

BNL-223637-2022-TECH C-A/AP/681

## Integrated Fields of Permanent Magnet Dipole Trims

S. Brooks

October 2021

Collider Accelerator Department Brookhaven National Laboratory

## **U.S. Department of Energy**

USDOE Office of Science (SC), Nuclear Physics (NP) (SC-26)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No.DE-SC0012704 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

## DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

## Integrated Fields of Permanent Magnet Dipole Trims

Stephen Brooks

October 26, 2021

To adjust the integrated dipole field of a magnet (through the longitudinal z axis), pairs of small permanent magnets may be added at the end, symmetrically above and below the axis. If these are sufficiently small, they may be approximated by magnetic dipole point sources. The field of a single dipole source located at the origin is

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{3\hat{\mathbf{x}}(\hat{\mathbf{x}} \cdot \mathbf{m}) - \mathbf{m}}{|\mathbf{x}|^3} = \frac{\mu_0}{4\pi} \frac{3\mathbf{x}(\mathbf{x} \cdot \mathbf{m}) - \mathbf{m}|\mathbf{x}|^2}{|\mathbf{x}|^5},$$

where  $\hat{\mathbf{x}} = \mathbf{x}/|\mathbf{x}|$  and  $\mathbf{m}$  is the magnetic dipole moment, defined as

$$\mathbf{m} = V\mathbf{M} = V\frac{\mathbf{B}_r}{\mu_0}$$

for a permanent magnet of volume V with residual field vector  $\mathbf{B}_r$ .

Suppose the source is a distance y vertically below the axis, then setting  $\mathbf{x} = (0, y, z)$  will give the fields on-axis as a function of z. To get a vertical dipole field, assume the source is magnetised in the y direction, so that  $\mathbf{m} = (0, VB_{ry}/\mu_0, 0)$ . The field is now given by

$$\mathbf{B}(0,y,z) = \frac{\mu_0}{4\pi} \frac{3(0,y,z)(yVB_{ry}/\mu_0) - (0,VB_{ry}/\mu_0,0)(y^2+z^2)}{(y^2+z^2)^{5/2}} \\ = \frac{VB_{ry}}{4\pi} \frac{3(0,y,z)y - (0,1,0)(y^2+z^2)}{(y^2+z^2)^{5/2}}.$$

The unwanted longitudinal field component can be eliminated by adding a second identical source a distance y above the axis, producing a combined field from the two sources of:

$$\mathbf{B}^{\pm y}(z) = \mathbf{B}(0, y, z) + \mathbf{B}(0, -y, z) = \frac{VB_{ry}}{2\pi} \frac{3(0, y^2, 0) - (0, 1, 0)(y^2 + z^2)}{(y^2 + z^2)^{5/2}},$$

which has only a vertical component

$$B_y^{\pm y}(z) = \frac{VB_{ry}}{2\pi} \frac{2y^2 - z^2}{(y^2 + z^2)^{5/2}}.$$

The maximum field is achieved at z = 0:

$$B_y^{\pm y}(0) = \frac{V B_{ry}}{2\pi} \frac{2}{|y|^3},$$

although note at far distances  $|z| > \sqrt{2}|y|$ , the field actually becomes negative but with a much smaller magnitude. The integrated field is

$$\int_{-\infty}^{\infty} B_y^{\pm y}(z) \, dz = \frac{V B_{ry}}{2\pi} \int_{-\infty}^{\infty} \frac{2y^2 - z^2}{(y^2 + z^2)^{5/2}} \, dz$$
$$= \frac{V B_{ry}}{2\pi} \left[ \frac{z(2y^2 + z^2)}{y^2(y^2 + z^2)^{3/2}} \right]_{-\infty}^{z=\infty}$$
$$= \frac{V B_{ry}}{2\pi} \frac{2}{y^2}.$$

This leads to the relation

$$\int_{-\infty}^{\infty} B_y^{\pm y}(z) \,\mathrm{d}z = B_y^{\pm y}(0)|y|,$$

in other words, integrated field is just peak field multiplied by the vertical distance to the magnets. This is useful empirically because it is often quicker to measure a peak field with a Hall probe than an integrated field through an infinitely-long axis.