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Observation and Measurement of Linear Coupling in the AGS

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Experimenter(s) W.K. van Asselt and C.J. Gardner

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References

1. G. Guiguard, The General Theory of All Sum and Difference Resonances in a Three-Dimensional Magnetic Field in a Synchrotron, CERN 76-06, ISR Division, March 23, 1976.
2. Kaji Takikawa, A Simple and Precise Method for Measuring the Coupling Coefficient of the Difference Resonance, CERN ISR-MA/75-34, July 30, 1975.

Purpose

The purpose of these studies was to observe and measure parameters associated with the linear coupling resonance in the AGS using the methods developed in Reference 2.

Theory

The theory of the linear coupling resonance is developed and discussed in detail in Reference 1. The basic parameter of this theory, for the case in which the horizontal and vertical tunes, Q_H and Q_V, are approximately equal, is the linear coupling coefficient

$$\kappa = \kappa_{skew} + \kappa_{solenoid}, \text{ where} \tag{1}$$

$$\kappa_{skew} = \frac{1}{4\pi R} \int_0^{2\pi} \sqrt{\beta_H(\theta)\,\beta_V(\theta)}\; K(\theta) \exp i\left[\mu_H(\theta) - Q_H\theta - \mu_V(\theta) + Q_V\theta\right] d\theta; \tag{2}$$

$$\kappa_{solenoid} = \frac{1}{8\pi} \int_0^{2\pi} \sqrt{\beta_H(\theta)\,\beta_V(\theta)}\; M(\theta)\, [\alpha_H(\theta)/\beta_H(\theta) -$$

$$\alpha_V(\theta)/\beta_V(\theta) - i/\beta_H(\theta) - i/\beta_V(\theta)]\; \exp\, i[\mu_H(\theta) - Q_H\theta -$$

$$\mu_V(\theta) + Q_V\theta]d\theta; \qquad (3)$$

$$\mu_H(\theta) = \int_0^\theta \frac{Rd\theta'}{\beta_H(\theta')}\;;\; \mu_V(\theta) = \int_0^\theta \frac{Rd\theta'}{\beta_V(\theta')}\;; \qquad (4)$$

$K(\theta)$ is proportional to the skew quadrupole gradient at azimuthal angle θ; $M(\theta)$ is proportional to the solenoid field at θ; R is the orbit circumference divided by 2π, and the other symbols have their usual meanings. (Note that Q_H and Q_V are the unperturbed tunes; i.e., the tunes one would measure in the absence of coupling.)

If $\kappa \neq 0$, then the betatron oscillations in the horizontal and vertical planes are coupled and the energy associated with a coherent oscillation excited in one plane will be periodically exchanged between the two planes as illustrated in Figure 1. Here the beam has been kicked in the horizontal plane by the tune meter kicker in the A-10 straight sectiion and the top and bottom traces show respectively the resulting filtered horizontal and vertical difference signals from the tune meter. The low-frequency envelopes of these traces show clearly the exchange of energy between the two planes. The period, T, of this energy exchange is easily measured directly from the oscilloscope traces and is given in terms of the parameters of the theory by

$$T = \frac{1}{f\sqrt{\Delta^2 + 4\|\kappa\|^2}}, \qquad (5)$$

where f is the revolution frequency, $\|\kappa\|$ is the modulus of κ, and

$$\Delta = Q_H - Q_V. \qquad (6)$$

Each signal shown in Figure 1 is a superposition of the two normal modes associated with the coupled betatron oscillations. The frequencies of these normal modes are

$$Q_1 = \frac{1}{2}(Q_H + Q_V) + \frac{1}{2}\sqrt{\Delta^2 + 4\|\kappa\|^2}$$

$$Q_2 = \frac{1}{2}(Q_H + Q_V) - \frac{1}{2}\sqrt{\Delta^2 + 4\|\kappa\|^2}. \qquad (7)$$

In general, an analysis of the frequency spectrum of the signals from the tune meter, for the case of coupled betatron oscillations, should yield peaks at the frequencies fQ_1 and fQ_2.

If the beam is kicked in the horizontal plane, the ratio of the minimum to maximum amplitude of the low frequency envelope of the resulting horizontal signal is

$$R = \frac{\|\Delta\|}{\sqrt{\Delta^2 + 4\|\kappa\|^2}} . \qquad (8)$$

Equation (8) can be used together with Equation (5) to obtain $\|\kappa\|$ and $\|\Delta\|$ in terms of T, R, and f:

$$\|\kappa\| = \frac{1}{2fT} \cdot \sqrt{1 - R^2}$$

$$\|\Delta\| = \frac{R}{fT} \qquad (9)$$

Thus, measurements of T, R, and f will yield the modulus of the coupling parameter and the separation of the unperturbed tunes. In general, it is difficult to obtain an accurate measurement of R (when $R \neq 0$) because the coherent betatron oscillations die away too rapidly. However, if the nu-quads are adjusted so that $\Delta = Q_H - Q_V = 0$, then $R = 0$ and $\|\kappa\|$ depends only on f and T which are both easily measured. Thus, the basic technique employed to determine $\|\kappa\|$ in these studies was to kick the beam in the horizontal plane, adjust the nu-quads so that the ratio $R = 0$, and measure the period T and revolution frequency f.

