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Synchronizing ESR and HSR for collisions at IP6 and IP8

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Synchronizing ESR and HSR for collisions at IP6 and IP8

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Abstract

Bunch circulation times in the ESR and HSR must be identical, whatever the electron and hadron energies. A particular bunch in either ring can only have collisions at either IP6 or IP8, but not at both. Synchronization is achieved by co-ordinating the design trajectory circumferences of the two rings, and by controlling the HSR and ESR path lengths from IP6 to IP8. Radial shift control of the HSR circumference is summarized. The conditions for synchronization are presented. The impact of operating HSR at low proton energy with 4 inner and 2 outer arcs is assessed. Changes of the ESR circumference due to the use of super-bends at the lowest electron energies are evaluated. The effect of J_x-tuning on the ESR circumference and its compensation by an HSR radial shift are discussed. Finally, the potential effects of using a Ring Cooler for hadron cooling are considered.

1 Matching ESR and HSR circumferences

Electrons in the ESR are ultra-relativistic, with $1 - \beta_e < 5.2 \times 10^{-9}$ over the energy range from 5 GeV to 18 GeV, where β_e is the electron speed divided by c, the speed of light. Their revolution period is virtually unchanged in all operational scenarios, with minor exceptions when the ESR super-bends are re-tuned for 5 GeV operation, and if J_x-tuning is used. (Both of these effects are discussed below.) In contrast, HSR hadrons are merely relativistic: $1 - \beta_p$ for protons varies from 44.0×10^{-6} to 5.8×10^{-6} over their nominal energy range from 100 GeV to 275 GeV.

The time periods of the HSR and ESR closed orbits must be the same. For the ESR, it is reasonable to assume a speed of c – only very minor corrections are required. For the HSR, proton beams with a total energy as small as 100 GeV are shifted radially inward in the arcs, while beams as high in energy as 275 GeV are shifted radially outward, so that the orbit period is held constant across the entire proton energy range.

The radial shift is turned off in the first stages of lattice design. The HSR then has a closed orbit length of $C_{\rm HSR}$. At 275 GeV, the beam is radially shifted outward in the arcs so as to increase the orbit length by ΔC_{275}. Similarly, inward radial shifts at 100 GeV decrease the closed orbit length by ΔC_{100} at 100 GeV. The conditions for the HSR period to match the ESR period at both ends of the proton energy range are therefore

$$C_{\rm ESR} \;=\; \frac{C_{\rm HSR} + \Delta C_{275}}{\beta_{275}} \;=\; \frac{C_{\rm HSR} - \Delta C_{100}}{\beta_{100}} \tag{1}$$

where $C_{\rm ESR}$ is the length of the design closed orbit in the ESR. Assuming (for now) that the absolute value of the circumference change is the same at both ends of the energy range,

$$\Delta C_{100} \;=\; \Delta C_{275} \;=\; \Delta C \tag{2}$$

then the ESR circumference is

$$C_{\rm ESR} \;=\; \frac{2}{\beta_{275} + \beta_{100}} C_{\rm HSR} \;=\; 1.00002492 \times C_{\rm HSR} \tag{3}$$

and the maximum HSR circumference change is

$$\Delta C \;=\; \frac{\beta_{275} - \beta_{100}}{\beta_{275} + \beta_{100}} C_{\rm HSR} \;=\; 73.226 \text{ mm} \tag{4}$$

Taking values from the lattice release HSR-220613a, the orbit period of a 275 GeV proton beam with no radial shift is

$$T_{\mathrm{HSR}} \;=\; 12.788546521\ \mu\mathrm{s} \tag{5}$$

corresponding to

$$C_{\mathrm{HSR}} \;=\; \beta_{275} c\, T_{\mathrm{HSR}} \;=\; 3833.887480\ \mathrm{m} \tag{6}$$

and

$$C_{\mathrm{ESR}} \;=\; 3833.983022\ \mathrm{m} \tag{7}$$

The circumference of the RCS

$$C_{\mathrm{RCS}} \;=\; \left(\frac{316}{315}\right) C_{\mathrm{ESR}} \;=\; 3846.154396\ \mathrm{m} \tag{8}$$

is in turn set by the ESR circumference.

