336	174	0	168.19734	225.14344
3	180	170	184.08966	199.07335	171	0	170.23614	227.49256
4	180	170	184.08967	199.07334	168	0	172.29233	229.84602
5	180	170	184.08966	199.07334	165	0	174.36709	232.20533
6	177	172	178.43276	200.72111	165	0	174.36707	232.20531
7	174	174	172.63753	202.52287	165	0	174.36709	232.20531
8	171	176	166.69559	204.46703	165	0	174.36712	232.20540
9	168	178	160.59563	206.54226	165	0	174.36710	232.20535
10	165	180	154.32192	208.73700	165	0	174.36709	232.20534

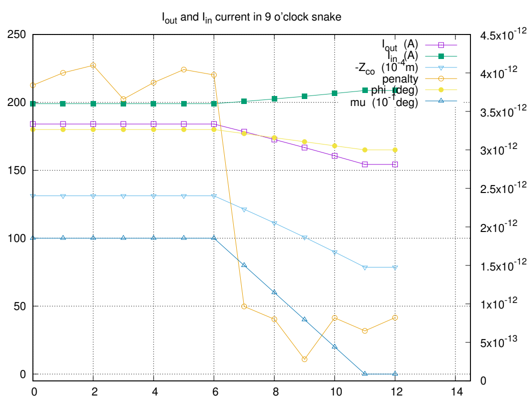


Figure 24: Case of 9 o'clock snake, step 0 (repeated) to 11: coil currents, snake angles, closed orbit coordinate at snake entrance, and (right vertical axis) FIT penalty

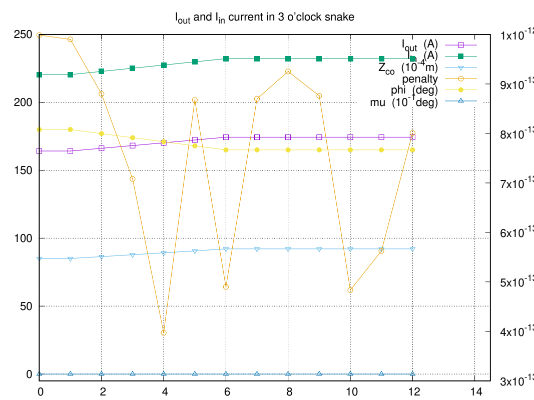


Figure 25: Case of 3 o'clock snake, step 0 (repeated) to 11: coil currents, snake angles, closed orbit coordinate at snake entrance, and (right vertical axis) FIT penalty

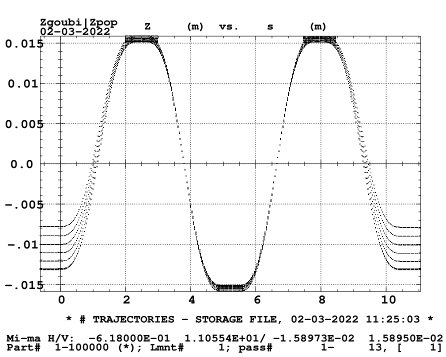


Figure 26: Vertical closed orbits over snake 9'o current scan full range.

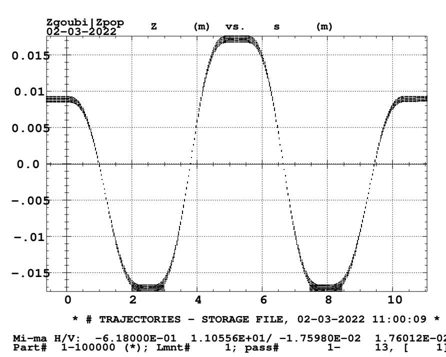


Figure 27: Vertical closed orbits over snake 3'o current scan full range.

2.3.2 Reversal of vertical polarization

Goal angles are in the table below. Coil currents for that are determined. Snakes are considered CCW, *i.e.* Yellow case.

All files for that simulation can be found in the dedicated RHIC repository
https://sourceforge.net/p/zgoubi/code/HEAD/tree/branches/exemples/RHIC/spinTransparency/snakeSettings_reversalVerticalPolar/

	9 o' snake, CCW				3 o' snake, CCW			
Step #	μ	ϕ	I_{out}	I_{in}	μ	ϕ	I_{out}	I_{in}
0	180	-10	184.08967	199.07333	180	0	164.16759	220.45335
1	180	-10	184.08967	199.07334	177.5	0	165.83915	222.40659
2	180	-10	184.08966	199.07335	175	0	167.52137	224.36109
3	180	-5	173.87827	209.78901	175	0	167.52137	224.36111
4	180	-2	167.99538	216.19137	175	0	167.52137	224.36111
5	180	0	164.16805	220.45281	175	0	167.52134	224.36106
6	180	2	160.41133	224.71226	175	0	167.52141	224.36113
7	180	5	154.90344	231.10352	175	0	167.52137	224.36111
8	180	10	146.03570	241.81008	175	0	167.52135	224.36106
9	180	10	146.03569	241.81009	177.5	0	165.83916	222.40660
10	180	10	146.03571	241.81005	180	0	164.16758	220.45335

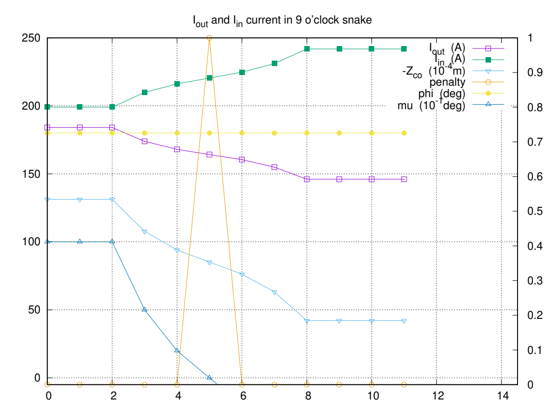


Figure 28: Case of 9 o'clock snake, step 0 to 11: coil currents, snake angles, closed orbit coordinate at snake entrance, and (right vertical axis) FIT penalty - note convergence problem at step 5 (m=180 / phi=0), reason TBFO

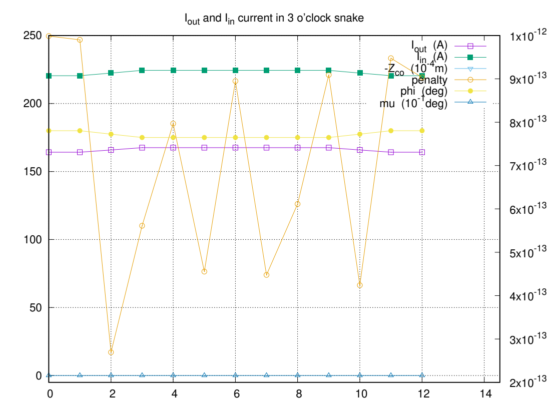


Figure 29: Case of 3 o'clock snake, step 0 to 11: coil currents, snake angles, closed orbit coordinate at snake entrance, and (right vertical axis) FIT penalty

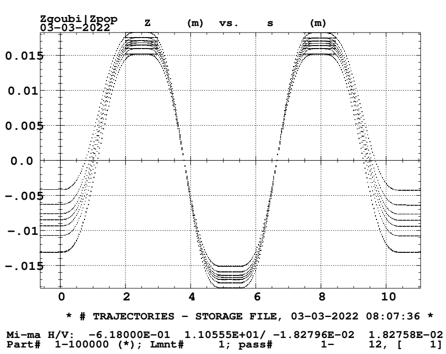


Figure 30: Vertical closed orbits over snake 9'o current scan full range. The orbit signs as simulated here (negative at snake ends in particular) concord with orbit sign in RHIC controls.

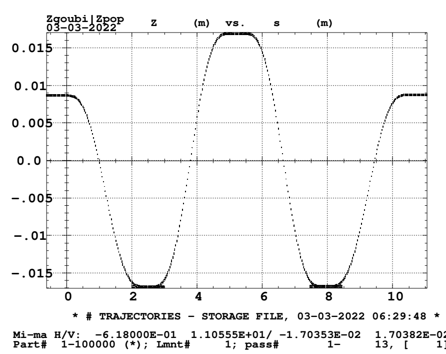


Figure 31: Vertical closed orbits over snake 3'o current scan full range. The orbit signs as simulated here (positive at snake ends in particular) concord with orbit sign in RHIC controls.

2.4 Coil current scan for constant spin tune

2.4.1 Vertical to radial polarization

Goal angles are in the table below. Coil currents for that are determined.
Snakes are considered CCW, 3OClock series is R+R-R+R-, all right-helicity, 9OClock series is R-R+R-R+. In particular, coil polarities for this spin transparency experiment are the same as in regular snake operation, see Sec. 3.1 for details.
The two files for that simulation, namely

snake9OCCW_scanMuPhi_adjRadPol_finerQsp.dat
snake3OCCW_scanMuPhi_adjRadPol_finerQsp.dat

can be found in the dedicated RHIC repository

https://sourceforge.net/p/zgoubi/code/HEAD/tree/branches/exemples/RHIC/spinTransparency/snakeSettings_adjustRadialPolar/

	9 o' snake, CCW				3 o' snake, CCW			
Step #	μ	ϕ	I_{out}	I_{in}	μ	ϕ	I_{out}	I_{in}
0.00	180.00	-10.0	184.0896	199.0735	180.00	0.	164.1676	220.4534
1.00	186.16	-9.00	179.0589	195.6699	173.84	0.	168.3086	225.2721
2.00	188.49	-8.00	175.7351	195.8453	171.51	0.	169.8851	227.0893
3.00	190.10	-7.00	172.7017	196.7030	169.90	0.	170.9876	228.3545
4.00	191.31	-6.00	169.8350	197.9463	168.69	0.	171.8204	229.3072
5.00	192.25	-5.00	167.0922	199.4548	167.75	0.	172.4628	230.0404
6.00	192.96	-4.00	164.4548	201.1643	167.04	0.	172.9552	230.6015
7.00	193.49	-3.00	161.9151	203.0357	166.51	0.	173.3209	231.0176
8.00	193.86	-2.00	159.4709	205.0437	166.14	0.	173.5739	231.3052
9.00	194.07	-1.00	157.1237	207.1716	165.93	0.	173.7227	231.4742
10.0	194.14	0.00	154.8777	209.4080	165.86	0.	173.7719	231.5299

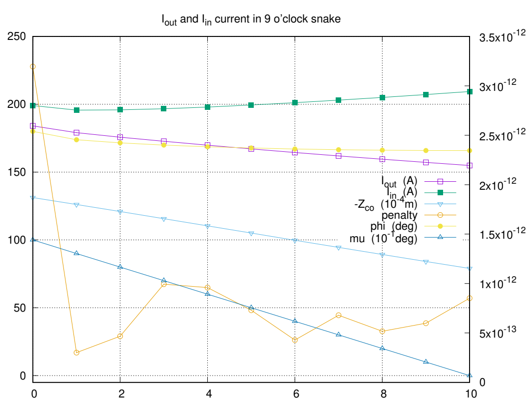


Figure 32: Case of 9 o'clock snake, step 0 (repeated) to 10: coil currents, snake angles, closed orbit coordinate at snake entrance, and (right vertical axis) FIT penalty

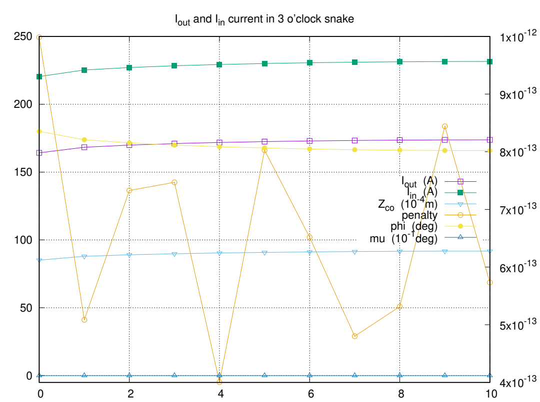
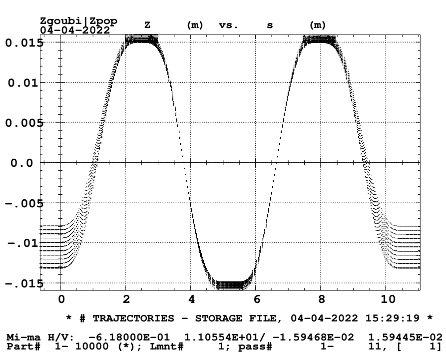
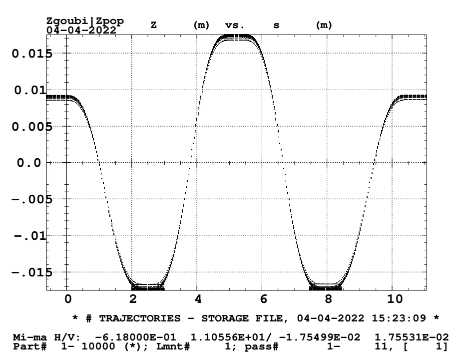


Figure 33: Case of 3 o'clock snake, step 0 (repeated) to 10: coil currents, snake angles, closed orbit coordinate at snake entrance, and (right vertical axis) FIT penalty





2.4.2 Reversal of vertical polarization

Goal angles are in the table below. Coil currents for that are determined. Snakes are considered CCW, *i.e.* Yellow case.