Observations and Results

Photographs of the filtered difference signals from the tune meter were taken at 2,000 Gauss clock counts ($P \simeq 1$ GeV/c) to determine $\|\kappa\|$ for several different settings of the zero-theta skew quads. These are shown in Figures 1-6 and the corresponding data are summarized in Table I. In each case the beam has been kicked in the horizontal plane and the upper and lower traces show respecitvely the resulting horizontal and vertical difference signals. The revolution frequency shown in the table was obtained from the rf frequency measured at 2,000 Gauss clock counts (GCC), and $\|\kappa\|$ was calculated using Equation (9) and the values of T obtained from the photographs. (The quoted error in $\|\kappa\|$ is due only to the error in the measurement of T; possible systematic effects are not included.)

A skew quad command of N counts in the table indicates that all 24 of the skew quads (located in straight sections 6 and 16 of each super-period) have been excited with 2N mA of current. In Figure 7, $\|\kappa\|$ is plotted as a function of the skew quad command, N, and a straight line has been fitted to the data. (For $N < 770$, $\|\kappa\|$ is plotted; for $N > 770$, $-\|\kappa\|$ is plotted.) The slope of the line is

$$\frac{\Delta\|\kappa\|}{\Delta N} = -3.12(4) \times 10^{-5}/\text{count} = -0.0156(2)/\text{Amp} \tag{10}$$

at a momentum of $P \simeq 1$ GeV/c (2,000 GCC). Using in Equation (2) the integrated strength of 21.37 Gauss/Amp for each skew quad (Jablonski and Buchanan, AGSCD Technical Note No. 128) and the values of β and μ obtained from the BEAM code, the contribution, κ_{SQ}, of the 24 skew quads to the coupling coefficient, κ, is found to be

$$\kappa_{SQ} = \left[0.019 + i(0.002)\right] I/P \tag{11}$$

where I is the current in Amps and P the momentum in GeV/c. The slope (10) calculated from our data agrees fairly well with this result.

The value , 0.024, of $\|\kappa\|$ measured at zero skew quad current is the magnitude of the intrinsic linear coupling coefficient of the machine at 1 GeV/c due to the skew quadrupole and/or solenoid fields inherently present in the machine. (These fields are presumably due to systematic tilting of the ring magnets, remanent fields, and the earth's magnetic field). At a skew quad command of 770 counts (1.5 Amps), Figure 4 shows that there is no net linear coupling between the horizontal and vertical planes, indicating that the intrinsic κ has been completely cancelled by the κ_{SQ} of the skew quads (at 2000 GCC).

By determining the amount of (zero theta) skew quad current necessary to null or minimize the linear coupling at a given GCC, we are effectively measuring the strengths of the fields responsible for the intrinsic coupling parameter of the machine at that point in the machine cycle. Since these fields contain a part which is constant over the machine cycle (due to remanent fields and the earth's field) and a part which scales linearly with the excitation current in the 240 ring magnets (due to systematic tilting of the magnets), we expect the amount of skew quad current required to null the coupling to vary linearly with the GCC during the machine cycle. (At high fields where the magnet iron becomes saturated, this is of course no longer true.)

Tables II and III list the amounts of skew quad current which were found neessary to either null or minimize the coupling at several different GCC during the machine cycle. (The data in Table II were obtained at various times in January and February while those in Table III were obtained during a dedicated study on March 1.) These data have been plotted in Figures 8 and 9 respectively and show clearly, with the exception of a few points, the expected linear dependence. The lines fitted to the data in the two figures are, respectively,

$$N = 550 + 0.084\ G \tag{12}$$

and

$$N = 615 + 0.111\ G, \tag{13}$$

where N is the skew quad command required to null (or minimize) the coupling and G is the number of GCC. Although it is not clear why it should matter, it should be noted that the apparently spurious point at 4500 GCC in Figure 8 was obtained on a 2.3 GeV/c flattop while all other points were obtained at non-zero $\dot{B}$ during the usual machine cycle. Also noteworthy is the fact that there are four iron-core skew quadruoples in the ring, used to correct coupling at high field, which may have been excited with different currents or have had different remanent fields at various times during these studies. This could possibly account for the departure of some of the points in Figure 8 from the linear relation obeyed by the other points. Since the data in

Figure 9 were all obtained during the same study period and appear more consistent than those in Figure 8, we shall use the fitted line (13) from Figure 9 in subsequent calculations.

Expressing the skew quad command, N, in terms of the skew quad current, I, (N = 500 I) and using the Gauss clock calibration

$$P(\mathrm{GeV/c}) = 0.054 + (5.074 \times 10^{-4})G \qquad (14)$$

obtained recently by L. Ahrens and P. Ingrassia in the range from 1400 to 5500 GCC, the fitted line (13) becomes

$$I = 1.206 + 0.438\ P \qquad (15)$$

where I is in Amps and P is in GeV/c. This result is consistent with data obtained by E. Raka ("Linear Coupling Correction in the Brookhaven AGS", AGS Div. 75-2, December 11, 1975). Now, since the integrated strength of each skew quad is 21.37 Gauss/Amp, the total integrated strength, S, for all 24 skew quads excited with the current I given in (15) is

$$S(\mathrm{Gauss}) = 24(21.37)I = 619 + 225\ P. \qquad (16)$$

This is the skew quad strength required to cancel the fields responsible for the intrinsic coupling parameter, κ, when the beam momentum is P. Since rotations of the 240 AGS magnets about the beam axis give rise to a zero-theta skew quadrupole field which scales with momentum, the second term in (16) can be used to estimate the angle of such rotations. Using R. Thern's field maps of the AGS magnets, the integrated gradient of these 240 magnets is found to be approximately $A + 8.43 \times 10^5\ P$ (Gauss), where A is a constant and P is in GeV/c. If all 240 magnets are rotated by angle θ about the beam axis, then the resulting zero-theta skew quadrupole strength will be

$$S = (A + 8.43 \times 10^5\ P) \sin 2\theta.$$

Comparing this with (16) we have

$$225 = (8.43 \times 10^5) \sin 2\theta, \text{ which gives } \theta = 0.13 \text{ milliradians.} \qquad (17)$$