Heavy ions must operate within the HSR radial shift range that is necessary for proton operations. Since the proton mass is almost identical to the atomic mass unit mass

$$\begin{aligned} m_p &= 0.938272\ \mathrm{GeV/c^2} \\ m_u &= 0.931494\ \mathrm{GeV/c^2} \end{aligned} \tag{9}$$

then protons with energy E GeV travel at almost the same speed as heavy ions of energy E GeV/u. Thus, the range of gold ion energies that can circulate in 3 inner and 3 outer arcs is

$$99.277\ \mathrm{GeV/u} \;\le\; E_{\mathrm{Au}} \;\le\; 110\ \mathrm{GeV/u} \tag{10}$$

with a minimum that is numerically close to the minimum proton energy of 100 GeV. The maximum gold ion energy is limited by the maximum rigidity of the HSR magnets.

2 RF system parameters

The highest RF frequency in both ESR and HSR is approximately 591 MHz, while crab cavities in the ESR and HSR operate at about 394 MHz and 197 MHz, respectively. The corresponding harmonic numbers

$$\begin{aligned} h_{591} &= 7560 = 3 \times (2^3 \cdot 3^2 \cdot 5 \cdot 7) \\ h_{394} &= 5040 = 2 \times (2^3 \cdot 3^2 \cdot 5 \cdot 7) \\ h_{197} &= 2520 = \quad (2^3 \cdot 3^2 \cdot 5 \cdot 7) \end{aligned} \tag{11}$$

indicate that 2520 buckets are available to accept bunches in both the ESR and the HSR, spaced in time by

$$T_0 \;=\; \frac{C_{\mathrm{ESR}}}{h_{197} c} \;=\; 5.074917\ \mathrm{ns} \tag{12}$$

during collisions using the HSR-220613a lattice. Although the bucket spacing is the same *in time* for ESR and HSR, there is a slight difference in their spacing *in space*, since electrons move faster than hadrons.

At injection into the HSR as many as 290 bunches are captured in consecutive buckets of a 24.6 MHz system, with a harmonic number of

$$h_{25} \;=\; 315 \;=\; 3^2 \cdot 5 \cdot 7 \tag{13}$$

At least 25 consecutive buckets are left empty for the HSR abort gap, for a total of 290 + 25 = 315 buckets. Injected bunches are later split into 4 parts for a maximum of 1160 bunches, spaced by 2 buckets in the 197 MHz system.

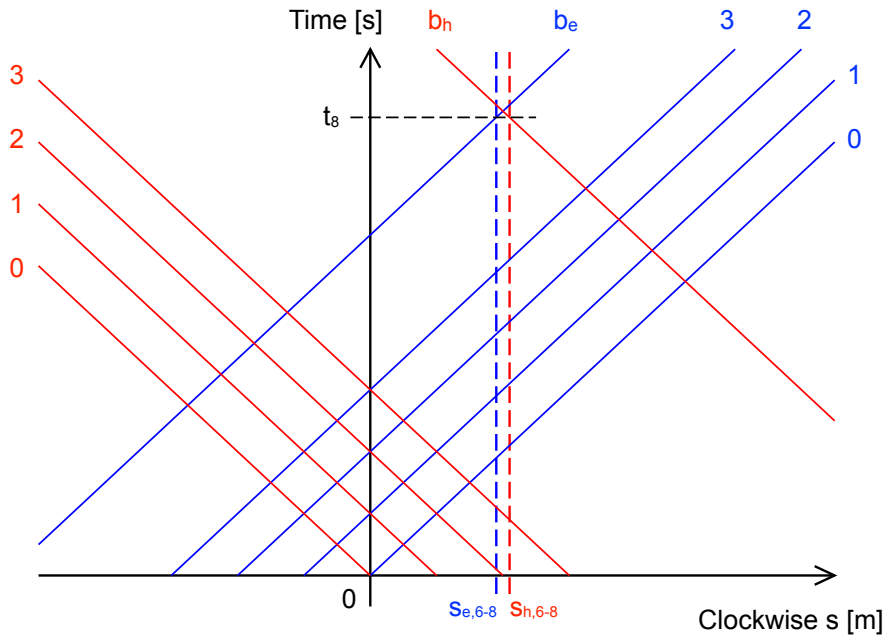


Figure 1: The space-time motion of available buckets in ESR (blue) and HSR (red), labeled respectively by integers b_e and b_h that run from 0 to 2519. Electron buckets move clockwise, so s (which in all cases increases in the clockwise direction) increases with time. Hadrons move in the counter-clockwise direction, and so for their buckets s decreases with time. In both rings bucket 0 passes $s = 0$ at IP6 when $t = 0$. The small difference between the two path lengths from IP6 to IP8, $s_{e,6\to 8}$ and $s_{h,6\to 8}$, is greatly exaggerated.