All files for that simulation can be found in the dedicated RHIC repository https://sourceforge.net/p/zgoubi/code/HEAD/tree/branches/exemples/RHIC/spinTransparency/snakeSettings_reversalVerticalPolar/

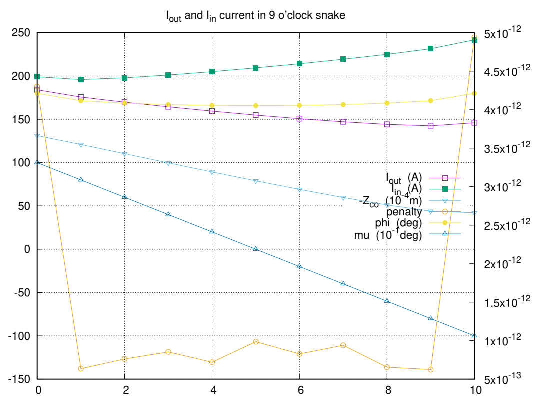


Figure 36: Case of 9 o'clock snake, step 0 to 10: coil currents, snake angles, closed orbit coordinate at snake entrance, and (right vertical axis) FIT penalty - note convergence problem at step 5 (m=180 / phi=0), reason TBFO

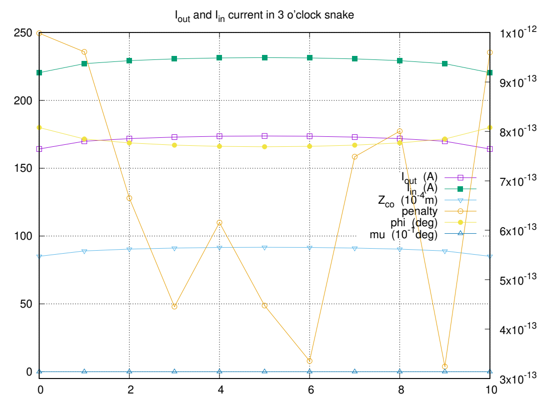


Figure 37: Case of 3 o'clock snake, step 0 to 10: coil currents, snake angles, closed orbit coordinate at snake entrance, and (right vertical axis) FIT penalty

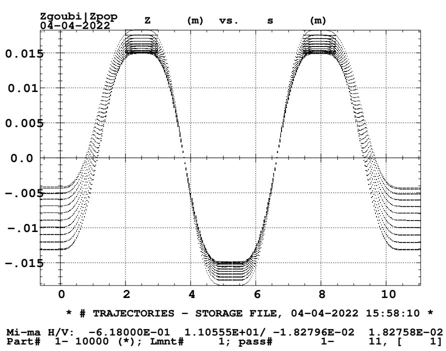


Figure 38: Vertical closed orbits over snake 9'o current scan full range. The orbit signs as simulated here (negative at snake ends in particular) concord with orbit sign in RHIC controls.

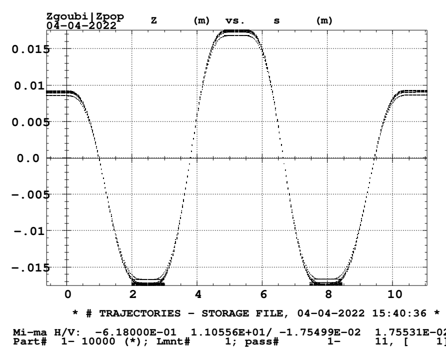


Figure 39: Vertical closed orbits over snake 3'o current scan full range. The orbit signs as simulated here (positive at snake ends in particular) concord with orbit sign in RHIC controls.

3 In RHIC Yellow CCW at Injection

First (Sec. 3.1) it is shown that snake current signs in these spin transparency simulations are the same as in regular RHIC Yellow operation simulations. Which means that in real life, snake polarities are unchanged, they are the same for normal operation and for this spin transparency experiment, no need to reverse connection polarities.

3.1 Preliminary: check ring optics and regular spin $\vec{n}_0$

CCW hypotheses, see Fig. 1:
- origin is at IP6,
- going CCW 3 o'clock snake is first met, then 9 o'clock snake.
- 3 o'clock snake module series is R+R-R+R-, all right-helicity,
- 9 o'clock snake module series is R-R+R-R+, all right-helicity.

By convention R+ (resp. R-) means positive (resp. negative) coil PS feed. This is the convention used as well in manipulating the OPERA field maps: referring to Tab. 1, an R+ module OPERA field is left as is, sign-wise (BNORM=+1), whereas an R- module OPERA field is assigned a negative sign (BNORM=-1), yielding input sequences given in Tab. 1. The actual field sign accounts for a scaling factor, namely the absolute value of the coil current, which is given in the SCALING command (Tab. 1).

Table 1: SCALING factors and field signs in OPERA field map modules in these simulations.

• SCALING factors (absolute value of the snake coil currents):

```
'SCALING'                                                                           4
1  4
TOSCA    snk1LowB
100.         !  low-field coils current (A)
TOSCA    snk1HighB
322.         !  high-field coils current (A)
TOSCA    snk2LowB
100.         !  low-field coils current (A)
TOSCA    snk2HighB
322.         !  high-field coils current (A)
```

• 3 o'clock snake, R+R-R+R- series input data:

```
'TOSCA'      snk2LowB
1.  1.  1.  1.                      ! BNORM= +1
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table

'TOSCA'      snk2HighB
 -1.  1.  1.  1.                      ! BNORM= -1
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table

'TOSCA'      snk2HighB
1.  1.  1.  1.                      ! BNORM= +1
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table

'TOSCA'      snk2LowB
 -1.  1.  1.  1.                      ! BNORM= -1
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table
```

• 9 o'clock snake, R-R+R-R+ series input data:

```
'TOSCA'      snk2LowB       44
-1.  1.  1.  1.                     ! BNORM= -1
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table

'TOSCA'      snk2HighB      46
 1.  1.  1.  1.                     ! BNORM= +1
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table

'TOSCA'      snk2HighB      49
 -1.  1.  1.  1.                      ! BNORM= -1
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table

'TOSCA'      snk2LowB       51
 1.  1.  1.  1.                     ! BNORM= +1
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table
```

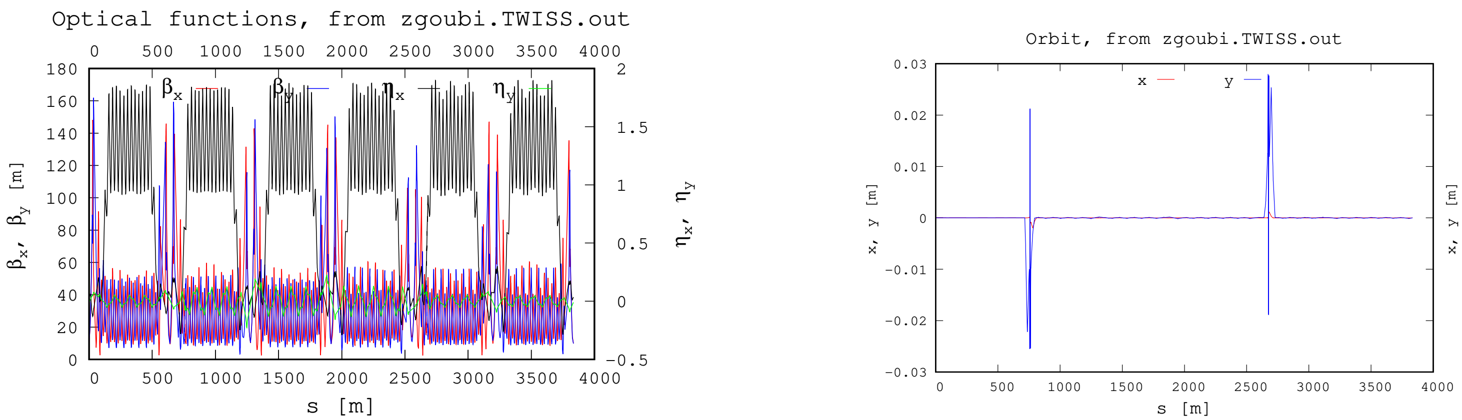


Figure 40: Optics and orbits, $G\gamma = 45.5$. Origin s=0 is at IP6, increasing s is CCW.

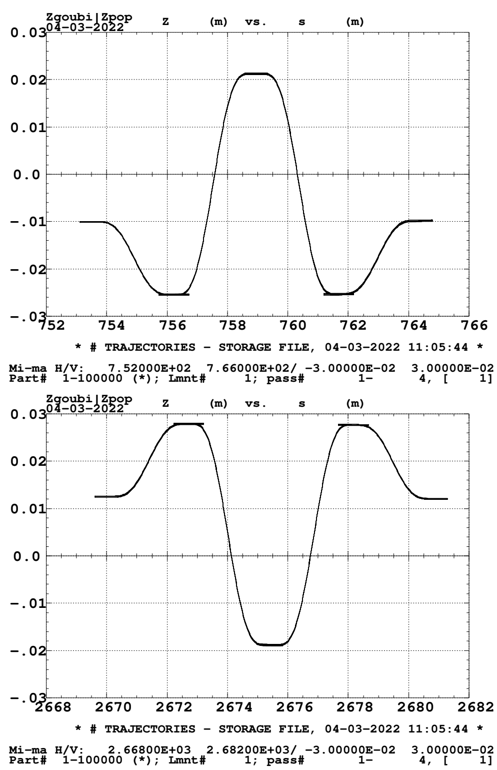


Figure 41: Orbits along 3OClock (top) and 9OClock (bottom) snakes at $G\gamma = 45.5$ in normal polarized pp RHIC operation, *i.e.*, $I_{\text{out}} = 100\,\text{A}$, $I_{\text{in}} = 322\,\text{A}$ (incidentally, orbit centering is a little loose; spin-wise this does not matter).

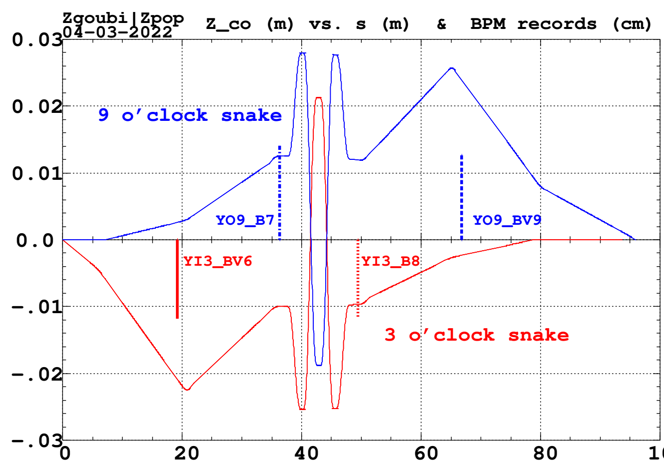


Figure 42: Closed orbit bumps at snakes, normal polarized pp operation, from simulations and, superimposed, BPM records that confirm that snake polarities in these simulations and in RHIC operation are the same. Note that BPM records at the center of the snakes ("7.1 BPM"s) are not shown here, however, consistently with simulations, they measure similar orbit amplitude with opposite sign to BPMs to the left and right.