In addition to determining the skew quad current necessary to null coupling, the skew quads were turned off and a measurement of $\|\kappa\|$ was made at each of the various momenta listed in Table III. Figures 10-14 show the photographs of the filtered difference signals from which $\|\kappa\|$ was determined, and the corresponding data are summarized in Table IV. As before, the beam has been kicked in the horizontal plane and the upper and lower traces show respectively the resulting horizontal and vertical difference signals. For each momentum $\|\kappa\|$ was calculated using in Equation (9) the measured revolution frequency, f, and the values of T obtained from the photographs. Since these values of $\|\kappa\|$ are the magnitudes of the intrinsic linear coupling parameter of the machine, they should agree with the values of $\|\kappa_{SQ}\|$ required to null the coupling at each momentum. The final column in Table IV lists the values of $\|\kappa_{SQ}\|$ obtained by substituting Equation (15) into Equation (11). These are in good agreement with the values of $\|\kappa\|$.

The normal mode frequencies, fQ_1, and fQ_2, present in the tune meter signals were measured using the fast-Fourier-transform (FFT) capability of the 125 MHz Digital oscilloscope (#9400) on loan from LeCroy. An example of the frequency spectrum obtained with this device is shown in Figure 15. Here the upper trace shows the filtered vertical difference signal from the tune meter when the beam has been kicked in the horizontal plane. The lower trace is the frequency spectrum of this signal. The two large peaks in the spectrum occur at frequencies of 248 and 284 kHz respectively and are presumably the normal mode frequencies. Both peaks have the same magnitude--a feature which is predicted by the theory. The two frequencies are in fact very close to the coherent frequencies fQ_H and fQ_V measured with the coupling nulled. This is due to the fact that for large separations of the unperturbed tunes (e.g., $Q_H >> Q_V$) the difference between Q_1 and Q_H and between Q_2 and Q_V becomes very small (see Equation (7)). A more careful study in which Q_1 and Q_2 are measured for different values of $\|\kappa\|$ and $\Delta = Q_H - Q_V$ is planned. Preliminary experience with the LeCroy device and its on-line FFT capability has shown it to be very powerful in analyzing signals in this frequency range. (The device has been ordered and should arrive in March.)