3 Synchronizing collisions at IP6 and IP8

The bunch intensities are so high that each bunch may only collide once per turn – either at IP6 or at IP8, but not at both. This is guaranteed by adjusting the electron and hadron path lengths from IP6 to IP8, as follows. Number the ESR and HSR buckets by integers b_e and b_h that vary from 0 to 2519, as illustrated in Figure 1. Hadron bunches will only occupy buckets with b_h even.

The time at which the center of electron bucket b_{e6} passes IP6 on turn N_e is given by

$$t_{e6}(b_{e6}, N_e) \;=\; t_{e06} + (b_{e6} + h_{197}N_e)T_0 \tag{14}$$

Similarly, the center of hadron bucket b_{h6} passes IP6 on turn N_h at

$$t_{h6}(b_{h6}, N_h) \;=\; t_{h06} + (b_{h6} + h_{197}N_h)T_0 \tag{15}$$

For high energy operation (when hadrons pass through 3 inner and 3 outer arcs), and assuming

$$t_{e06} \;=\; t_{h06} = 0 \tag{16}$$

then collisions will occur at IP6 when

$$t_{e6} \;=\; t_{h6} \tag{17}$$

requiring

$$\begin{aligned} b_{e6} &= b_{h6} \\ N_e &= N_h \end{aligned} \tag{18}$$

Define $T_{e,6\to 8}$ to be the time for an electron to travel from IP6 to IP8, and $T_{h,8\to 6}$ to be the time for a hadron to travel from IP8 to IP6. Then the arrival times of the electron and hadron bucket centers at IP8

are, respectively,

$$t_{e8}(b_{e8}, N_e) = t_{e06} + (b_{e8} + h_{197}N_e)T_0 + T_{e,6\to 8} \quad (19)$$
$$t_{h8}(b_{h8}, N_h) = t_{h06} + (b_{h8} + h_{197}[N_h + 1])T_0 - T_{h,8\to 6} \quad (20)$$

For bunches to collide at IP8

$$t_{e8} = t_{h8} \quad (21)$$

With the condition $t_{e06} = t_{h06} = 0$ from above, then those collisions require

$$T_{e,6\to 8} + T_{h,8\to 6} = pT_0 \quad (22)$$

where p is the integer

$$p = b_{h8} - b_{e8} + h_{197}(N_h + 1 - N_e) \quad (23)$$

The indices of buckets that collide at IP8 are therefore related by

$$b_{h8} = \mathrm{mod}(b_{e8} + p,\ h_{197}) \quad (24)$$

They collide when either

$$N_e = N_h \quad \text{or} \quad (25)$$
$$N_e = N_h + 1$$

depending on whether b_{e8} is greater or smaller than b_{h8}. The integer p must be odd, in order for bunches that collide at IP8 to not collide at IP6, at least if almost all hadron buckets are occupied. (Note that p need not be precisely an integer if there is no detector at IP8, although it should be nearly so, in order to minimize the effect of parasitic long-range beam-beam collisions.)