• Beam matrix at IP6, tunes, chroms (QF, QD, SXF, SXD usually need be tweaked at the level of respectively 0.01% and 1% so to restore original MAD model values). Note the substantial coupling introduced by the vertical snake bumps, the coupling strength is greater than the the distance between the unperturbed tunes ($\nu_Y = 0.69698743$, $\nu_Z = 0.68998100$), this precludes reaching the exact original MAD values.

```
 Reference particle (#     1), path length :   383384.96     cm  relative momentum :    1.00000

                TRANSFER  MATRIX  ORDRE  1  (MKSA units)
        -0.181574        -9.39891       -2.549241E-02   -0.212229         0.00000        6.801730E-02
         9.708524E-02   -0.471680       -9.249190E-05   -3.898306E-02     0.00000        8.188140E-04
        -2.690189E-02    0.222204       -0.171039        -9.05345         0.00000        1.034660E-02
        -4.449469E-03   -8.734771E-03    9.972439E-02   -0.565072         0.00000        4.890427E-03
        -6.944800E-03    2.594781E-02   -2.297087E-03   -3.714435E-02     1.00000         7.34042
          0.00000         0.00000         0.00000         0.00000         0.00000         1.00000

         DetY-1 =      -0.0018595763,    DetZ-1 =      -0.0005013863

         R12=0 at   -19.93     m,        R34=0 at   -16.02     m

     First order symplectic conditions (expected values = 0) :
       -6.3590E-04    4.7276E-04    2.6011E-03    6.1653E-02    9.5208E-04   -9.5103E-03


               TWISS  parameters,  periodicity  of   1  is  assumed
                                  -  COUPLED  -

      Beam  matrix  (beta/-alpha/-alpha/gamma) and  periodic  dispersion  (MKSA units)

           9.880264     0.155865     0.000000     0.000000     0.000000     0.035036
           0.155865     0.103671     0.000000     0.000000     0.000000     0.002805
           0.000000     0.000000     9.799700     0.213729     0.000000    -0.009851
           0.000000     0.000000     0.213729     0.106705     0.000000     0.002382
           0.000000     0.000000     0.000000     0.000000     0.000000     0.000000
           0.000000     0.000000     0.000000     0.000000     0.000000     0.000000

                         Betatron  tunes  (Q1 Q2 modes)
                 NU_Y =  0.70008816          NU_Z =  0.68688027
                                                                      MODE 1               MODE 2
     FRACTIONAL PART OF THE BETATRON TUNES IN THE DECOUPLED FRAME:       0.70008816           0.68688027

     UNPERTURBED HORIZONTAL AND VERTICAL TUNES:                          0.69698743           0.68998100

     HAMILTONIAN PERTURBATION PARAMETERS:
                          - COUPLING STRENGTH OF THE DIFFERENCE LINEAR RESONANCE:      0.01119636
                          - DISTANCE FROM THE NEAREST SUM LINEAR RESONANCE:            0.38696843

                                  Transition gamma  =  2.28540662E+01

                                  Chromaticities :
                 dNu_y / dp/p =   1.0975937                  dNu_z / dp/p =   2.2153153
```

3.2 Coil current scan

3.2.1 Vertical to radial polarization: ring data

Direct triedra (X,Y,Z): long. (s increases going CCW), radial (Y points towards center of Yellow), vertical points up. Stable spin precession direction in Figs. 43, 44 is obtained from 1-turn spin matrix.

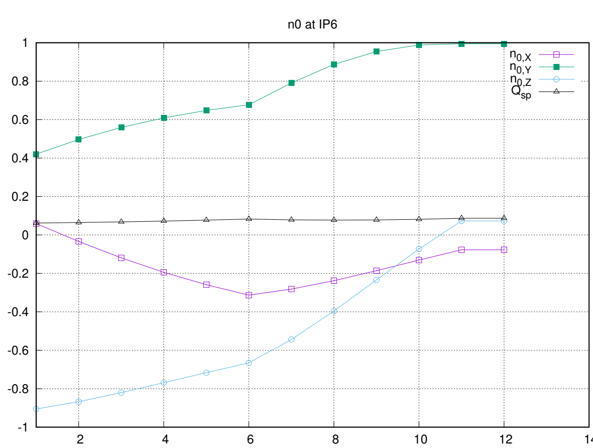


Figure 43: Components of stable spin precession direction at IP6, and spin tune.

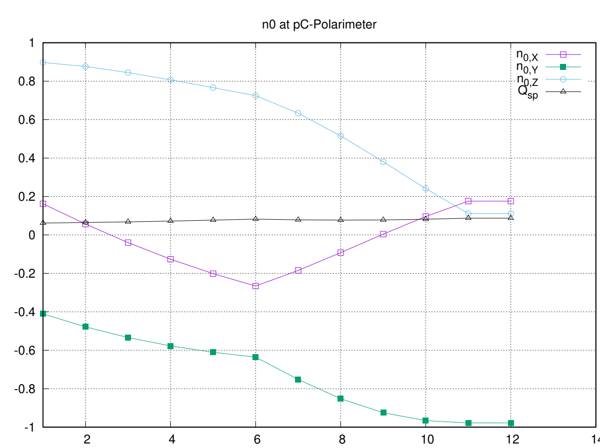


Figure 44: Components of stable spin precession direction at pC-polarimeter.

Table 2: Spin tune, and $\vec{n}_0$ vectors at STAR and at pC-polarimeter.

step #	$\nu_{\rm sp}$	STAR			pC-polarimeter		
		$n_{0,X}$	$n_{0,Y}$	$n_{0,Z}$	$n_{0,X}$	$n_{0,Y}$	$n_{0,Z}$
0	0.06214	-5.93769E-02	-0.42047	0.90535	0.16196	-0.41048	0.89737
1	0.06462	3.41321E-02	-0.49669	0.86724	5.70525E-02	-0.47834	0.87631
2	0.06803	0.11922	-0.55944	0.82024	-3.97697E-02	-0.53371	0.84473
3	0.07223	0.19409	-0.60933	0.76879	-0.12603	-0.57722	0.80681
4	0.07710	0.25857	-0.64798	0.71642	-0.20122	-0.61043	0.76608
5	0.08252	0.31339	-0.67741	0.66548	-0.26579	-0.63535	0.72503
6	0.07858	0.28135	-0.79081	0.54354	-0.18461	-0.75165	0.63318
7	0.07700	0.23797	-0.88697	0.39578	-9.20225E-02	-0.85199	0.51539
8	0.07792	0.17404	-0.97216	0.26762	4.43482E-03	-0.92483	0.38034
9	0.08127	0.13066	-0.98872	7.30632E-02	9.58899E-02	-0.96566	0.24149
10	0.08678	7.71128E-02	-0.99432	-7.30264E-02	0.17598	-0.97807	0.11131

3.2.2 Reversal of vertical polarization: ring data

Direct triedra (X,Y,Z): long. (s increases going CCW), radial (Y points towards center of Yellow), vertical points up. Stable spin precession direction in Figs. 45, 46 is obtained from 1-turn spin matrix.

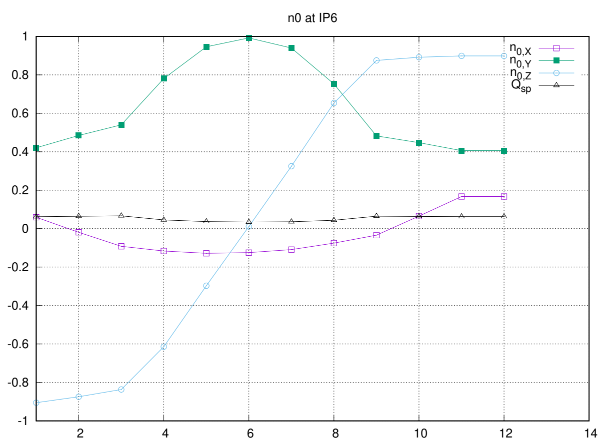


Figure 45: Components of stable spin precession direction at IP6, and spin tune.

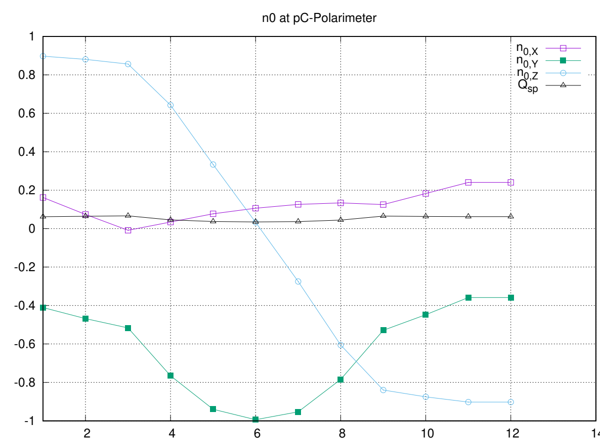


Figure 46: Components of stable spin precession direction at pC-polarimeter.

Table 3: Spin tune, and $\vec{n}_0$ vectors at STAR and at pC-polarimeter.

step #	$\nu_{\rm sp}$	STAR			pC-polarimeter		
		$n_{0,X}$	$n_{0,Y}$	$n_{0,Z}$	$n_{0,X}$	$n_{0,Y}$	$n_{0,Z}$
0	0.06207	0.0594	0.4205	-0.9054	0.1619	-0.4105	0.8974
1	0.06407	-0.0341	0.4967	-0.8673	0.0740	-0.4679	0.8807
2	0.06672	-0.1192	0.5594	-0.8203	-0.0086	-0.5166	0.8562
3	0.04535	-0.1941	0.6093	-0.7688	0.0343	-0.7655	0.6425
4	0.03703	-0.2586	0.6480	-0.7164	0.0770	-0.9396	0.3335
5	0.03502	-0.3134	0.6774	-0.6655	0.1062	-0.9938	0.0331
6	0.03650	-0.2813	0.7908	-0.5435	0.1259	-0.9532	-0.2748
7	0.04428	-0.2380	0.8870	-0.3958	0.1338	-0.7844	-0.6056
8	0.06528	-0.1860	0.9543	-0.2337	0.1253	-0.5287	-0.8395
9	0.06355	-0.1307	0.9887	-0.0730	0.1827	-0.4479	-0.8752
10	0.06254	-0.0771	0.9943	0.0730	0.2405	-0.3583	-0.9021

3.3 Determine coil current from fitting, imposing constant spin tune

The fitting procedure has 4 variables: the 4 sake currents. It has 4 constraints: the 3 components of the stable spin precession direction at pC-polarimeter, and the spin tune.

3.3.1 Vertical to radial polarization: ring data

Direct triedra (X,Y,Z): long. (s increases going CCW), radial (Y points towards center of Yellow), vertical points up.

Stable spin precession direction in Fig. 47 is obtained from 1-turn spin matrix.