Fig. 1

Horizontal Signal

Skew Quads = 200

Vertical Signal

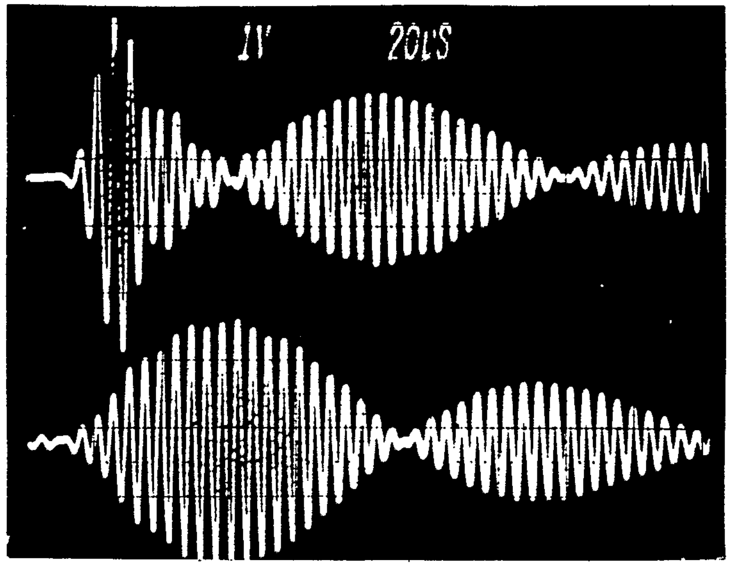


Fig. 2

Horizontal Signal

Skew Quads = 0

Vertical Signal

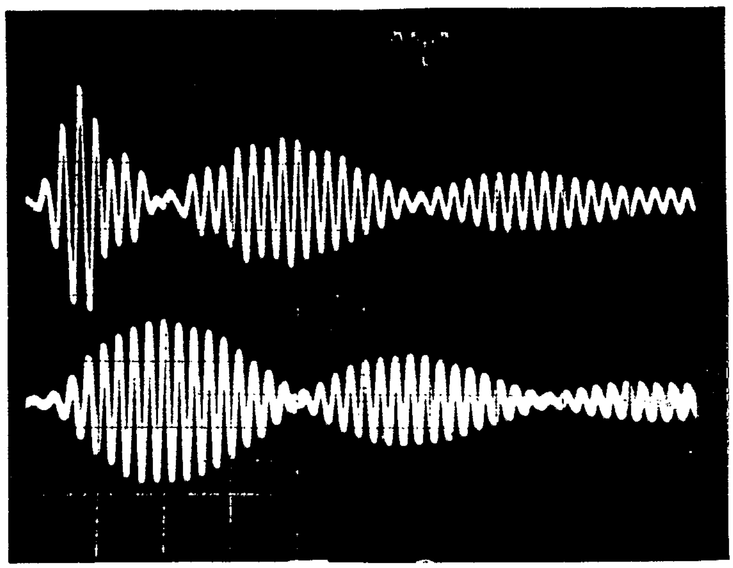


Fig. 3

Horizontal Signal

Skew Quads = 400

Vertical Signal

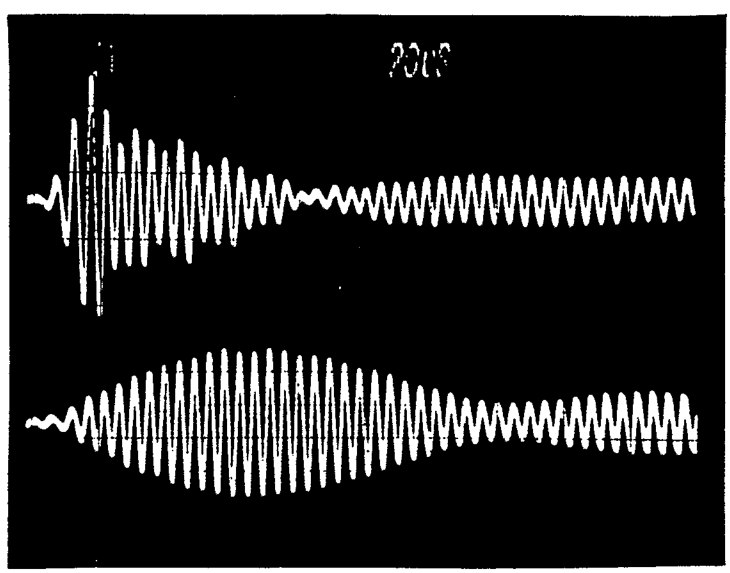


Fig. 4

Horizontal Signal

Skew Quads = 770

Vertical Signal

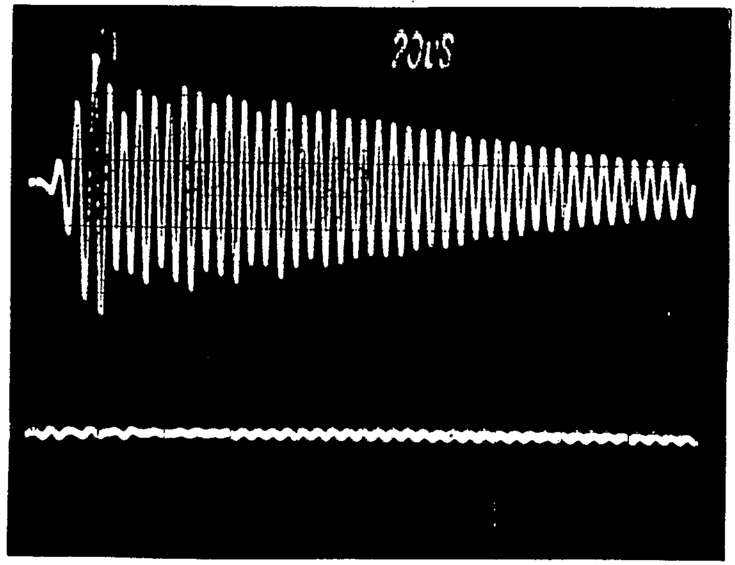


Fig. 5

Horizontal Signal

Skew Quads = 1200

Vertical Signal

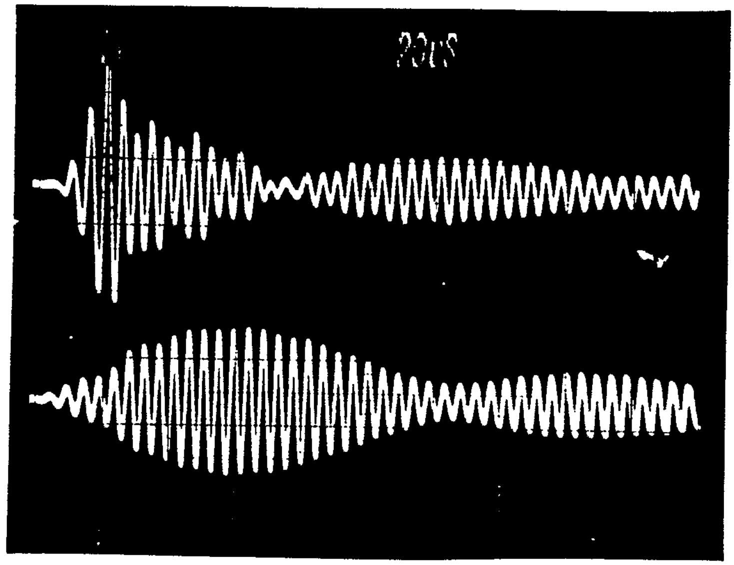
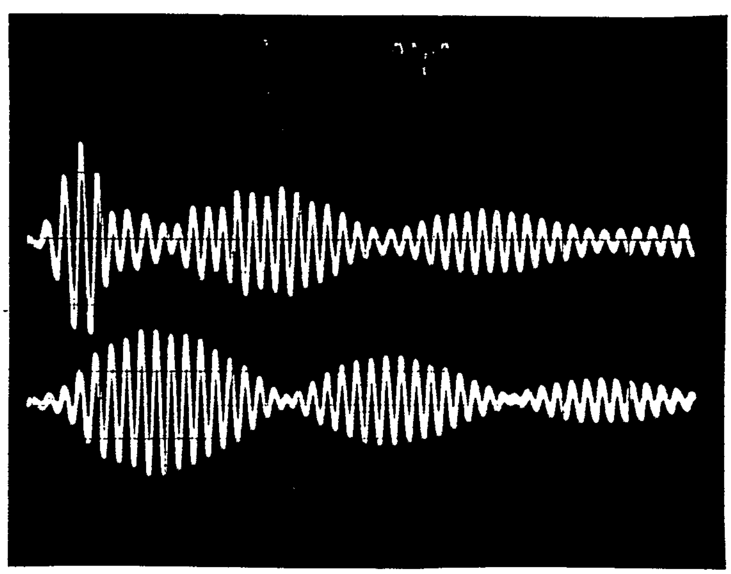