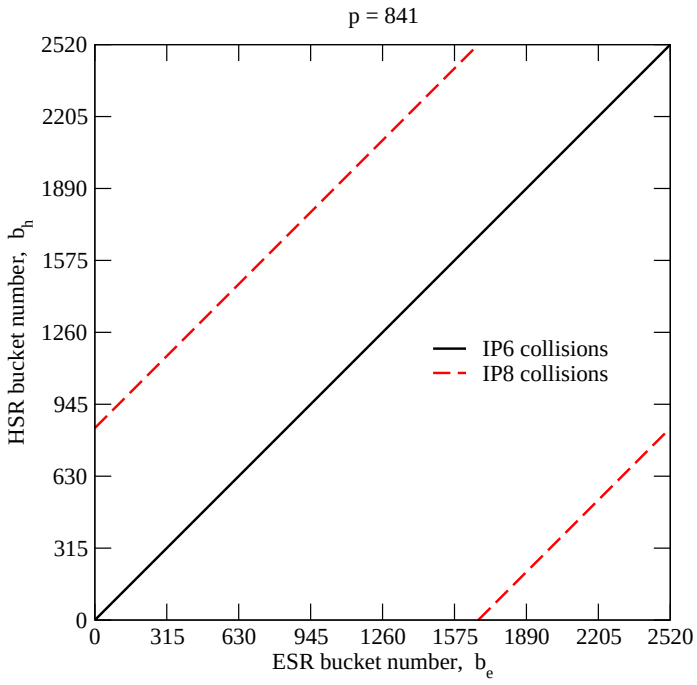


Figure 2: Colliding 197 MHz buckets at IP6 and IP8, following Equations 18 and 24.

Since the distance between IP6 and IP8 is approximately 1/6 of the orbit circumference, then

$$p \approx h_{197}/3 = 840 \quad (26)$$

In fact (see below) current lattice designs satisfy Equation 22 with p very close to 841. Figure 2 sketches the bucket collision conditions of Equations 18 and 24.

4 Adjusting the path lengths between IP6 and IP8

The preceding section shows that $T_{e,6\to8} + T_{h,8\to6}$ should be an odd integer multiple of T_0 when bunches collide at IP8, and nearly so if bunches do not collide at IP8. Since the positions of the HSR arcs are fixed in their RHIC locations, and the HSR IR6 layout is not flexible, there are only three ways to adjust $T_{e,6\to8} + T_{h,8\to6}$:

1. Change the radial position of the ESR arc between IP6 and IP8.
2. Shift both the IP8 collision point and the detector center, longitudinally.
3. Shift the collision point longitudinally, within an unmoved detector.

There is a limited range that the ESR arc can move radially, and moving the collision point within the detector is undesirable. We will therefore move the IP8 detector longitudinally, in the strongly-favored clockwise direction.

Call $C_{h,8\to6}$ the length of the orbit in the HSR between IP6 and IP8, without a radial shift. Then, assuming that the radial shift leads to an equal path length change $\Delta C_{h,8\to6}$ at 100 GeV and 275 GeV,

$$\begin{aligned} T_{h,8\to6} &= \frac{\Delta z_8 + C_{h,8\to6} + \Delta C_{8\to6}}{\beta_{275}c} \\ &= \frac{\Delta z_8 + C_{h,8\to6} - \Delta C_{8\to6}}{\beta_{100}c} \\ &= \frac{2(\Delta z_8 + C_{h,8\to6})}{(\beta_{100}+\beta_{275})c} \end{aligned} \tag{27}$$

where Δz_8 is the positive longitudinal shift of IP8, in the clockwise direction. (Note that this is only approximate, since the longitudinal shift would likely have a slightly more complicated geometric effect.)

The required longitudinal shift is found by solving the path length condition

$$T_{h,8\to6} + T_{e,6\to8} = \frac{2\Delta z_8}{(\beta_{100}+\beta_{275})c} + \frac{2C_{h,8\to6}}{(\beta_{100}+\beta_{275})c} + \frac{\Delta z_8}{c} + \frac{C_{e,6\to8}}{c} = pT_0 \tag{28}$$

where

$$T_{h,8\to6} = 2.129674705\ \mu\text{s} + \frac{2\Delta z_8}{(\beta_{100}+\beta_{275})c} \tag{29}$$

in the HSR-220613a lattice, and

$$C_{e,6\to8} = 640.94\ \text{m} \tag{30}$$

in ESR lattice version 5.6. The path-length condition of Equation 28 still offers some flexibility, since it is the *sum* of $T_{e,6\to8}$ and $T_{h,6\to8}$ that matters. It is therefore possible to modify the values of both $T_{e,6\to8}$ and $T_{h,6\to8}$ by adjusting Δz_8. These current ESR and HSR lattices have

$$\frac{2C_{h,8\to6}}{(\beta_{100}+\beta_{275})c} + \frac{C_{e,6\to8}}{c} = 840.924\,T_0 \tag{31}$$

leading to the conclusion that p should become 841 by moving the IP8 detector by

$$\Delta z_8 = \left(841\,cT_0 - \frac{2C_{h,8\to6}}{\beta_{100}+\beta_{275}} - C_{e,6\to8}\right)\frac{\beta_{100}+\beta_{275}}{2+\beta_{100}+\beta_{275}} = 57.673\ \text{mm} \tag{32}$$