Spin tune 0.06 - a first series of constraint weights - Variables and constraints (and their weight) in the FIT/REBELOTE sequence for step by step scan is as follows:

```
  'FIT2'
 4    save fitVals_n0_pCPol
 6 28 0 .9        ! snake 1 (9 o'clock) low B current
 6 32 0 .9        ! snake 1 (9 o'clock) high B current
 6 36 0 .9        ! snake 2 (3 o'clock) low B current
 6 40 0 .9        ! snake 2 (3 o'clock) high B current
 4
 10.3   1 1 #End  0.  10. 0      ! maintain SX~0.; loose constraint
 10.3   1 2 #End  0.  1.  0      ! start with SY=0, REBELOTE will increase to 1.
 10.3   1 3 #End  1.  1.  0      ! start with SZ=1, REBELOTE will decrease to 0.
 10.4   1 1 #End .06   .1 0      ! Spin tune maintained constant, strong constraint

'FAISCEAU'
'SPNPRT'   MATRIX PRINT

'REBELOTE'
10  0.1 0 1                                    ! 10 additional steps
4
FIT2   74   0.1:1.                             ! SY constraint, increase from .1 to 1.
FIT2   75   1. 5. 10. 20. 40. 40. 20. 10. 5. 1. ! its weight
FIT2   84   0.9:0.                             ! SZ constraint, decrease from .9 to 0.
FIT2   85   1. 5. 10. 20. 40. 40. 20. 10. 5. 1. ! its weight
```

Results are displayed in Figs. 47, 48.

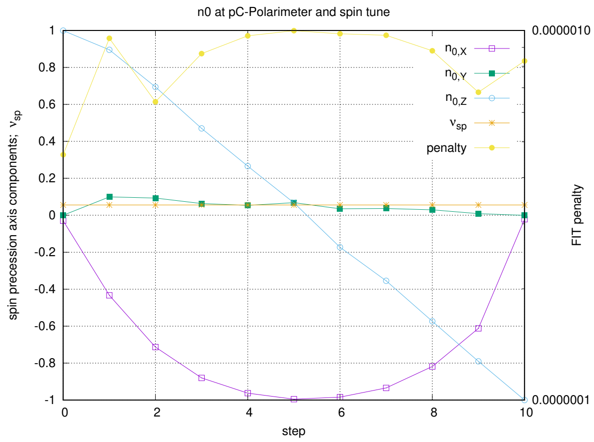


Figure 47: Components of stable spin precession direction at pCPol, and spin tune.

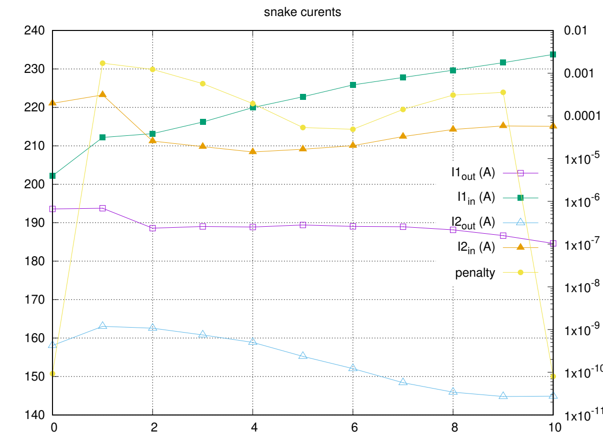


Figure 48: Snake currents (can be compared with Figs. 24, 25 page 8)

Spin tune 0.06 - a second series of constraint weights - Relax on X component and on intermediate Y and Z spin $\vec{n}_0$ component values, relax as well on penalty:

```
 'FIT2'
 4    save fitVals_n0_pCPol
 6 28 0 .9        ! snake 1 (9 o'clock) low B current
 6 32 0 .9        ! snake 1 (9 o'clock) high B current
 6 36 0 .9        ! snake 2 (3 o'clock) low B current
 6 40 0 .9        ! snake 2 (3 o'clock) high B current
 4 1e-6 300
 10.3   1 1 #End  0.  1.e3 0     ! maintain SX~0.; loose constraint
 10.3   1 2 #End  0.  1.   0     ! start with SY=0, REBELOTE will increase to 1.
 10.3   1 3 #End  1.  1.   0     ! start with SZ=1, REBELOTE will decrease to 0.
 10.4   1 1 #End .06  .1   0     ! Spin tune maintained constant, strong constraint

'FAISCEAU'
'SPNPRT'   MATRIX PRINT

'REBELOTE'
10  0.1 0 1                              ! 10 additional steps
4
FIT2   74   0.1:1.                       ! SY constraint, increase from .1 to 1.
FIT2   75   5. 20. 40. 80. 200. 80. 40. 20. 10. 1. ! its weight
FIT2   84   0.9:0.                       ! SZ constraint, decrease from .9 to 0.
FIT2   85   5. 20. 40. 80. 200. 80. 40. 20. 10. 1. ! its weight
```

Results are displayed in Figs. 49, 50, detailed snake current values are given in Tab. 4.

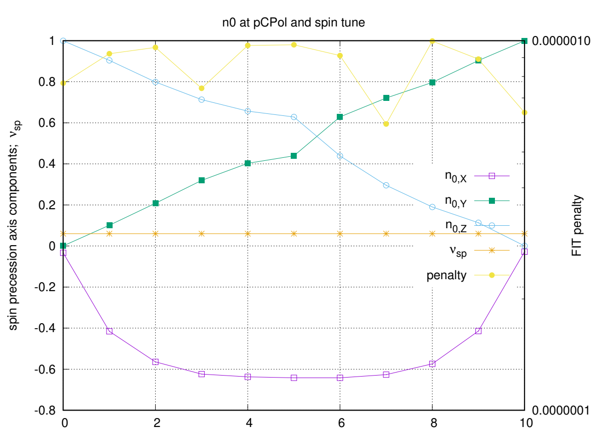


Figure 49: Vertical to radial polarization exchange: components of stable spin precession direction at pCPol, with constant spin tune $\nu_{\rm sp} = 0.06$.

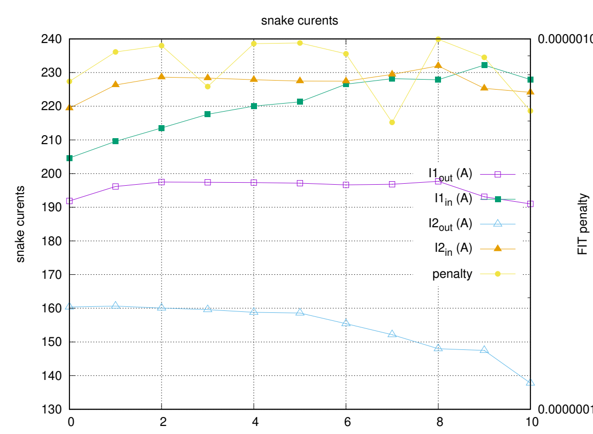


Figure 50: Vertical to radial polarization exchange: snake currents (can be compared with non-constant $\nu_{\rm sp}$ case of Figs. 24, 25, page 8). Detailed values are given in Tab. 4.

Table 4: Vertical to radial polarization exchange: snake currents (correspond to Figs. 49, 50)

	9 oclock snake		3 oclock snake	
step #	$I1_{out}$	$I1_{in}$	$I2_{out}$	$I2_{in}$
0.00	191.87844	204.64519	160.39338	219.43131
1.00	196.17300	209.56110	160.64605	226.32252
2.00	197.49818	213.54697	160.05965	228.64614
3.00	197.41452	217.59918	159.59859	228.36775
4.00	197.33729	220.03104	158.80599	227.86840
5.00	197.13436	221.31062	158.57964	227.47993
6.00	196.63208	226.56090	155.41987	227.43498
7.00	196.83126	228.17767	152.15385	229.45722
8.00	197.69624	227.87016	147.97539	231.97321
9.00	193.10422	232.22474	147.48606	225.32010
10.0	190.98916	227.93243	137.77474	224.12903

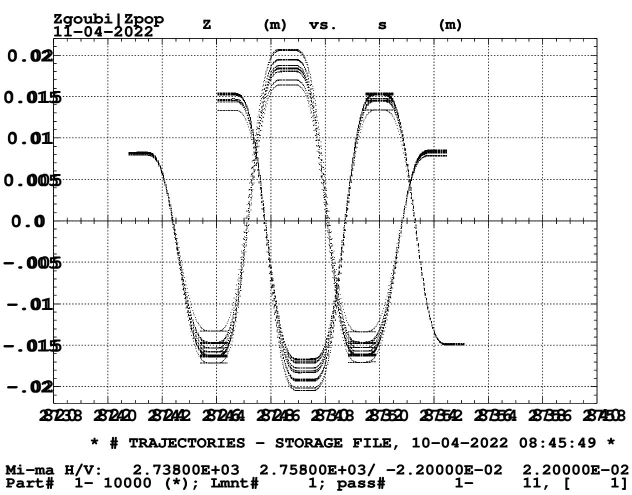


Figure 51: Vertical to radial polarization exchange: orbits along 3OClock and 9OClock snakes at $G\gamma = 45.5$ over the ten steps of vertical to radial adjustment (cf. Figs. 49, 50)

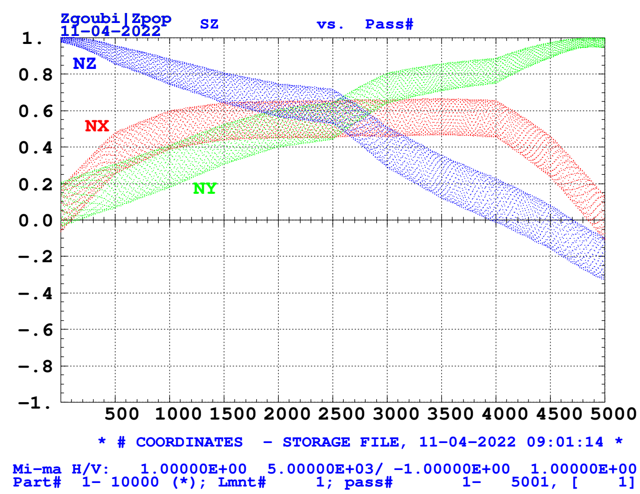


Figure 52: Vertical to radial polarization exchange, case of $\nu_{\text{sp}} = 0.06$; motion of spin $\vec{n}_0$ at IP6, in the case of a continuous, 5000 turn ramp through the 10 steps, starting with $\vec{n}_0$ vertical IP6. Case of defect-free RHIC lattice

Spin tune 0.056 - Variables and constraints (and their weight) for a FIT/REBELOTE sequence for step by step scan is as follows:

```
 'FIT2'
 4    save fitVals_n0_pCPol
 6 28 0 .9        ! snake 1 (9 o'clock) low B current
 6 32 0 .9        ! snake 1 (9 o'clock) high B current
 6 36 0 .9        ! snake 2 (3 o'clock) low B current
 6 40 0 .9        ! snake 2 (3 o'clock) high B current
 4 1e-6 300
 10.3   1 1 #End  0.  1.e3 0      ! maintain SX~0.; loose constraint
 10.3   1 2 #End  0.  1.   0      ! start with SY=0, REBELOTE will increase to 1.
 10.3   1 3 #End  1.  1.   0      ! start with SZ=1, REBELOTE will decrease to 0.
 10.4   1 1 #End .056  .1   0      ! Spin tune maintained constant, strong constraint

'FAISCEAU'
'SPNPRT'   MATRIX PRINT

'REBELOTE'
10  0.1 0 1                              ! 10 additional steps
4
FIT2   74   0.1:1.                       ! SY constraint, increase from .1 to 1.
FIT2   75   5. 20. 40. 80. 200. 80. 40. 20. 10. 1. ! its weight
FIT2   84   0.9:0.                       ! SZ constraint, decrease from .9 to 0.
FIT2   85   5. 20. 40. 80. 200. 80. 40. 20. 10. 1. ! its weight
```

Results are displayed in Figs. 53, 54, detailed snake current values are given in Tab. 5.
A 5000 turn tracking for cross-check is displayed in Fig. 55.