Fig. 6

Horizontal Signal

Skew Quads = 1600

Vertical Signal



LINEAR COUPLING COEFFICIENT AT 2000 GCC (1 GEV/C)
SLOPE= -0.003115 +/- 0.000037
INTERCEPT= 2.398901 +/- 0.028729

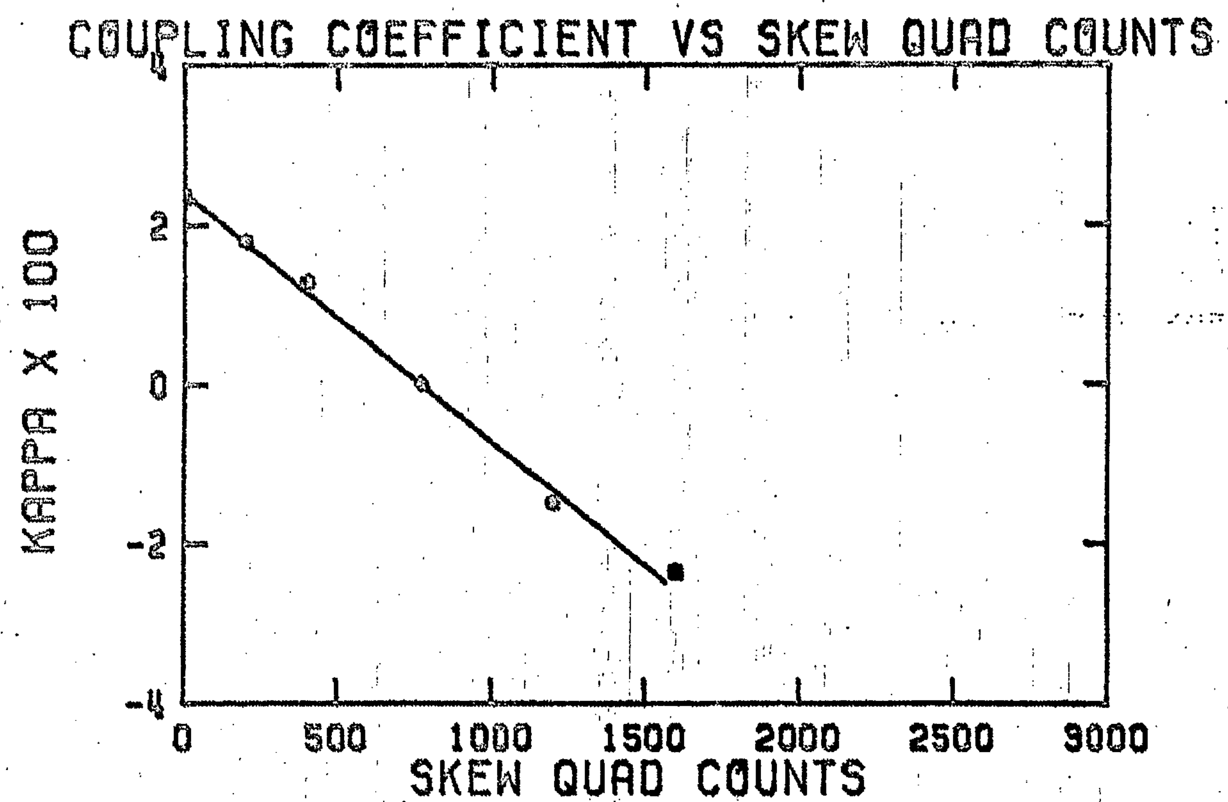


Figure 7

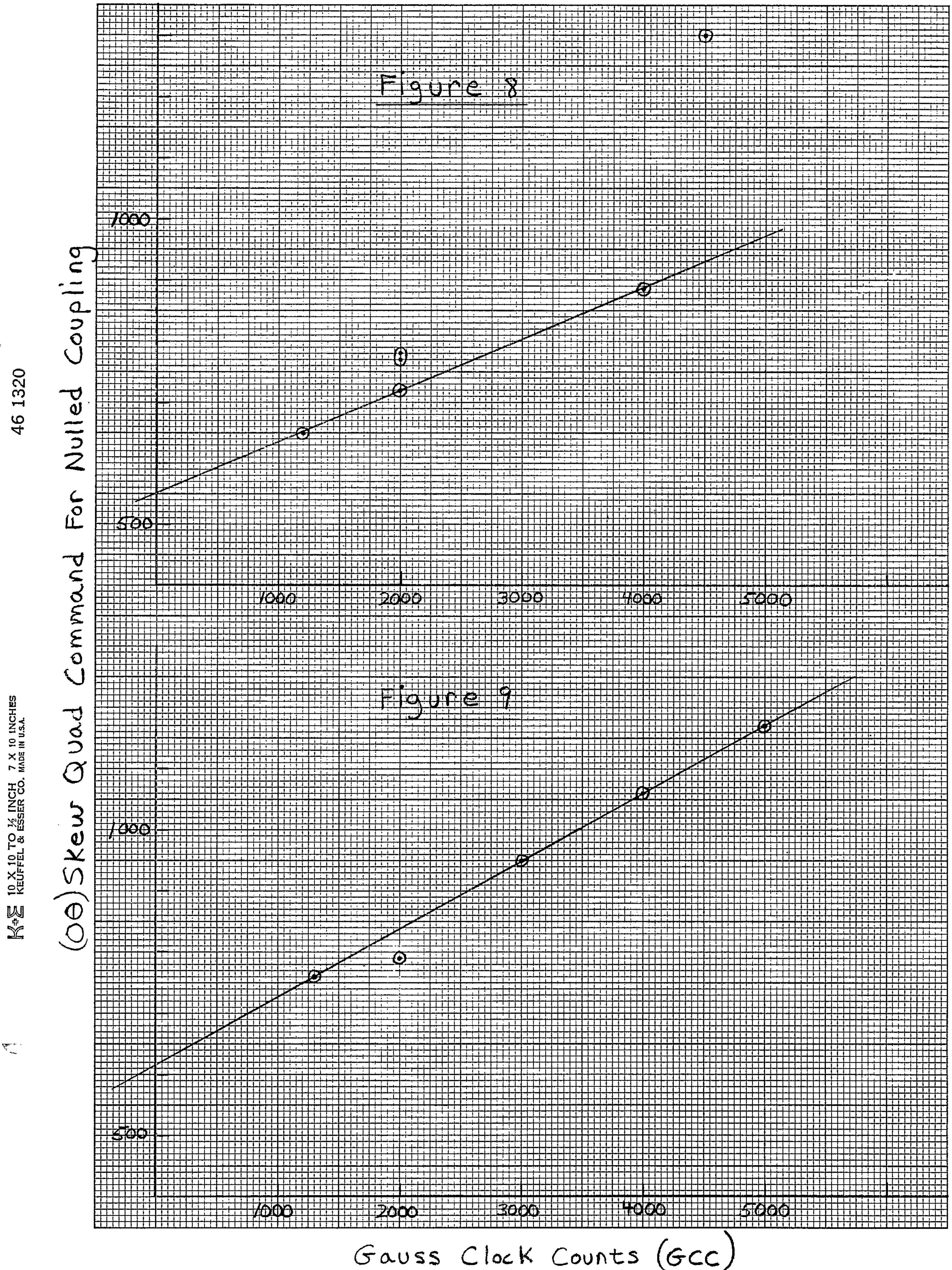


Figure 8
1000
500
1000
2000
3000
4000
5000
Figure 9
1000
500
1000
2000
3000
4000
5000