The required HSR radial shift orbit circumference change between IP6 and IP8 (assuming equal but opposite changes at 100 GeV and 275 GeV) is

$$\Delta C_{h,8\to6} = \frac{\beta_{275}-\beta_{100}}{\beta_{275}+\beta_{100}}(\Delta z_8 + C_{h,8\to6}) = 12.195\ \text{mm} \tag{33}$$

This is very close to $\Delta C/6$, taking ΔC from Equation 4. Nonetheless, differences in the the details of the radial shift for each sextant will likely require that the radial shift between IP6 and IP8 is controlled independently of the radial shift in the other 5 arcs.

5 Bunch patterns

Bunches injected into the HSR are later split into 4 parts, located in 197 MHz buckets with even bucket indices b_h. Not all of the even b_h buckets contain bunches, due to the need for an abort gap and also depending on the spectrum of operational needs. Figure 3 (left) illustrates how even b_h values, combined with Equation 18, mean that only electrons in even ESR buckets b_e may collide at IP6. Similarly, Figure 3 (right) and Equation 24 show how only electron bunches in odd b_e buckets may collide at IP8, since p is odd.

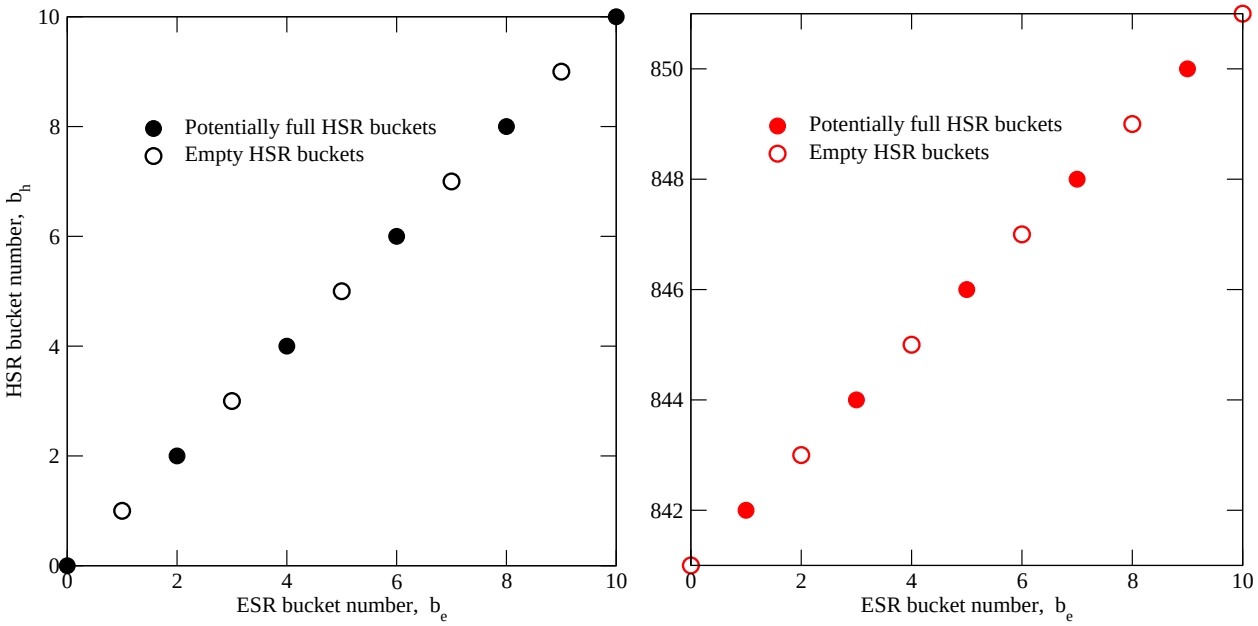


Figure 3: Only HSR buckets with an even index number b_h can contain bunches. Left: Since $b_h = b_e$ for collisions at IP6, only electron bunches in even b_e buckets may collide at IP6. Right: Because $p = 841$ is an odd integer, only electron bunches in odd b_e buckets may collide at IP8.

