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Radial-offset Optics in EIC Hadron Lattices. Trajectory Lengthening

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Abstract

This Tech. Note describes a method for a radial orbit offset in the arcs of the EIC hadron lattices, based on the perturbation dB/B of the magnetic field of the arc main bend magnets. This can be done independently for each arc (allowing multiple rigidities) and requires the use of pairs of orbit correctors at the extremities of the arc to ensure that the radially shifted orbit is on the *zero-orbit* (*i.e.* with the horizontal coordinates $x = 0$, $p_x = 0$) at the exit and entry point of the respectively upstream and downstream interaction region (IR). The goal is to produce appropriate orbit lengthening as required for time of flight adjustments between the electron and hadron beams as part of EIC operations. An *ad hoc* parameter is introduced: a "geometric compaction factor" $\alpha_B = (d\mathcal{C}/\mathcal{C})/(dB/B)$, to quantify the orbit lengthening, or equivalently the radial orbit shift $dR = R \cdot \alpha_B \cdot dB/B$, from bending perturbation. Detailed numerical results are produced and discussed.

Contents

1 Introduction

The EIC electron-hadron collider project, currently in its conceptual design stage, calls for repurposing parts of the existing RHIC to serve as the EIC hadron lattices in order to transport hadron bunches towards the collision point IP6 with electron bunches inside of IR6 where the STAR detector is located. The EIC is being designed for a wide range of center of mass energies, with $\sqrt{s_{e-h}} = 29$–140 GeV, with Tab. 1 listing all design beam energies for hadron species being considered.

Table 1: Design beam energies for the species planned for EIC operations at $\sqrt{s_{e-h}}$ = 29–140 GeV.

Species	Polarized electrons	Polarized protons	Heavy ions Au
Energy [GeV, GeV/u]	5, 10, 18	41, 100, 275	41, 100

Electron-ion collision at IP6 requires the revolution frequency of the hadron bunches to match that of the electron bunches; from a practical standpoint, and given the energies listed in Tab. 1, it will not be possible to have bunches circulating through the magnetic center of the arc magnets for all modes of operation if the revolution frequency is to be matched across all cases. This change of circumference has to take place in the arcs of the EIC hadron lattices in order to preserve the integrity of each IR by keeping the circulating beam on the magnetic center of all dipole and quadrupole magnets that constitute the IR layout.

Since modifying the circumference of a hadron ring using only the arcs is equivalent to changing the radius of the dipole magnets in these arcs, there are two ways that one can apply this change:

- design the hadron lattices as if the circulating beam is off-momentum: this would take care of the circumference change since the shifted orbit would be proportional to the dispersion function, however it does not address the need to maintain a zero-orbit (for which $x = 0$, $p_x = 0$) in the IRs, and furthermore any $\delta p/p \neq 0$ creates discrepancies with the rest of the EIC machine design;

- apply a small relative deviation dB/B to all dipole magnets of a given arc (most direct way to change the bending radius of a given section of the machine) and use pairs of horizontal orbit correctors at each end of the considered arc to set the circulating beam back on the zero-orbit going out of and into the neighboring IRs.

The second method is the one being studied in this document, with the goal to generate hadron lattices for a number of different values of dB/B in order to establish the correlation between this variable and the resulting relative change in lattice circumference $d\mathcal{C}/\mathcal{C}$.

The established codes MAD-X and zgoubi are used to set up the radial-shift lattices:
- using two codes is just for convenience, mostly because one of the authors is a MAD-X user,
- the complete study discussed here may anyway be achieved with whatever code can handle orbit offset;
- on the other hand the EIC hadron ring is translated to zgoubi from its original MAD-X files, thus any ancillary work (as re-tuning the IRs in the present case, see below) can readily be performed using these existing MAD-X files;
- a third, obvious advantage of this opportunity to use two codes concurrently, is in its allowing tight cross-checks.