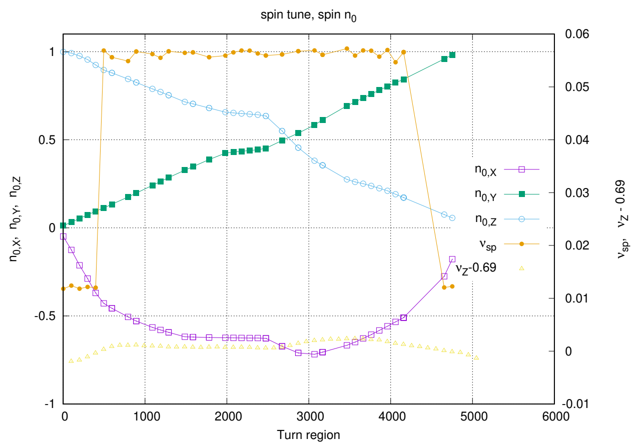


Figure 53: Components of stable spin precession direction at pCPol, and spin tune.

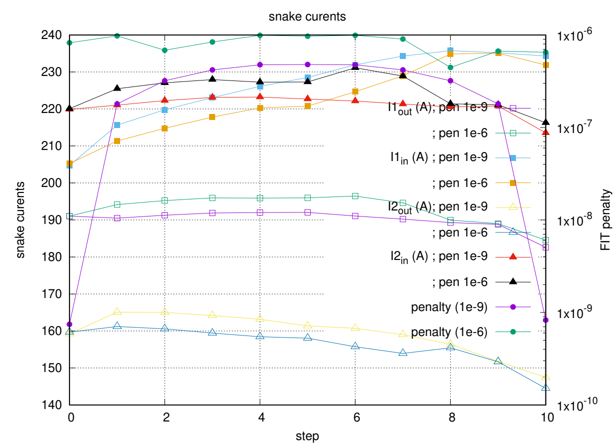


Figure 54: Snake currents (can be compared with Figs. 24, 25 page 8). Two values of FIT penalty are tried, 1e-6 and 1e-9, they provide slightly different paths.

Table 5: Vertical to radial polarization exchange; snake currents (correspond to Figs. 53, 54)

	9 oclock snake		3 oclock snake	
step #	$I1_{out}$	$I1_{in}$	$I2_{out}$	$I2_{in}$
0.00	191.05221	205.23546	159.67288	220.06857
1.00	194.15039	211.35839	161.18629	225.47141
2.00	195.23940	214.77308	160.56429	227.04852
3.00	195.94735	217.79968	159.42372	227.95135
4.00	195.90515	220.29429	158.44052	227.23955
5.00	195.98908	220.78342	158.06495	227.28741
6.00	196.46859	224.64421	155.72585	231.13759
7.00	194.62559	228.98185	153.93455	228.89063
8.00	189.96476	234.85141	155.45070	221.39170
9.00	188.99741	235.07534	151.64309	221.12475
10.0	184.54184	231.87676	144.47979	216.27449

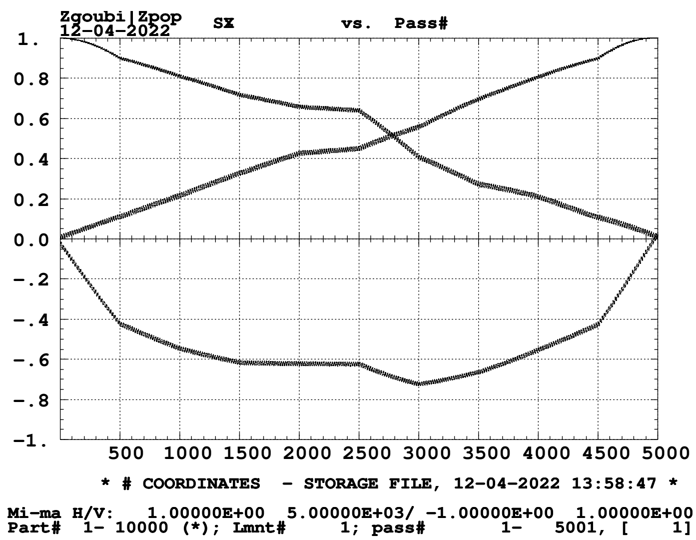


Figure 55: Vertical to radial polarization exchange, case $\nu_{\rm sp} = 0.056$; motion of spin $\vec{n}_0$ observed at pC-polarimeter, in the case of a continuous, 5000 turn ramp through the 10 steps, starting with $\vec{n}_0$ vertical at pC-polarimeter, Case of defect-free RHIC lattice

Spin tune 0.03 - Results are displayed in Figs. 56, 57, detailed snake current values are given in Tab. 6.

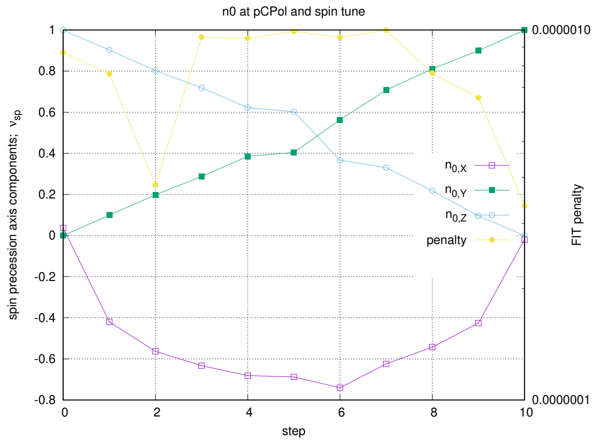


Figure 56: Vertical to radial polarization exchange; components of stable spin precession direction at pCPol, with constant spin tune $\nu_{\rm sp} = 0.03$.

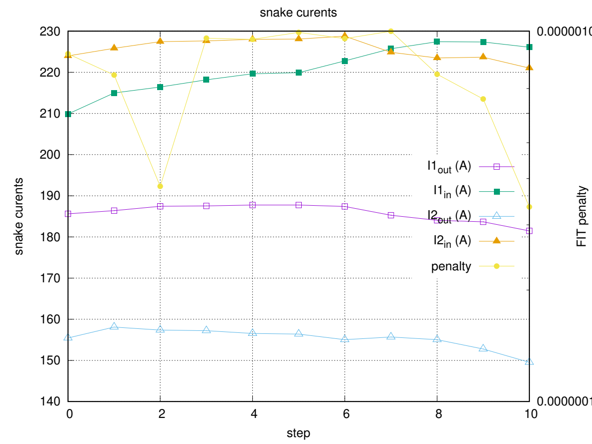


Figure 57: Vertical to radial polarization exchange; snake currents (can be compared with non-constant $\nu_{\rm sp}$ case of Figs. 24, 25, page 8). Detailed values are given in Tab. 6

Table 6: Vertical to radial polarization exchange; snake currents (correspond to Figs. 56, 57)

	9 oclock snake		3 oclock snake	
step #	$I1_{out}$	$I1_{in}$	$I2_{out}$	$I2_{in}$
0.00	185.60569	209.87763	155.45401	223.97781
1.00	186.39633	214.97334	158.10399	225.83712
2.00	187.45756	216.42659	157.37571	227.44329
3.00	187.54336	218.17720	157.21401	227.62576
4.00	187.74851	219.63344	156.54356	228.02416
5.00	187.74937	219.90594	156.39506	228.07882
6.00	187.39842	222.77682	155.04998	228.78774
7.00	185.26239	225.71885	155.69754	224.86447
8.00	184.04235	227.42992	155.03585	223.49143
9.00	183.67640	227.35986	152.78126	223.68435
10.0	181.47245	226.08769	149.53608	221.02084

3.3.2 Reversal of vertical polarization

Spin tune 0.056 - Variables and constraints (and their weight) in the FIT/REBELOTE sequence for step by step scan is as follows:

```
 'FIT2'
 4    save fitVals_n0_pCPol
 6 28 0 .9        ! snake 1 (9 o'clock) low B current
 6 32 0 .9        ! snake 1 (9 o'clock) high B current
 6 36 0 .9        ! snake 2 (3 o'clock) low B current
 6 40 0 .9        ! snake 2 (3 o'clock) high B current
 4 1e-6 300
 10.3   1 1 #End  0.  1.e3 0      ! maintain SX~0.; loose constraint
 10.3   1 2 #End  0.  1.   0      ! start with SY=0, REBELOTE will increase to 1.
 10.3   1 3 #End  1.  1.   0      ! start with SZ=1, REBELOTE will decrease to 0.
 10.4   1 1 #End .056  .1   0      ! Spin tune maintained constant, strong constraint

'FAISCEAU'
'SPNPRT'   MATRIX PRINT

'REBELOTE'
10  0.1 0 1                                    ! 10 additional steps
4
FIT2   74   0.1:0.                            ! SY constraint, ends up 0 as at start
FIT2   75   5. 20. 40. 80. 200. 80. 40. 20. 10. 1. ! its weight
FIT2   84   0.9:-1.                           ! SZ constraint, ends up flipped
FIT2   85   5. 20. 40. 80. 200. 80. 40. 20. 10. 1. ! its weight
```

Results are displayed in Figs. 58, 59 and Tab. 7.
A 5000 turn tracking check is displayed in Fig. 61.

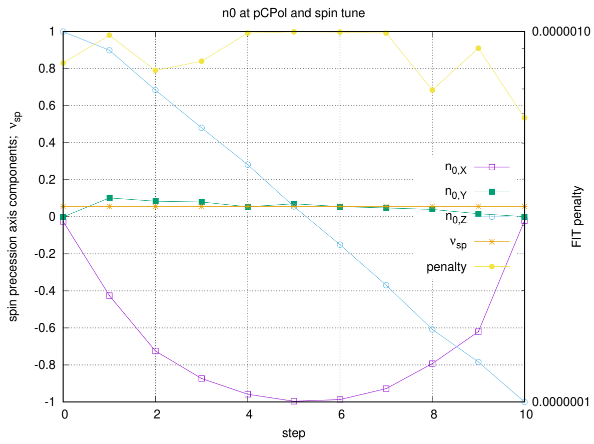


Figure 58: Spin flip; components of stable spin precession direction at pCPol, and spin tune.

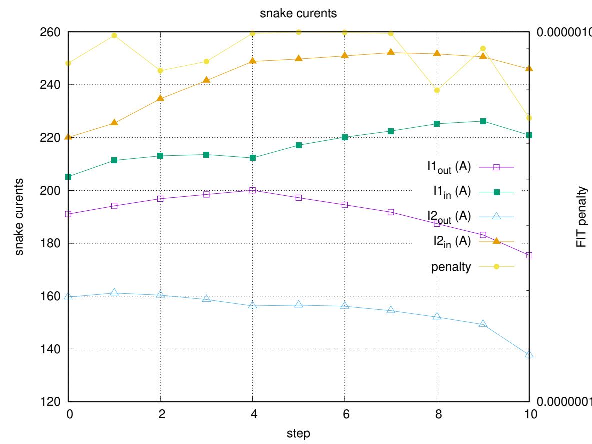


Figure 59: Spin flip; snake currents

Table 7: Vertical to radial polarization exchange; snake currents (correspond to Figs. 58, 59)

	9 oclock snake		3 oclock snake	
step #	$I1_{out}$	$I1_{in}$	$I2_{out}$	$I2_{in}$
0.00	191.11102	205.27089	159.70140	220.11138
1.00	190.97453	215.23148	164.65156	221.73555
2.00	192.21764	219.10160	165.14016	228.10751
3.00	191.89136	222.29549	165.36883	233.08808
4.00	193.29926	221.73338	162.63299	239.71622
5.00	192.79550	223.41891	161.03232	243.05032
6.00	190.46811	225.60525	159.84549	245.79852
7.00	187.98431	228.00066	158.63360	246.32461
8.00	184.24024	230.33758	156.51299	246.32642
9.00	178.23768	232.98493	154.28758	243.79868
10.0	168.17614	229.96669	146.02882	236.94956