(OO) Skew Quad Command For Nulled Coupling
Gauss Clock Counts (GCC)
46 1320
K+E 10 X 10 TO ½ INCH 7 X 10 INCHES
KEUFFEL & ESSER CO. MADE IN U.S.A.

Fig. 10

Horizontal Signal

(1300 GCC)

Vertical Signal

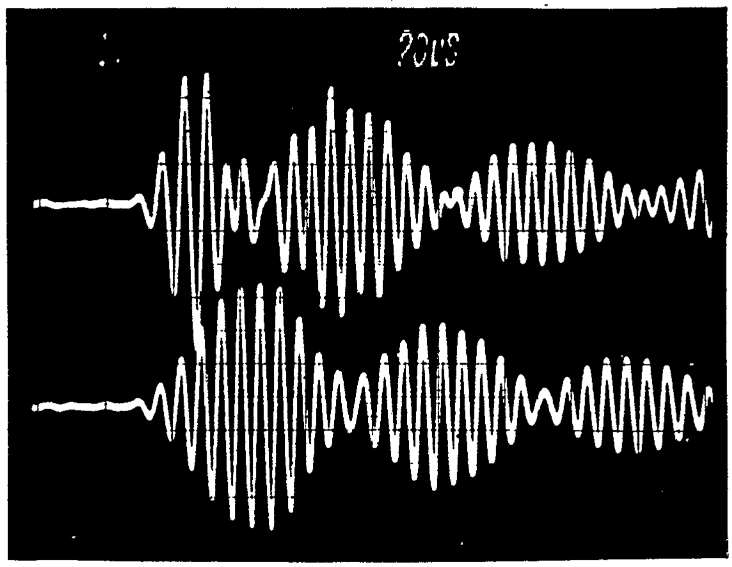


Fig. 11

Horizontal Signal

(2000 GCC)

Vertical Signal

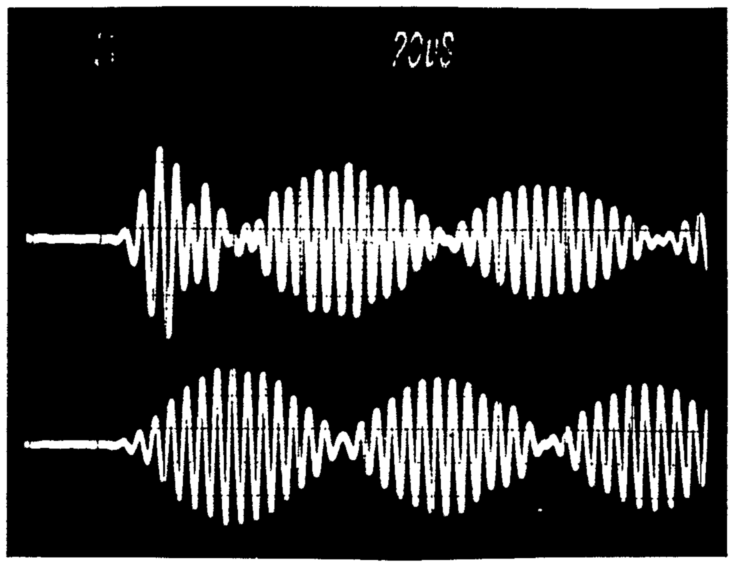


Fig. 12

Horizontal Signal

(3000 GCC)