Using the two codes, the procedure is as follows:

1. with the MAD-X sequence file for the design (on-momentum, zero-orbit) lattice, generate a Twiss file of the entire ring, as a reference;

2. using this sequence file and the Twiss file as input for zgoubi, generate a zgoubi Twiss file for consistency check on the main linear optics parameters (circumference, tunes, chromaticities, optical functions);

3. isolate each arc and apply a value of dB/B, taken in the following as [-0.01, -0.005, 0, 0.005, 0.01];

4. calculate the required strengths of the horizontal orbit correctors selected to put the circulating beam back on the zero-orbit at the edges of the arc being modified (see Tab. 2 for the full list of utilized correctors);

5. recalculate the periodic solution of the linear optics functions along with the shifted orbit (zero at both ends);

6. with the new boundary conditions ($\beta_{x,y}$, $\alpha_{x,y}$, D_x) set by these periodic solutions, rematch the linear optics of each IR - this is readily done in MAD-X;

7. in order to keep the tunes constant, calculate the total changes in phase advances in the arcs due to the shifted orbit (using the initial Twiss files from the design lattice), and adjust the phase advances in each IR accordingly;

8. translate the rematched IRs, with updated quadrupole strengths, to zgoubi and produce the periodic solutions of the shifted orbit (radially offset in the arcs, zero in the IRs, as a consequence of the previous steps) and linear optics functions, together with tunes, chromaticities and other paraxial parameters.

9. a small rematch of the ring, for exact tune and chromaticity values, may be needed. It will be shown that this can be performed with just two families of lenses (main QF and QD quadrupoles for tunes, main SF and SD sextupoles for chromaticities), with moreover very small tweak of their strengths, in the $10^{-2} - 10^{-3}$ range, relative. Note that in the present study, this global ring re-match leaves the quadrupoles and sextupoles of the IRs untouched, which may introduce a modulation of the resulting optical functions - including the dispersion - and subsequently the orbit offset in the arcs. This can be refined, if needed, as part of an iteration of the process.

Once all these steps are completed, one can calculate such quantities as
- the resulting relative change in lattice circumference $d\mathcal{C}/\mathcal{C}$ under the effect of a dB/B perturbation of the main magnet field in one or more arcs,
- a "geometric compaction factor" $\alpha_B = (d\mathcal{C}/\mathcal{C})/(dB/B)$ so the radial offset then writes $dR/(dB/B) = R \cdot \alpha_B$.

In addition, a new MAD-X sequence file can be generated that would include the value of dB/B in the form of a specific syntax when defining the SBEND arc dipoles, along with the corresponding strengths of orbit correctors as calculated by zgoubi to obtain a closed, shifted orbit. This MAD-X sequence file could then be used in Y. Luo's SimTrack for dynamic aperture tracking.

2 Lattice Version V200512_Mod of 08/10/2020

This is the lattice used in this study. Its origin is the hadron lattice V200512 taken from the "EIC HCR Task Force" sharepoint, set to proper tunes and chromaticities, namely $Q_x = 28.228,\ Q_y = 29.21$ and $\xi_x = \xi_y = 1$.

Working hypotheses

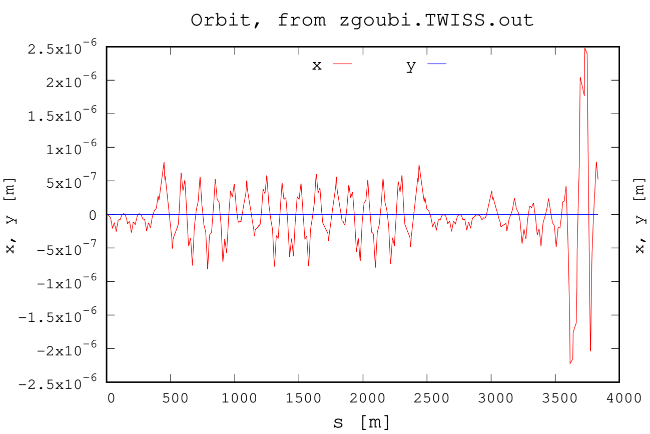
All ak1l (quadrupole component) and ak2l (sextupole) in BEND are removed, because they cause orbit. That jeopardizes both the tunes and the chromaticities. QF* and QD* are then tweaked to get back to QX=28.228 and Qy=29.21: this is a minor scaling, about 1.5e-3 relative, see page 4. SF* and SD* are tweaked to recover chromaticities close to 1, this is a small scaling factor, see page 4.