Three sets of indices therefore define the electron and hadron bunch patterns:

- b_{6k}, with k ranging from 1 to N_6, are the ESR and HSR indices of full buckets that collide at IP6. The b_{6k} are always even.
- b_{h8k}, with k ranging from 1 to N_8, are the HSR indices of full buckets that collide at IP8. The b_{h8k} are always even.
- b_{e8k} are the ESR indices of full buckets that collide at IP8, derived from the HSR bucket indices b_{h8k} via Equation 24. The b_{e8k} are always odd.

There is a great deal of freedom in the choice of which bunches collide at each IP. The most straightforward and flexible scheme is to first choose b_{6k} and b_{h8k} values (even only) that are less than 840. Then for each such bunch in the first third of the circumference, one *may* add bunches in the second and third thirds – in buckets $b_{6k} + 840$ and $b_{6k} + 1680$ for collision at IP6, and in $b_{h8k} + 840$ and $b_{h8k} + 1680$ for collisions at IP8. One can choose to not add bunches in those buckets, for instance to create the abort gap. ESR bunches are placed in the corresponding buckets for collisions at the correct IP, as described above.

Two examples illustrate the possibilities. Both have the same HSR bunch pattern, with 1160 bunches in every even bucket and an abort gap from buckets 1580 through 1778. First, consider a sequence of collisions at a given IP in sequential available buckets, followed by a long gaps without collisions. In this case:

$$
\begin{aligned}
b_{6k} &\in \{0, 2, \ldots, 418\} \cup \{840, 842, \ldots, 1258\} \cup \{1780, 1682, \ldots, 2098\} \\
b_{h8k} &\in \{420, 422, \ldots 838\} \cup \{1260, 1262, \ldots 1578\} \cup \{2100, 2102, \ldots, 2518\} \\
b_{e8k} &\in \{419, 421, \ldots, 737\} \cup \{1259, 1261, \ldots, 1677\} \cup \{2099, 2101, \ldots, 2517\}
\end{aligned}
$$

The ESR train has two long gaps, one from bucket 738 through bucket 839, then another from bucket 1678 through bucket 1779. There are also three bunch pairs that are 1 bucket apart and one bunch pair that is 3 buckets apart.

Second, consider a bunch pattern that collides sequential HSR buckets at alternating IPs:

$$\begin{aligned} b_{6k} &\in \{0, 4, \ldots, 1576\} \cup \{1780, 1784, \ldots, 2516\} \\ b_{h8k} &\in \{2, 6, \ldots, 1578\} \cup \{1782, 1786, \ldots, 2518\} \\ b_{e8k} &\in \{1, 5, \ldots, 737\} \cup \{941, 945, \ldots, 2517\} \end{aligned}$$

This results in an ESR train with bunches spaced by 4 buckets from buckets 740 through 940 and from buckets 1577 through 1777, and bunches alternately spaced by 1 and 3 buckets elsewhere.

6 HSR operations with 4 inner and 2 outer arcs

Low energy protons at approximately 41 GeV have a sufficiently low speed that they must use the inner arc between IR10 and IR12, rather than the outer arc, in order to synchronize with the ESR. The length of the on-axis orbit for them is

$$C_{hL} \;=\; 3832.947189 \text{ m} \tag{34}$$

in the HSR-220613a lattice. Synchronizing with low energy (5 GeV) electrons is somewhat further complicated by the need to decrease the ESR circumference slightly, due to the super-bend tuning described below.

If a proton energy of exactly 41 GeV is required, then a global radial shift is required to ensure synchronization, with an HSR path length change of

$$\Delta C_{h41} \;=\; (C_{\rm ESR} + \Delta C_{eL})\beta_{41} - C_{hL} \;=\; 21.759 \text{ mm} \tag{35}$$

with the HSR-220613a lattice, assuming an ESR circumference reduction of

$$\Delta C_{eL} \;=\; -10 \text{ mm} \tag{36}$$

Alternatively, the low energy proton beam can be kept on-axis. In that case the proton speed must be

$$\beta_{hL} \;=\; \frac{C_{hL}}{C_{ESR} + \Delta C_{eL}} \tag{37}$$

This corresponds to a proton energy of

$$E_{hL} \;=\; 40.563 \text{ GeV} \tag{38}$$

again with $\Delta C_{eL} = -10$ mm and with the HSR-220613a lattice.