Vertical Signal

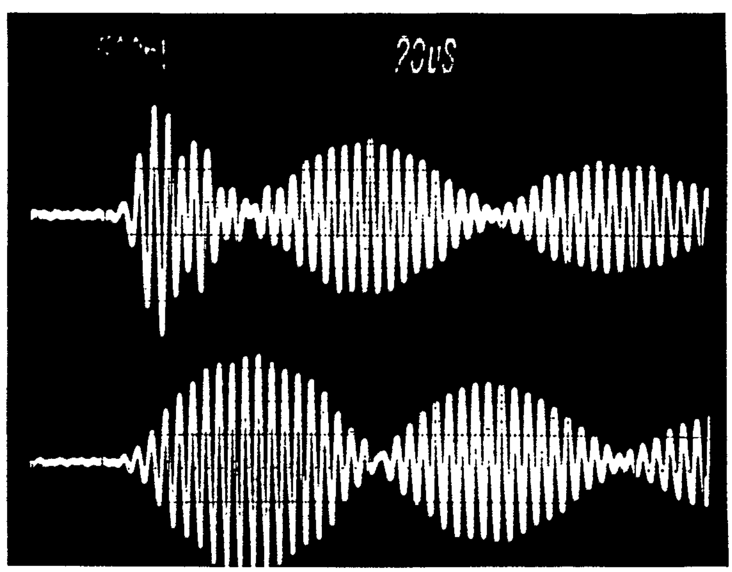


Fig. 13

Horizontal Signal

(4000 Gcc)

Vertical Signal

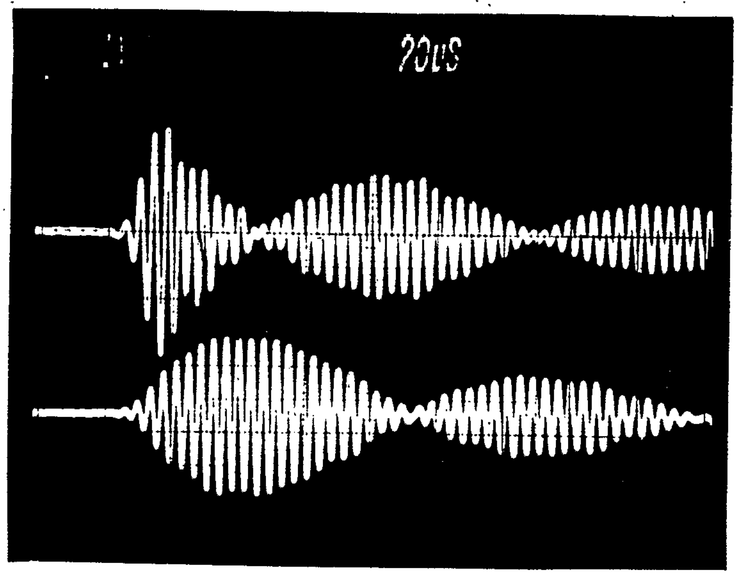
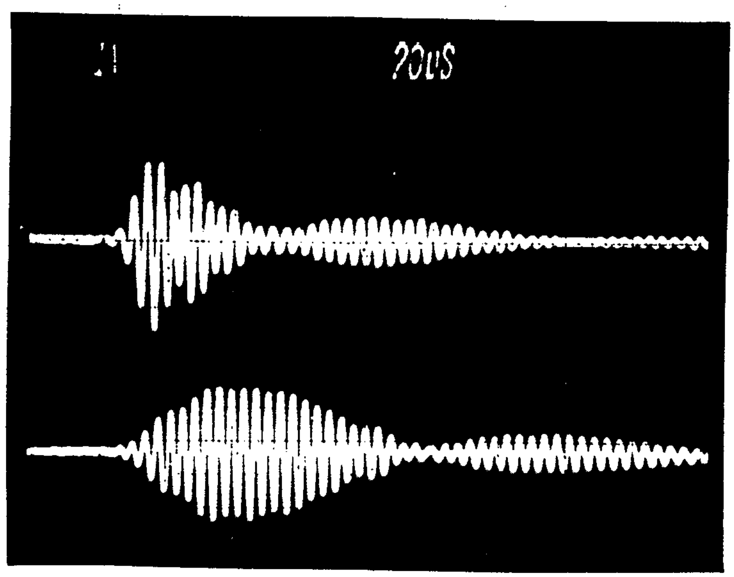


Fig. 14

Horizontal Signal

(5000 Gcc)

