

## Methodology for designing Complex Bend lattice

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Every cell has the following properties:

	K1	K2	K3	K4	type
	0	0	0	0	drift
	1	0	0	0	dipole
	0	1	0	0	quad
	0	0	1	0	sextupole
	0	0	0	1	octupole
	1	1	0	0	CB

The subject of optimization is to select best parameters of each of the 264 elements. Elements with the same properties may be combined. The ranges for the 3 CB elements and the BM sources are defined in a special excel document by Bernie Kosciuk.

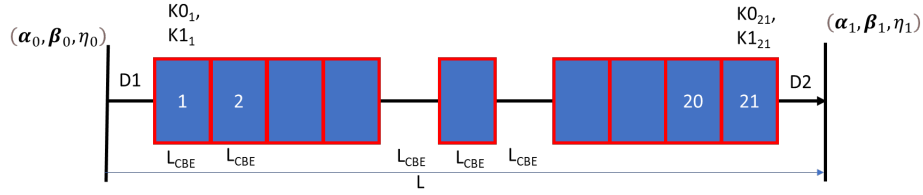
The range of a single cell will be split into 6 regions with approximate count of elements as follows:

- ID straight with matching triplet (doublet), 53 elements,
- Outer CB 1, 17 elements,
- Dispersion bump 1, 36 elements,
- Central CB element with a placeholder for the source for a BM beamline, 52 elements,
- Dispersion bump 2, 36 elements,
- Outer CB 2, 17 elements,
- ID straight with a matching triplet (doublet), 53 elements.

Each region should be optimized separately using MAD, ELEGANT or minimizers in Python. Below we look at optimization of the 3 CB elements, focusing on the central one.

## 2. Optimizing the central CB element

- Define the s-range of central CB
- Define the location of BM source in this s-range
- Determine vector  $(\alpha, \beta, \eta)$  at the beginning and end of the s-range
  - Take these values from ESRF-EBS or APS-U
  - Subject to further optimization
- Break s-range into 21 elements and 2 drifts in the following way CB : LINE=(D1,21xCBE,D2)
  - Drifts D1 and D2 may not be necessary for initial runs
  - CBEs have the same L and different parameters K0 and K1
  - CBE = CBE magnet +CBE gap
  - Some CBEs can have K0=K1=0, which means drift for BM magnet, vacuum, diagnostics or correction elements



- Approximate 2 subsequences of CBEs with  $K0(s) = \sum_0^3 A0_n s^n$  and  $|K1(s)| = \sum_0^3 |A1_n| s^n$ 
  - This gives 10 parameters (4 A0, 4 A1, D1(L), D2(L)), which are used as tuning knobs
  - K0s are of the same polarity, while K1s are of alternative polarity
- Matrix representation of CB line
  - Transport Matrix element:  $M(s_n) = \prod_1^n MCBE_n$
  - Twiss functions  $M(s_n) = f(\alpha, \beta, \eta)$
  - Average values  $\alpha_n, \beta_n, \eta_n$ : will use these for estimates
- 3 constraints
  - Overall length  $L=21 * L_{CBE} + D_1 + D_2$
  - Overall angle  $\rightarrow \frac{L_{CBE}}{BR} \sum_1^{21} K0_n = \text{CB angle}$
  - $K0 \leq 0.5 \text{ T}, K1 \leq 130 \text{ T/m}$
- 10 optimization goals  $G_i$  (not independent)
  - Tune throughout CB for sextupole / octupole compensation:  $\mu_0$
  - Low emittance:  $I_5 = \int_0^L \frac{\eta^2 + (\beta\eta' + \alpha\eta)^2}{\beta|\rho|^3} ds \rightarrow \min, \rho = \frac{BR}{K0}$
  - Stable machine:  $I_4/I_2 < 0.5$
  - Low chromaticity:  $\xi = \frac{-1}{4\pi} \int_0^L \mathbf{K1}\beta ds \rightarrow \min$
  - Matching condition:  $(\alpha, \beta, \eta)$  at  $s=L$
- Solve 10 x 10 system and determine optimal D1, D2, K0\_n, K1\_n
- Note: actual number of coefficients may vary depending on the number and the specific arrangement of the CB elements.
- Every lattice is expressed in a 264 vector of the cells (inputs) and lattice parameters (output). Then the lattice parameters  $G_i$  are summed up with weights to create a unique qualifier or a merit parameter for a given lattice candidate:  $F = \sum G_i^2 / W_i$ .

### 3. Optimizing the outer CB elements

The difference between optimization of the central and outer CB elements is in the specific conditions for dispersion at the beginning of the first outer CB element and the end of the second outer CB element. The dispersion function needs to be set to zero in these two points, while at the edges of the central CB element the dispersion is non-zero, while it's derivative should be maximized.

### 4. Dispersion bumps

Following the MBA concept discovered [5] and implemented at ESRF-EBS [6] the TCBA lattice uses intra-cell compensation of the driving terms. Therefore, a sing TCBA cell contains two regions with

non-zero dispersion for compensation of chromaticity, aka Dispersion Bumps. These regions contain a triplet or a quadruplet of quadrupoles and nonlinear lattice elements. One of the major constraints for a Dispersion Bump region is a high value of dispersion to minimize the required strength of the chromatic sextupoles for the chromaticity reduction.

One of the remarkable features of TCBA as compared with the MBA design is the absence of longitudinal gradient bends. The power of the CB concept should enable integration of the LGB and several quads in the straights into the two CB elements, thereby saving substantial amount of space in the lattice, while reducing the number of components and power supplies.

## 5. Conclusions

In this note we discussed a new approach for the lattice design. The methodology explores the power of Complex Bend concept and offers the following benefits:

- Binning of the lattice space reduces the number of free variables associated with the element length and their placement. Once the robust lattice is obtained, we can change the length of the elements to optimize the gradients and adjust spacings between the former for finely tuned lattice performance.
- Standardization of the elements into 6 cell types permits straightforward initial lattice design working with optimization of Twiss parameters, chromaticity correction and assessment of DA and MA.
- Complex Bends combine dipole and quadrupole components create many densely spaced knobs for tailoring Twiss parameters according to the constraints.
- Using polynomials in the Complex Bends knobs permits to reduce the number of variables in the optimization problem and match the number of knobs to the number of optimization constraints (goals).
- Computation of the values of optimization constraints is simple for the binned lattice and they can be expressed in simple sums. Synchrotron integrals can be calculated for separate CBs and with the 3 values per cell, the whole ring lattice performance can be calculated.
- Every lattice candidate is expressed in a 264 vector of the cells (inputs) and lattice parameters (output). Then the lattice parameters are summed up with weights to create a unique qualifier for the candidate. The best candidate wins the race.

Constraints coming from the nonlinear particle dynamics will be discussed in the next tech note.

## References

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