Low energy protons travel from IP8 to IP6 in the same beamline as high energy protons. If the low energy protons are on-axis, then their travel time between IP8 and IP6 increases by

$$\Delta T_{hL,8\to6} \;=\; \frac{\Delta z_8 + C_{h,8\to6}}{\beta_{hL} c} - T_{h,8\to6} \;=\; 0.516938 \text{ ns} \tag{39}$$

This is a small but significant fraction of T_0. If detectors at IP6 and IP8 were to take data concurrently, then a radial shift in the arc between IP6 and IP8 would need to change the path length by an unrealistically large value of about 155 mm. Thus we are left with two possibilities:

1. Collide bunches at only one IP at a time.
2. Shift the collision points within the detectors closer longitudinally to each other by about 77.5 mm.

7 ESR super-bends

Each of the 192 super-bends in the ESR arcs contains a series of three dipole magnets. The field of the shorter central magnet is reversed at the lowest electron energy of 5 GeV, in order to add bending that increases both the radiation damping decrement and also the natural emittance. Such super-bend tuning decreases the path-length in each arc cell by about 100 μm, and between IP6 and IP8 by about 2 mm. The total circumference of the ESR decreases by about 10 mm. When colliding 100 GeV protons with 5 GeV electrons, we could either have an even larger global radial shift to the inside, or maintain the same radial shift to the inside and run the protons at a slightly higher energy corresponding to a velocity of

$$\beta_M \;=\; \frac{C_{\rm HSR} - \Delta C}{C_{\rm ESR} + \Delta C_{eL}} \tag{40}$$

which, for $\Delta C_{eL} = -10$ mm and the HSR-220163a lattice corresponds to an energy of

$$E_M \;=\; 103.101 \text{ GeV} \tag{41}$$

If the maximum HSR circumference reduction is $\Delta C = -63.226$ mm (before compensation of the super-bend effect), then the range of gold energies in 3 inner and 3 outer arcs is narrowed, to become

$$102.356 \text{ GeV/u} \;\leq\; E_{Au} \;\leq\; 110 \text{ GeV/u} \tag{42}$$

It may be desirable to slightly lower E_{p0}, the central energy, in order to achieve lower gold energies.

8 ESR J_x-tuning

The nominal on-momentum ESR circumference is related to the center of the range of radially-shifted HSR circumferences by Equation 3. However, the ESR retains the ability to tune its horizontal emittance by operating slightly off-momentum.

Off-momentum parameter $\delta = \Delta p/p$	**Partition number MAD-X** J_x	**Partition number fit** J_x	**Circumference shift fit** ΔC [mm]
One low-beta collision point			
$-.000762$		1.25	-1.912
$-.0005$	1.176	1.176	-1.254
0	1.036	(1.035)	0
.0005	0.894	0.894	1.254
.001011		0.75	2.536
Two low-beta collision points			
$-.000963$		1.250	-2.359
$-.0005$	1.136	1.136	-1.225
0	1.013	(1.013)	0
.0005	0.890	0.890	1.225
.001069		0.750	2.619

Table 1: Partition number and circumference shifts in an 18 GeV ESR lattice with 90 degrees of phase advance per FODO cell, during J_x-tuning. TOP: One low-beta collision point, with a momentum compaction factor of 0.0006544. BOTTOM: Two low-beta collision points, with a momentum compaction factor of 0.000639.