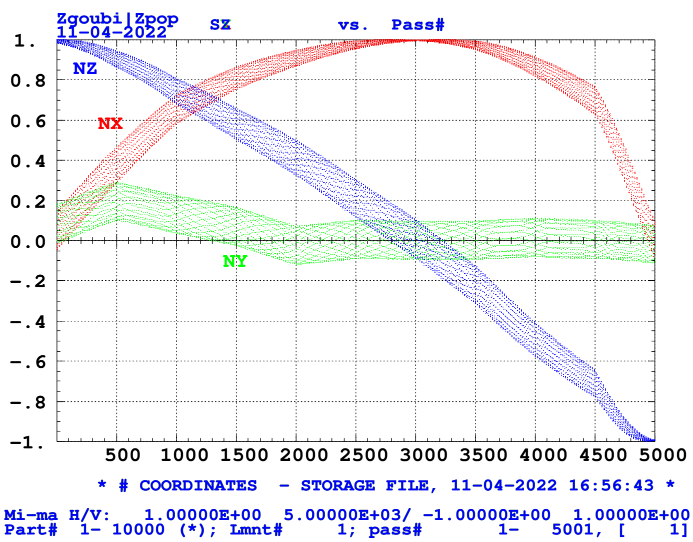


Figure 60: Spin flip, case $\nu_{\mathrm{sp}} = 0.056$; motion of spin $\vec{n}_0$ observed at IP6, in the case of a continuous, 5000 turn ramp through the 10 steps, starting with $\vec{n}_0$ vertical at IP6. Case of defect-free RHIC lattice

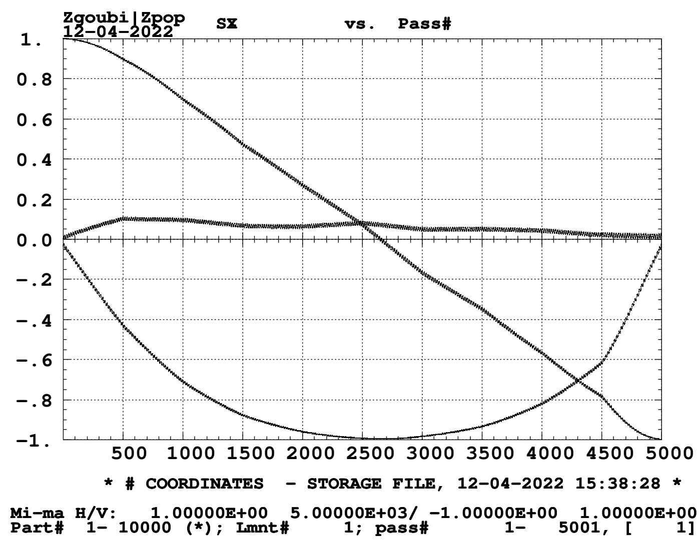


Figure 61: Spin flip, case $\nu_{\mathrm{sp}} = 0.056$; motion of spin $\vec{n}_0$ observed at pC-polarimeter, in the case of a continuous, 5000 turn ramp through the 10 steps, starting with $\vec{n}_0$ vertical at pC-polarimeter, Case of defect-free RHIC lattice

Spin tune 0.03 - Variables and constraints (and their weight) in the FIT/REBELOTE sequence for step by step scan is as follows:

```
 'FIT2'
 4    save fitVals_n0_pCPol
 6 28 0 .9        ! snake 1 (9 o'clock) low B current
 6 32 0 .9        ! snake 1 (9 o'clock) high B current
 6 36 0 .9        ! snake 2 (3 o'clock) low B current
 6 40 0 .9        ! snake 2 (3 o'clock) high B current
 4 1e-6 300
 10.3   1 1 #End  0.  1.e3 0      ! maintain SX~0.; loose constraint
 10.3   1 2 #End  0.  1.   0      ! start with SY=0, REBELOTE will increase to 1.
 10.3   1 3 #End  1.  1.   0      ! start with SZ=1, REBELOTE will decrease to 0.
 10.4   1 1 #End .03  .1   0      ! Spin tune maintained constant, strong constraint

'FAISCEAU'
'SPNPRT'   MATRIX PRINT

'REBELOTE'
10  0.1 0 1                                ! 10 additional steps
4
FIT2   74   0.1:0.                        ! SY constraint, ends up 0 as at start
FIT2   75   5. 20. 40. 80. 200. 80. 40. 20. 10. 1. ! its weight
FIT2   84   0.9:-1.                       ! SZ constraint, ends up flipped
FIT2   85   5. 20. 40. 80. 200. 80. 40. 20. 10. 1. ! its weight
```

Results are displayed in Figs. 62, 63 and Tab. 8.

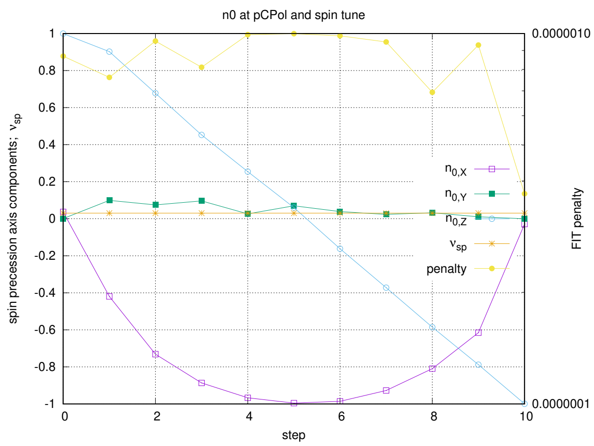


Figure 62: Spin flip, case $\nu_{sp} = 0.03$; components of stable spin precession direction at pCPol, and spin tune.

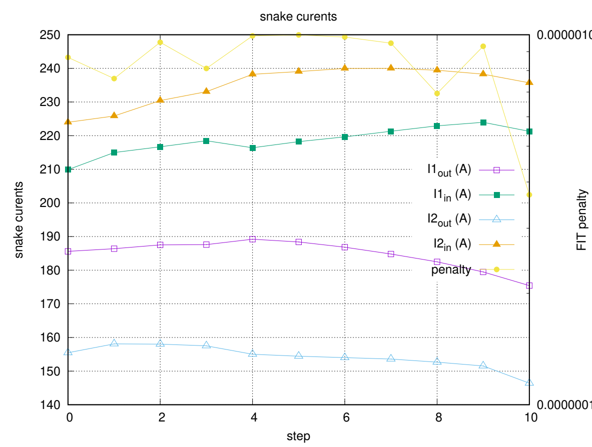


Figure 63: Spin flip, case $\nu_{sp} = 0.03$; snake currents

Table 8: Spin flip, case $\nu_{sp} = 0.03$; snake currents (correspond to Figs. 62, 63

	9 oclock snake		3 oclock snake	
step #	$I1_{out}$	$I1_{in}$	$I2_{out}$	$I2_{in}$
0.00	185.60569	209.87763	155.45401	223.97781
1.00	186.39633	214.97334	158.10399	225.83712
2.00	187.53374	216.68856	158.02268	230.42388
3.00	187.60721	218.49601	157.51042	233.05578
4.00	189.21352	216.40050	155.00736	238.22951
5.00	188.40176	218.21973	154.42654	239.04661
6.00	186.83031	219.64834	154.01458	239.98166
7.00	184.80642	221.30386	153.56985	240.01427
8.00	182.48285	222.91981	152.65502	239.45064
9.00	179.49039	223.96664	151.53702	238.28708
10.0	175.41416	221.28520	146.41859	235.69575

Snake current SCALING function for a sweep (Fig. 66):

```
TOSCA    snk1LowB             ! snk1 = 9'o
11
185.60569  186.39633  187.53374  187.60721  189.21352  188.40176  186.83031  184.80642  182.48285  179.49039  175.41416
 1         501        1001       1501       2001       2501       3001       3501       4001       4501       5001
TOSCA    snk1HighB             ! snk1 = 9'o
11
209.87763 214.97334 216.68856 218.49601 216.40050 218.21973 219.64834 221.30386 222.91981 223.96664 221.28520
1501       1001       1501       2001       2501       3001       3501       4001       4501       5001
TOSCA    snk2LowB             ! snk2 = 3'o
11
  155.45401 158.10399 158.02268 157.51042 155.00736 154.42654 154.01458 153.56985 152.65502 151.53702 146.41859
1          501        1001       1501       2001       2501       3001       3501       4001       4501       5001
TOSCA    snk2HighB             ! snk2 = 3'o
11
  223.97781 225.83712 230.42388 233.05578 238.22951 239.04661 239.98166 240.01427 239.45064 238.28708 235.69575
1          501        1001       1501       2001       2501       3001       3501       4001       4501       5001
```

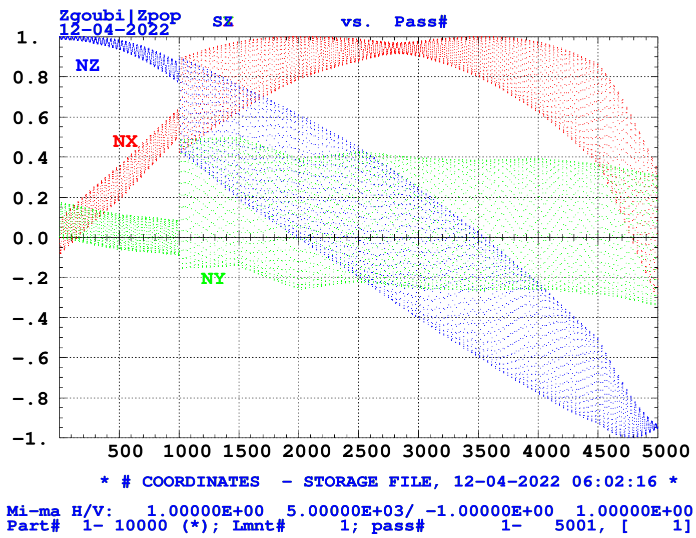


Figure 64: Spin flip, case $\nu_{\text{sp}} = 0.056$; motion of spin $\vec{n}_0$ at IP6, in the case of a continuous, 5000 turn ramp through the 10 steps, starting with $\vec{n}_0$ vertical at IP6. Case of defect-free RHIC lattice

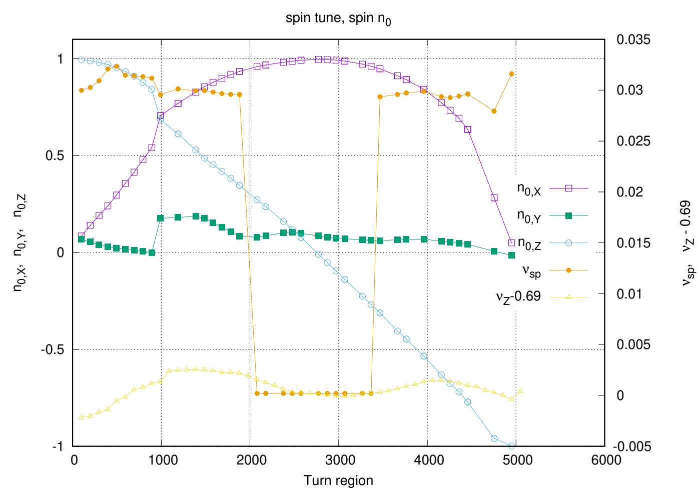


Figure 65: Spin components at IP6 (left vertical axis), spin tune and betatron tune from Fourier analysis (right V axis). $\nu_{\text{sp}} = 0$ in the region [2000,3500] is probably an artifact, TBC.