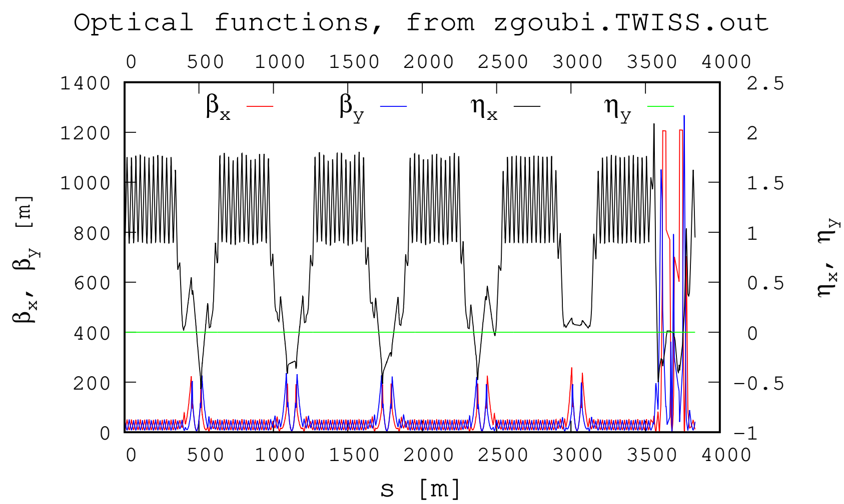
• SCALING factor settings to meet nominal tunes and chromaticities (an excerpt from the input data file):

```
MULTIPOL QF*
0.99841335
MULTIPOL QD*
1.0014330
MULTIPOL SF*
  1.005000000E+00
MULTIPOL SD*
  1.025000000E+00
MULTIPOL MULT
 -2.00000000E-01
```

• Twiss file header:

```
@ LENGTH            3833.864914
@ ALFA             0.1873813140E-02
@ ORBIT5                          -0
@ GAMMATR           23.10132340
@ Q1               0.2279989283      [fractional]
@ Q2               0.2099931863      [fractional]
@ DQ1               1.121545684
@ DQ2              0.9511037990
@ DXMAX             2.08901372E+00    @ DXMIN              -5.65589170E-01
@ DYMAX             0.00000000E+00    @ DYMIN               0.00000000E+00
@ XCOMAX            2.48096733E-04    @ XCOMIN             -2.22126673E-04
@ YCOMAX            0.00000000E+00    @ YCOMIN              0.00000000E+00
@ BETXMAX           1.20917125E+03    @ BETXMIN             2.11749487E-01
@ BETYMAX           1.26772319E+03    @ BETYMIN             1.06715632E-01
@ XCORMS            3.55396598E-01
```





3 Method for an Orbit Offset $\propto \eta_x$ in the Arcs

The orbit offset is made to scale the dispersion function.

• This is a simple way to get $\Delta R/R\,/\,\Delta B/B$ close to $\Delta R/R\,/\,\Delta p/p$.

• A double pair of steerers, located at the both ends of the arc, are used to force the orbit bump (which is excited along the arc, due to $\Delta B/B$) to follow the chromatic orbit.

• In this method, the arcs are treated independently, one by one, each as a periodic structure.