Operating off-momentum affects the horizontal partition number J_x, which is related to the unnormalised RMS emittance ϵ_x by

$$\epsilon_x = \frac{C_q}{J_x}\gamma^2 \frac{\langle G^3\mathcal{H}\rangle}{\langle G^2\rangle} \tag{43}$$

where $C_q = 3.84 \times 10^{-13}$ m, γ is the Lorentz factor, G is the inverse of the bending radius, and

$$\mathcal{H} = \gamma_x \eta^2 + 2\alpha_x \eta\eta' + \beta_x \eta'^2 \tag{44}$$

Here α_x, β_x, and γ_x are the Courant-Snyder parameters and η is the horizontal dispersion. A J_x tuning range of

$$0.75 \; < J_x \; < \; 1.25. \tag{45}$$

is reasonable. Such tuning is most likely to be necessary at the highest electron energy of about 18 GeV, naturally corresponding to HSR proton operation at 275 GeV. Smaller J_x-tuning may also be used at lower electron energies. Table 1 shows the results of MAD-X calculations of how far δ and the ESR circumference must vary, using ESR lattice version 5.6 with low-beta optics at either one or two collision points.

In conclusion, the HSR must be able to accommodate an additional circumference range of approximately $\Delta C \approx \pm 2.5$ mm, in order to enable J_x-tuning in the ESR. This is almost negligible, compared to the total range of $\Delta C = \pm 73.226$ mm that is available.

9 Impact of a Ring Cooler

The current EIC project baseline includes Strong Hadron Cooling using Coherent electron Cooling (SHC-CeC), symmetrically arranged on either side of HSR IP2. However, it is possible that a Ring Cooler will be used instead. Implementing a Ring Cooler brings the HSR design trajectory towards the radial center of the accelerator site. This could decrease the path length across IP2 – and the total HSR circumference – by as much as about 60 mm, as shown in Table 2.

The ESR circumference would also have to decrease by 60 mm or so. This could be done locally, with the ESR more or less following the HSR layout around IP2. Or it could be done globally, for example by shrinking the ESR layout radially inwards by the same amount in all 6 arcs.

The RCS layout would also have to be modified around IP2, ideally while maintaining its high levels of symmetry.

Parameter		**SHC-CeC PLD25**	**Ring Cooler layout 4**	**5**
Cooling straight length	m	–	150.1	199.3
Q10-to-Q10 length	m	313.1007	313.0602	313.0393
Path length change	mm	0	-40.5	-61.4

Table 2: Preliminary estimates of the shortening of the HSR path length across IP2 that will occur if a Ring Cooler is used instead of Coherent electron Cooling. The numerical label of the ring cooler layout ("4" or "5") refers to the number of half-cells in the dispersion suppressor on either side of IP2.

10 Summary

1. If the HSR circumference without radial shift is $C_{\rm HSR}$, then the ESR and RCS circumferences are

$$\begin{aligned} C_{\rm ESR} &= 1.00002492\, C_{\rm HSR} \\ C_{\rm RCS} &= \left(\frac{316}{315}\right) C_{\rm ESR} \end{aligned} \tag{46}$$

2. EIC bunch intensities are so high that each bunch may only collide once per turn – at IP6 or at IP8, but not at both. This is guaranteed by adjusting the two path lengths from IP6 to IP8.

3. The IP8 collision point and detector must be moved clockwise by about 57.673 mm relative to their locations at the time of writing, if $p = 841$ as defined in Equation 22.

4. Hadrons may only be stored in even-numbered buckets b_h of the 2520 buckets in the 197 MHz RF system, because of the RF beam-splitting gymnastics performed after injection.

5. Bunches in even-numbered buckets b_{e6} and b_{h6} collide at IP6 if

$$b_{h6} \;=\; b_{e6} \tag{47}$$

   Electrons in odd-numbered buckets b_{e8} and hadrons in even-numbered buckets b_{h8} collide at IP8 if

$$b_{h8} \;=\; \mathrm{mod}(b_{e8} + 841,\; 2520) \tag{48}$$

6. It is not yet clear whether HSR operations with 4 inner and 2 outer arcs (with a proton energy of around 41 GeV) can support simultaneous collisions at both IP6 and IP8.

7. ESR super-bend tuning at 5 GeV decreases its total circumference of the design orbit by about 10 mm. For 100 GeV protons colliding with 5 GeV electrons, this must be compensated either by tuning the HSR radial shift or by increasing the proton energy.

8. ESR J_x-tuning requires the HSR to accommodate an additional circumference range of approximately ± 2.5 mm, a value that is almost negligible.

9. Incorporating a Ring Cooler in HSR IR2 could decrease the HSR circumference by as much as about 60 mm, requiring the ESR and RCS circumferences to follow suit.