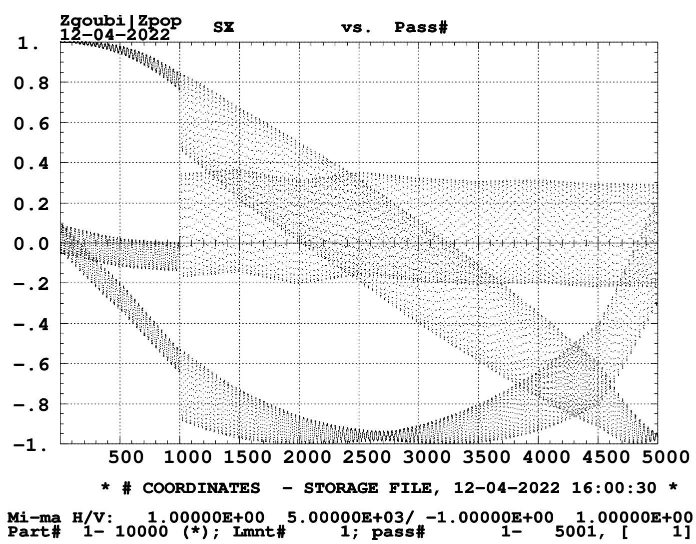


Figure 66: Spin flip, case $\nu_{\text{sp}} = 0.056$; motion of spin $\vec{n}_0$ at pC-polarimeter, in the case of a continuous, 5000 turn ramp through the 10 steps, starting with $\vec{n}_0$ vertical at pC-polarimeter. Case of defect-free RHIC lattice

4 APEX results: tentative simulation approach

4.1 Synchronized ramp (March 30th APEX)

Measured currents are displayed in Fig. 67, measurement data provided by Haixin.

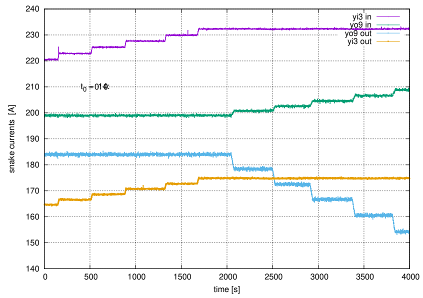


Figure 67: Measured snake currents, March 30th APEX, vertical to radial polarization.

4.1.1 Polarization vector, expectations

• To check that working hypotheses are ok, a scan is performed first. Using SPINR for a change, results in TOSCA case are detailed in earlier Sections: field maps (using TOSCA) are replaced by pure spin rotation, SPINR. Orbits are maintained at entrance of snakes, canceled downstream, Fig. 68, as with field maps.

A sanity check: both methods yield same $\vec{n}_0$ results, Fig. 69, a good point.

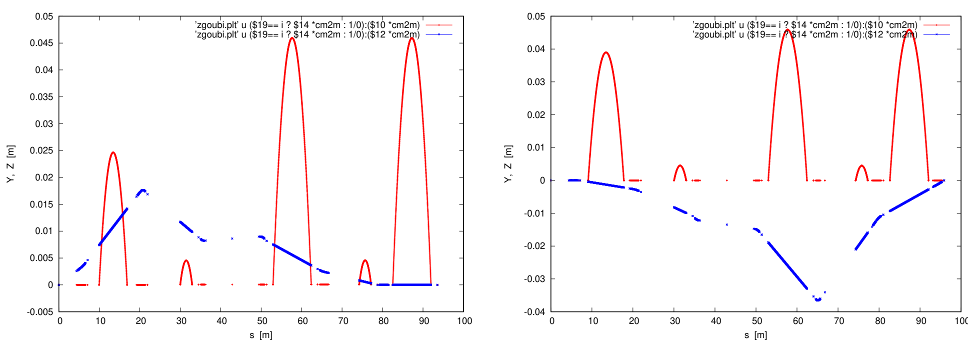


Figure 68: Local orbit bumps in SPINR segments, similar to field map case

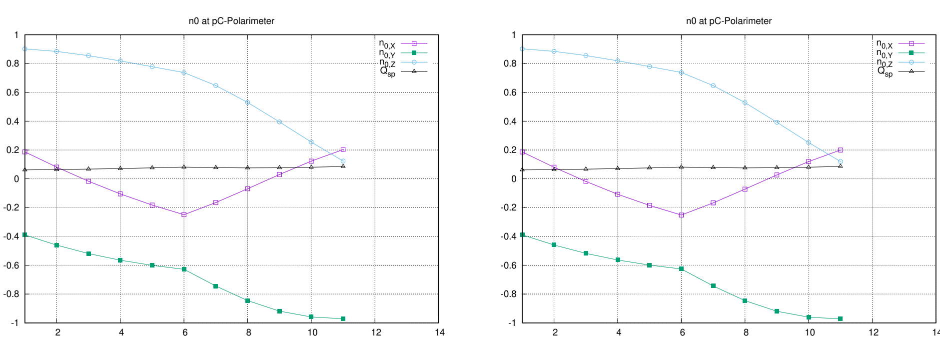


Figure 69: A scan of polarization vector at the 11 snake angle steps of the vertical to radial APEX. Left: using SPINR; right: using field maps.

• Back to using field maps, snake coils power supplies are now ramped following March 30th APEX operation, still just 1 particle launched on RHIC closed orbit. Expected polarization vector motion so obtained is displayed in Fig. 70.

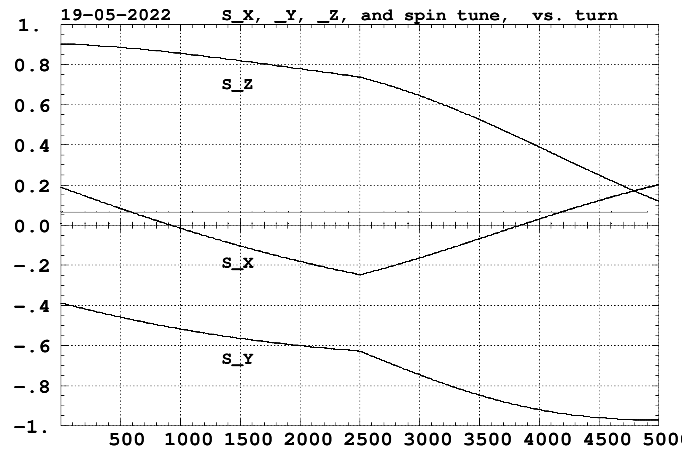


Figure 70: Motion of spin $\vec{n}_0$ observed at pC-polarimeter, case of a single particle on closed orbit over 5,000 turns. Continuous vertical to radial polarization exchange over 5000 turns. Case of $\nu_{\mathrm{sp}} = 0.066$, constant. Linear interpolation of snake currents between steps. Observation is at pC-polarimeter, starting spin is $\vec{n}_0$
Snake coil currents are changed continuously over 5,000 turns. Figure 70 is fully consistent with the stationary scan using SPINR in Fig. 69. Note that ramp speed does not matter much: 500 turns yield similar result to the present 5000 turn ramp.

4.1.2 6D bunch tracking, 15,000 turns, using field maps

The initial 6D bunch launched in this tracking is shown in Fig. 71.

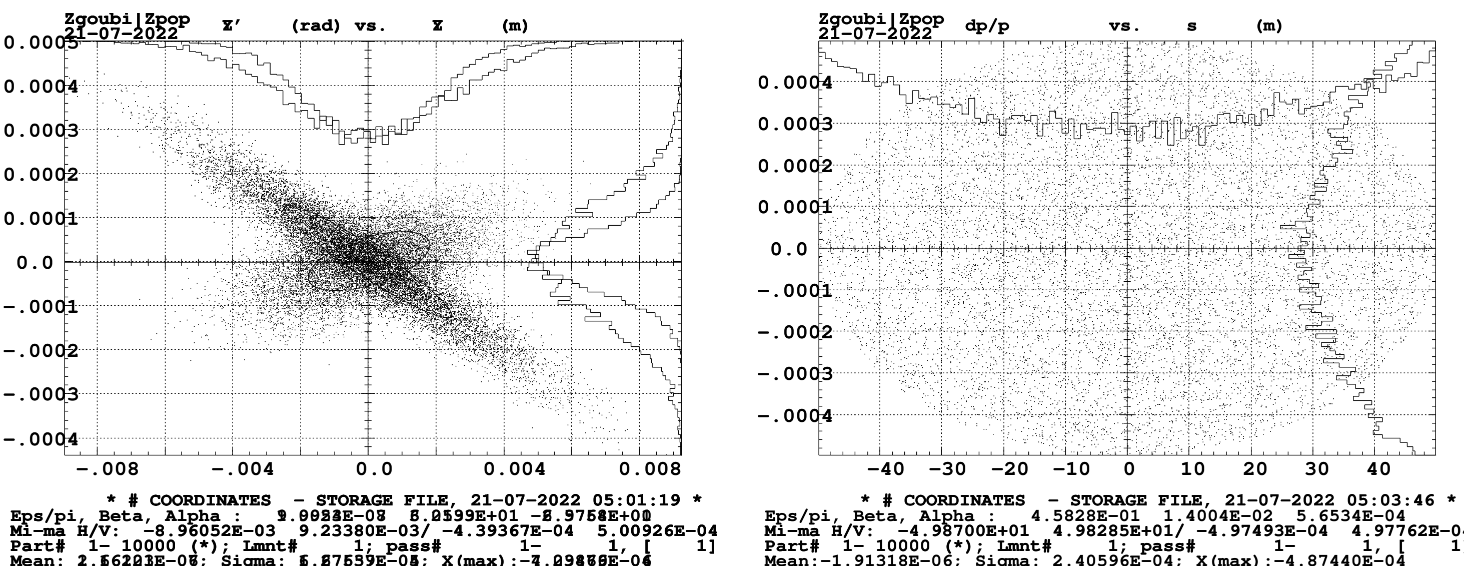


Figure 71: Left: transverse phase-spaces, 2.5 μm normalized. Right: longitudinal, dp/p in $\pm 5\,10^{-4}$, dt=$\pm$50/c=$\pm$80 ns $\Rightarrow$ $23\,10^9 \times 0.0005 * 80\,10^{-9} \approx 0.9\,\pi$eV.s (not matched to the RF bucket).

Double-RF parameters:

```
'CAVITE'
2
3833.84857 { 120. , 2520. }       ! Orbit length, harmonic numbers.
{ 17.d3, 11.d3 } 3.14159265359    !  Volts, synch. phase.
```

Outcomes of the 5000 turn tracking:

- transverse phase spaces (initially as shown in Fig. 71) are preserved, $2.5\,\pi\mu$m norm. all the way. This is an indication that tracking using snake field maps (TOSCA) behaves well - a possible double-check: re-do using SPINR (pure spin rotation) and compare particle dynamics;
- longitudinal phase space over 15,000 turns is displayed in Fig. 72.
- Figure 73 displays a few individual particle spin motion, their turn-by-turn average over 1,000 particles, and polarization (i.e., the average value of the projection of individual spins on the average spin vector).

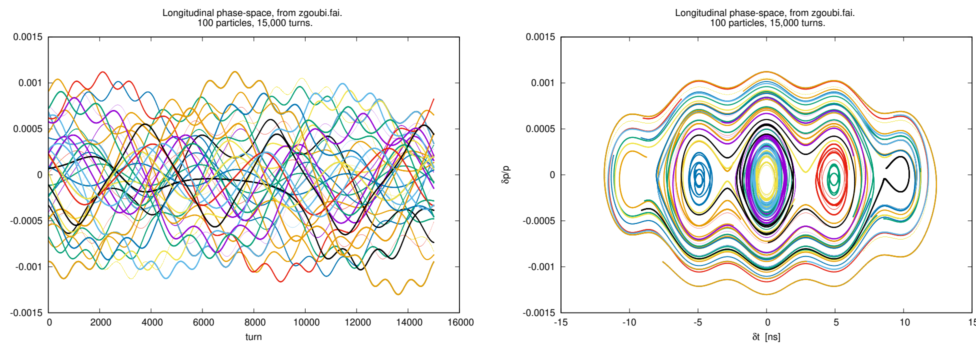


Figure 72: Sample particle motion in longitudinal phase space over 5,000 turn coasting followed by 10,000 turn snake supplies ramp. Full length is about 25 ns, momentum excursion is $\pm 10^{-3}$.