Strategy for geometric orbit to follow the chromatic orbit:

- Hill's equation for magnetic defect $\Delta B(s) = \Delta B_y$, or for momentum offset $\delta p/p$, have the same form, respectively:

$$x'' + Kx = -\frac{\Delta B_y}{B\rho} \qquad \text{and} \qquad x'' + Kx = \frac{1}{\rho}\frac{\Delta p}{p} \tag{1}$$

- thus, the geometric orbit $x_{co}(s)$ will follow $-\eta_x$ along the arc if its coordinates at the origin of the arc (say, s=0) satisfy

$$x_{co}(s=0) = -\eta_x \frac{\Delta B}{B} \qquad \text{and} \qquad x'_{co}(s=0) = -\eta'_x \frac{\Delta B}{B} \tag{2}$$

The latter are the two constraints applied at the downstream steerer of the upper pair (next to the upstream IR), with both these upstream steerers as variables in the matching procedure. Zero orbit and angle are the constraints applied at the downstream steerer of the downstream pair (just before the downstream IR), with both these downstream steerers as variables.

By analogy with the momentum compaction factor $\alpha_p = d\mathcal{C}/\mathcal{C}/\,dp/p$ to quantify the trajectory lengthening due to momentum offset, we introduce here a "geometric compaction" factor α_B to quantify the trajectory lengthening due to geometric (ΔB induced) orbit offset.

$$\alpha_B = d\mathcal{C}/\mathcal{C}/dB/B$$

4 Orbit Offset in the Arcs; Optical Functions

• Table 2 displays the strengths of the orbit offset steerers, for the four values $\mathrm{dB/B} = \pm 0.05 \text{ and } \pm 0.01$. It can be seen that the steerers response to dB/B is close to linear.

Table 2: Strengths of the orbit offset steerers (in units of 10^{-7} rad), for the four values $\mathrm{dB/B} = \pm 0.05 \text{ and } \pm 0.01$.

	Steerer	dB/B			
Arc	name	-0.01	-0.005	+0.005	+0.01
5-4	BI5_TH11	-366.12237	-181.23473	177.58894	351.61844
	BI5_TH13	3372.6357	1762.8166	-1913.8606	-3977.4355
	BI4_TH13	3753.3446	1857.6984	-1840.8107	-3747.1045
	BI4_TH11	-421.07912	-219.78372	333.82121	869.69872
3-2	BO3_TH10	-597.03541	-301.57065	307.70386	621.64203
	BO3_TH12	3715.0344	1853.0320	-1844.2605	-3679.4619
	BO2_TH12	3283.7361	1724.3471	-1911.7728	-3960.2274
	BO2_TH10	-192.04674	-171.75164	227.22489	418.70345
1-12	YO1_TH10	-474.49310	-235.62872	232.33654	461.53095
	YO1_TH12	3364.4473	1759.1452	-1911.7317	-3973.9201
	YO12_TH12	3443.0085	1790.1885	-1845.2136	-3689.3433
	YO12_TH10	-675.48512	-326.73983	313.25103	666.51374
11-1	BO11_TH10	-585.02791	-295.54350	301.58831	609.34504
	BO11_TH12	3667.5721	1829.6686	-1821.0114	-3633.3650
	BO10_TH12	3322.0473	1747.0506	-1932.9582	-3992.5256
	BO10_TH10	-233.14673	-184.67045	224.34123	399.27264
9-8	BI9_TH11	-393.62525	-194.96887	191.30175	379.01094
	BI9_TH13	3375.2837	1765.2944	-1918.7588	-3989.5041
	BI8_TH13	3764.6706	1866.5856	-1854.5600	-3777.1755
	BI8_TH11	-445.60501	-230.70777	340.20023	875.87467
7-6	YI7_TH11	-532.34972	-268.97830	274.60244	554.97529
	YI7_TH13	3684.7731	1838.4168	-1830.7798	-3653.4931
	YI6_TH13	3489.2447	1814.6292	-1854.8456	-3663.3062
	YI6_TH11	-435.94046	-202.53789	161.67522	331.84953