Vertical Signal



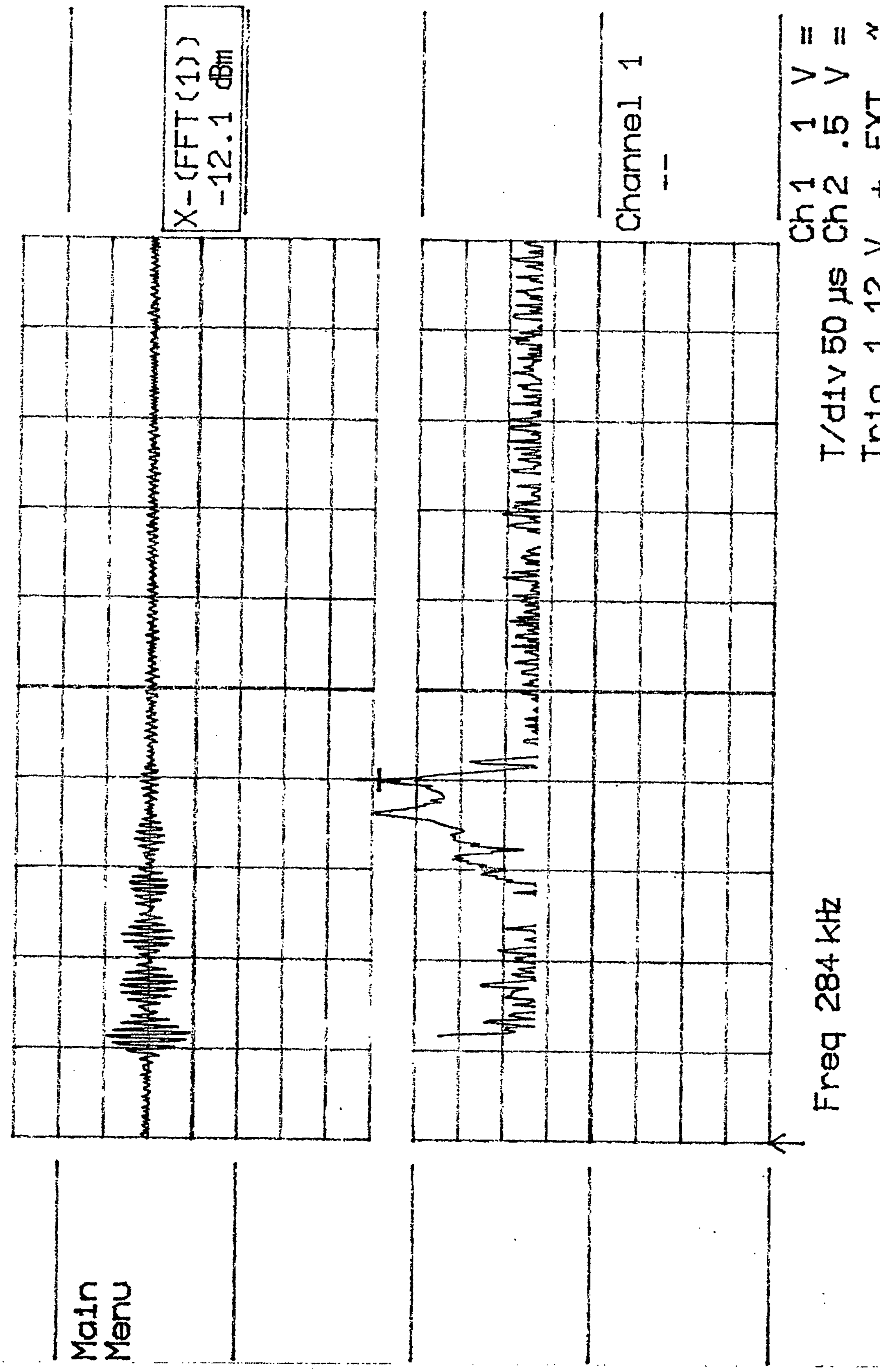

X-(FFT(1))
-12.1 dBm
Channel 1
--
Ch 1 1 V =
Ch 2 .5 V =
T/div 50 µs
Trig 1.12 V + EXT
Freq 284 kHz
Main
Menu

Figure 15

Table I

Figure	Skew Quad Command	T (μs)	f (MHz)	‖κ‖
1	200	100 ± 2	0.2787	0.0179(4)
2	0	76 ± 2	0.2790	0.0236(6)
3	400	140 ± 4	0.2790	0.0128(4)
4	770	∞	0.2790	0
5	1200	120 ± 4	0.2790	0.0149(5)
6	1600	76 ± 2	0.2790	0.0236(6)

Table II

GCC	P(GeV/c)*	Skew Quad Command For Nulled Coupling
1200	0.66	650 (1.30 Amps)
2000	1.07	770 (1.54 Amps)
2000	1.07	780 (1.56 Amps)
2000	1.07	720 (1.44 Amps)
4000	2.08	885 (1.77 Amps)
4500	2.34	1300 (2.60 Amps)

*These momenta and those in Tables III and IV, were calculated using the Gauss clock calibration $P(GeV/c) = 0.054 + (5.074 \times 10^{-4})GCC$ obtained recently by L. Ahrens and P. Ingrassia in the range from 1400 to 5500 GCC.

Table III

GCC	P(GeV/c)*	Skew Quad Command for Nulled Coupling
1300	0.71	760 (1.52 Amps)
2000	1.07	790 (1.58 Amps)
3000	1.58	950 (1.90 Amps)
4000	2.08	1060 (2.12 Amps)
5000	2.59	1170 (2.34 Amps)

Table IV

Figure	Gcc	P(GeV/c)*	T (μs)	F (MHz)	$\|\kappa\|$	$\|\kappa_{SQ}\|$
10	1300	0.71	60 ± 4	0.2246	0.037(2)	0.041
11	2000	1.07	64 ± 2	0.2761	0.0283(9)	0.030
12	3000	1.58	72 ± 2	0.3188	0.0218(6)	0.023
13	4000	2.08	84 ± 2	0.3385	0.0176(4)	0.019
14	5000	2.59	92 ± 2	0.3489	0.0156(3)	0.017

*See comment under Table II.