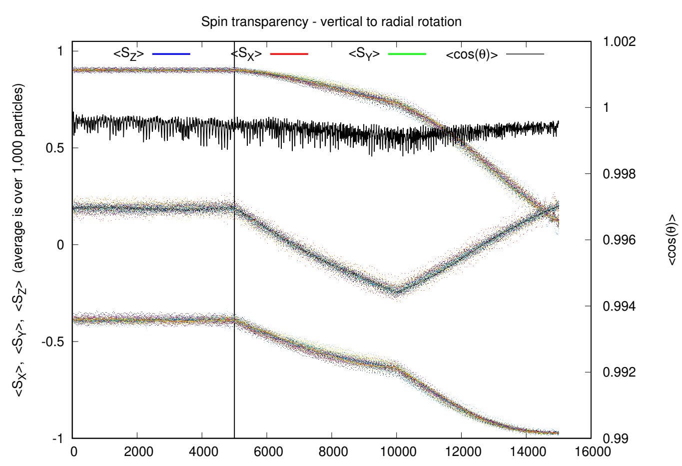


Figure 73: Polarization rotation from vertical to radial, a few individual particle spin motion, and spin component averages. An initial 5000 turns is allowed to allow momentum spread and polarization to stabilize. Snake currents then are ramped smoothly over 10,000 turns. Initial bunch is shown in Fig. 71: transverse emittances $2.5\,\pi\mu$m normalized, Gaussian, momentum spread dp/p in $\pm 10^{-3}$, longitudinal motion is shown in Fig. 72.

Appendix

A Optical sequences for 9 o'clock snake in Yellow, going clockwise

The file below includes the FIT command to get low-field (coils 1 and 4) and high-field (coils 2 and 3) currents, given spin precession angle and precession vector constrained values.

```
Snake 1. 9 o'clock snake, in Yellow going clockwise.
'OBJET'
79.366778931425273 * 1d3
1
3  1 1 1 1 1
0.00.   0.00.   0.00.   0.00.   0.00  0.000000E+00
0.  0.  0.85093379  0.  0.  1.

'PARTICUL'
PROTON

'SPNTRK'
4
1.00000000E+00  0.00000000E+00  0.00000000E+00
0.00000000E+00  1.00000000E+00  0.00000000E+00
0.00000000E+00  0.00000000E+00  1.00000000E+00

'SCALING'
1  2
TOSCA    snk1LowB
-1
164.16759    !  low-field coils current (A)
1
TOSCA    snk1HighB
-1
220.45332    !  high-field coils current (A)
1

! start snake 1

'MARKER'  SNK1
'DRIFT'
-78.2

'DRIFT'
3.6
'DRIFT'
16.4

'TOSCA'          snk1LowB
0  0002  ! .plt
+1.  1.0000.  1.0000.  1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table
0 0 0 0
2
.3
2   0.0000.  0.0000.  0.00000000E+00

'DRIFT'    2.162m_C1toC2
-98.800000

'TOSCA'          snk1HighB
0  0002  ! .plt
-1.  1.0000.  1.0000.  1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table
0 0 0 0
2
.3
2   0.0000.  0.0000.  0.00000000E+00

'DRIFT'    YO9_B7.1  VMON  ! center of 9 oclock snake
-37.6

'DRIFT'    2.162m_C2toC3
-37.6

'TOSCA'          snk1HighB
0  0002  ! .plt
+1.  1.000.  1.000.  1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table
0 0 0 0
2
.3
2   0.000.  0.000.  0.00000000E+00

'DRIFT'    2.162m_C3toC4
-98.800000

'TOSCA'          snk1LowB
0  0002  ! .plt
-1.  1.000.  1.000.  1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table
0 0 0 0
2
.3
2   0.000.  0.000.  0.00000000E+00

'DRIFT'
16.4

'DRIFT'
-78.2

! end snake 1

'FAISCEAU'
'SPNPRT'  MATRIX

'FIT2'
3
1 42 0  .1                     ! Z_0, vertical position of reference orbit at OBJET.
4  4 0  .1                                                        ! Low B coil current.
4  8 0  .1                                                       ! High B coil current.
5  1e-12
7.3  1 4 12    0.             1. 0       ! Vertically center traj #1 in 3rd helix;
10.2 1 0 #End  3.14159265359   1. 0         ! D-group 1. Spin rotation angle = pi;
10.3 1 1 #End  1.  1. 0                ! X component of rotation axis = cos 0deg;
10.3 1 2 #End  0.  1. 0                ! Y component of rotation axis = sin 0deg.
10   1 4 #End  1.  .1 0                                                    ! |S|==1.

! FIT DATA FOR PRECESSION AXIS AT 10DEG TO X AXIS:
! 'FIT2'
! 1 42 0  .2                   ! Z_0, vertical position of reference orbit at OBJET.
! 4  4 0  .2                                                      ! Low B coil current.
! 4  8 0  .2                                                     ! High B coil current.
! 5  1e-12
! 7.3  1 4 12    0.             1. 0     ! Vertically center traj #1 in 3rd helix;
! 10.2 1 0 #End  3.14159265359   1. 0       ! D-group 1. Spin rotation angle = pi;
! 10.3 1 1 #End  0.984807753012  1. 0   ! X component of rotation axis = cos 10deg;
! 10.3 1 2 #End  0.173648177667  1. 0   ! Y component of rotation axis = sin 10deg;
! 10   1 4 #End  1. .1 0                                                   ! |S|==1.

'END'
```

B Optical sequences for μ and ϕ scan for transparency experiment

The file below includes the FIT+REBELOTE commands. They result in Figs. 24, 25.

```
Snake 1. 9 o'clock snake in Yellow, going counter-clockwise.
'OBJET'
79.366778931425273 * 1d3
1
3  1 1 1 1 1
0.0    0.0    0.0    0.0    0.00  0.000000E+00
0.  0. -1.3206397  0.  0.  1.
'PARTICUL'
PROTON
'SPNTRK'
4
1.000   0.000   0.00000000E+00
0.000   1.000   0.00000000E+00
0.000   0.000   1.00000000E+00

'SCALING'
1  2
TOSCA    snk1LowB
-1
184.47838    !  low-field coils current (A)
1
TOSCA    snk1HighB
-1
198.70954    !  high-field coils current (A)
1

'TOSCA'          snk1LowB
0  0002  ! .plt
-1.  1.000   1.000   1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table
etc.
'DRIFT'    2.162m_C1toC2
-98.800000
'TOSCA'          snk1HighB
0  0002  ! .plt
+1.  1.000   1.000   1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table
etc.
'DRIFT'    YO9_B7.1  VMON  ! center of 9 oclock snake
-37.6
'DRIFT'    2.162m_C2toC3
-37.6
'TOSCA'          snk1HighB
0  0002  ! .plt
-1.  1.000   1.000   1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table
etc.
'DRIFT'    2.162m_C3toC4
-98.800000
'TOSCA'          snk1LowB
0  0002  ! .plt
+1.  1.000   1.000   1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table
etc.

'FIT2'
3  save ! Save variable values, penalty, etc., to zgoubi.FITVALS.out.
1 42 0  1.5     ! Z_0, vertical position of reference orbit at OBJET.
4  4 0  1.5                                      ! Low B coil current.
4  8 0  1.5                                     ! High B coil current.
5  1e-12
7.3  1 4 10    0.              1. 0 ! Vertically center traj #1 in 3rd helix;
10.2 1 0 #End  3.14159265359   1. 0   ! D-group 1. Spin rotation angle = 180;
10.3 1 1 #End -0.984807753012  1. 0  ! X component of rot. axis=cos(-10deg);
10.3 1 2 #End  0.173648177667 .01 0  ! Y component of rot. axis=sin(-10deg);
10   1 4 #End  1. .1 0                                          ! |S|==1.

'FAISCEAU'
'SPNPRT'  MATRIX  PRINT

'REBELOTE'
12  0.1 0 1
3     !  1         2           3           4           5           6
FIT2 64  3.1415926 3.14159265  3.14159265  3.14159265  3.14159265  3.14159265
3.0892327  3.0368728  2.9845130  2.932153143  2.87979326579   2.87979326579
FIT2 74 -0.9848077 -.98480775 -.984807753 -.984807753 -.98480775 -.9848077530
-0.9902680687 -0.9945218953 -0.997564050    -0.999390827  -1.          -1.
FIT2 84  0.17364817 0.17364817  0.17364817  0.173648177  0.17364817  0.17364817
0.13917310096   0.104528463268  0.0697564737441  0.0348994967025 0.         0.

'SPNPRT'  MATRIX  PRINT            ! Shows final case results, just for check.
'SYSTEM'
1
gnuplot <./gnuplot_FITVALS.gnu
'END'
```

```
Snake 2. 3 o'clock snake in Yellow, going counter-clockwise.
'OBJET'
79.366778931425273d3
1
3  1 1 1 1 1
0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.00  0.000000E+00
0.   0.  0.85357994 0.   0.00  1.00000000E+00
'PARTICUL'
PROTON
'SPNTRK'
4
1.00000000E+00  0.00000000E+00  0.00000000E+00
0.00000000E+00  1.00000000E+00  0.00000000E+00
0.00000000E+00  0.00000000E+00  1.00000000E+00

'SCALING'
1  2
TOSCA    snk2LowB
-1
164.43807    !  low-field coils current (A)
1
TOSCA    snk2HighB
-1
220.65730    !  high-field coils current (A)
1

'TOSCA'      snk2LowB
0  002  ! .plt
+1.  1.00000000E+00  1.00000000E+00  1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table
etc.
'DRIFT'    2.162m_C1toC2
-98.800000
'TOSCA'      snk2HighB
0  002  ! .plt
-1.  1.00000000E+00  1.00000000E+00  1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table
etc.
'DRIFT'    YI3_B7.1  VMON  ! center of 3 oclock snake
-37.6
'DRIFT'    2.162m_C2toC3
-37.6
'TOSCA'      snk2HighB
0  002  ! .plt
+1.  1.00000000E+00  1.00000000E+00  1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.246305418719e-4   ! = 1/4.060e4
b_model3a2a322a-x-4_4_y-4_4_z-180_180-integral.table
etc.
'DRIFT'    2.162m_C3toC4
-98.800000
'TOSCA'      snk2LowB
0  002  ! .plt
-1.  1.00000000E+00  1.00000000E+00  1.00000000E+00
HEADER_8   RHIC_helix
361 81 81  15.1  0.748502994012e-4     ! = 1/1.3360e4
b_model3a2a-x-4_4_y-4_4_z-180_180-integral.table
etc.

'FIT2'
3  save ! Save variable values, penalty, ... to zgoubi.FITVALS.out.
1 42 0  1.5   ! Z_0, vertical position of reference orbit at OBJET.
4  4 0  1.5                                     ! Low B coil current.
4  8 0  1.5                                    ! High B coil current.
5  1e-12
7.3  1 4 10    0.              1. 0 ! Vertically center traj #1 in 3rd helix;
10.2 1 0 #End  3.14159265359   1. 0   ! D-group 1. Spin rotation angle = 180;
10.3 1 1 #End  1.              1. 0  ! X component of rotation axis = cos(0);
10.3 1 2 #End  0.              .1 0  ! Y component of rotation axis = sin(0);
10   1 4 #End  1. .1 0                                               ! |S|==1.

'FAISCEAU'
'SPNPRT'  MATRIX  PRINT

'REBELOTE'
12  0.1 0 1
1     ! 0             1             2             3             4
FIT2 64 3.1415926535  3.0892327760  3.0368728984  2.9845130209  2.93215314335
2.87979326 2.87979326 2.87979326 2.87979326 2.87979326 2.87979326 2.879793265

'SPNPRT'  MATRIX  PRINT           ! Shows final case results, just for check.
'SYSTEM'
1
gnuplot <./gnuplot_FITVALS.gnu
'END'
```

References

[1] F. Méot, Gupta, R., Huang, H., Ranjbar, V., Robert-Demolaize, G.: Re-visiting RHIC snakes: OPERA fields, $\vec{n}_0$ dance. C-A/AP/590; BNL-114379-2017-IR (Sept. 2017). https://technotes.bnl.gov/PDF?publicationId=42159