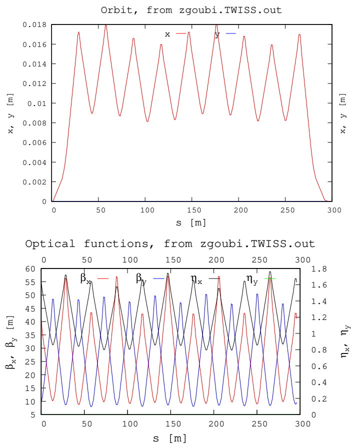
• The graphs in pp. 6, 7 show the orbit offset in the arcs (treated each one as a separate, periodic structure), for the four values $\mathrm{dB/B} = \pm 0.05 \text{ and } \pm 0.01$. It can be seen that the orbit excursion reacts essentially linearly to dB/B. The exact values of orbit excursion extrema are given in pp. 8 and 9 under the "Twiss output file header".

Once the arcs are individually set (closed orbit bump and periodic optics, for each arc concerned), the IRs are re-tuned to match them, and the 12 segments so obtained (6 arcs and 6 IRs, in the present case where orbit has been offset in all 6 arcs) are re-assembled:

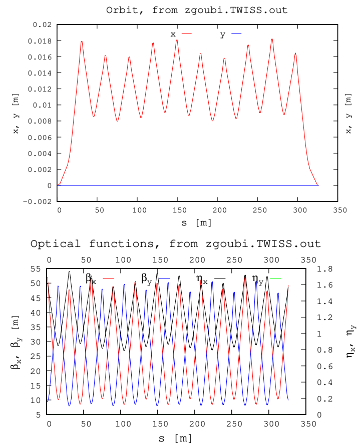
• The graphs and tables in pp. 8, 9 show the global, periodic ring optics parameters, closed orbits, and optical functions, in the presence of orbit steerer settings (Tab. 2) yielding the arc orbit offsets of pp. 6, 7. One can observe some modulation of the dispersion function extrema, greater for greater dB/B, and, by correlation (Eq. 2), of the orbit offset extrema. The probable cause has been discussed in item 9 page 3. This is however a minor effect and can be taken care of with an iteration of the process.

dB/B = -0.01

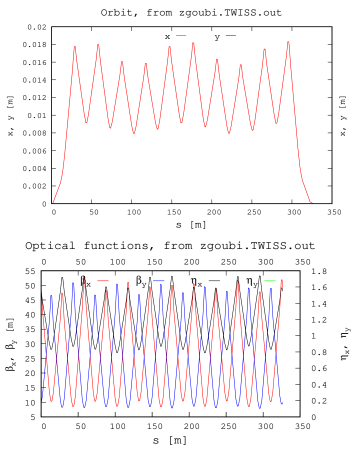
arc 5 - arc 4



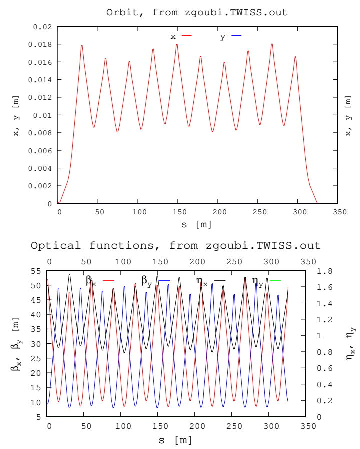
arc 3 - arc 2



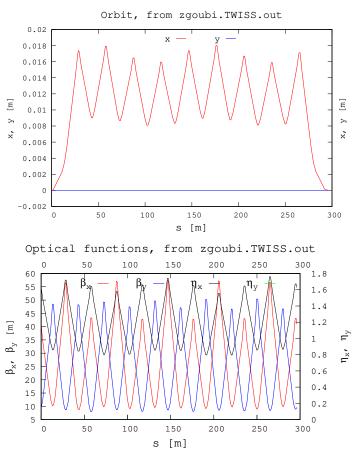
arc 1 - arc 12



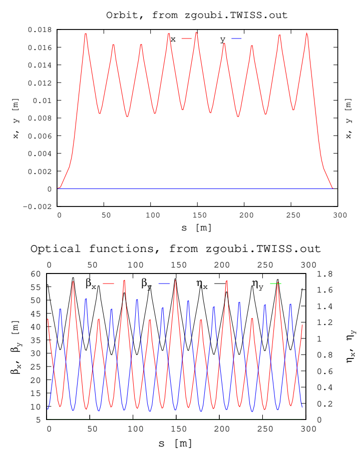
arc 11 - arc 10



arc 9 - arc 8

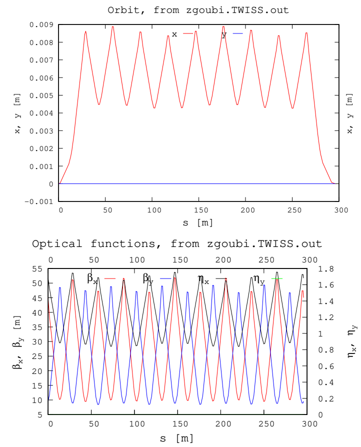


arc 7 - arc 6

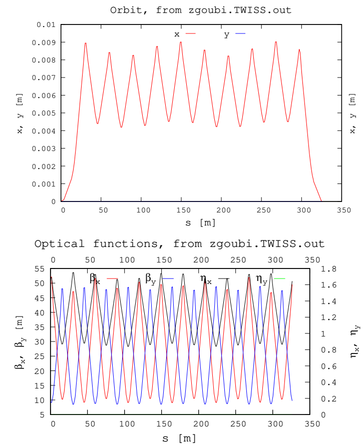


dB/B = -0.005

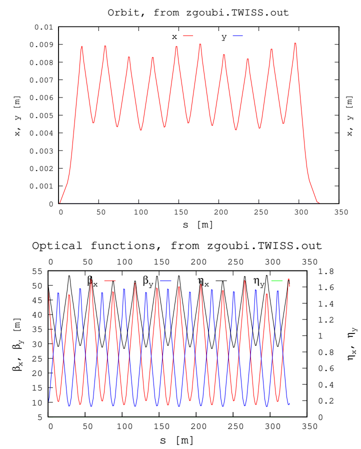
arc 5 - arc 4



arc 3 - arc 2



arc 1 - arc 12



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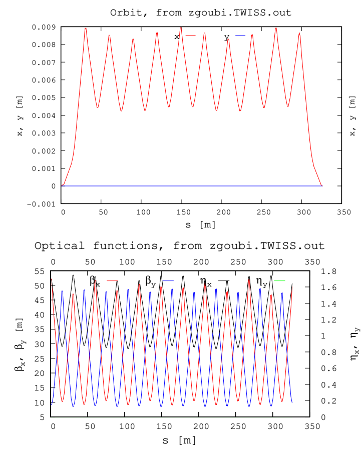
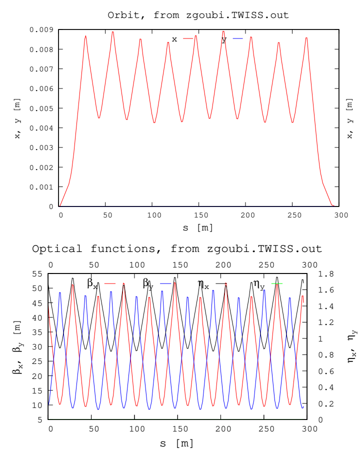
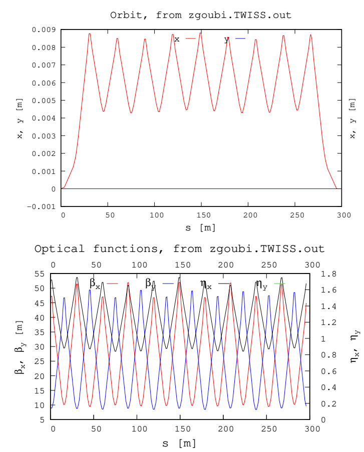


arc 9 - arc 8



arc 7 - arc 6



dB/B = +0.005

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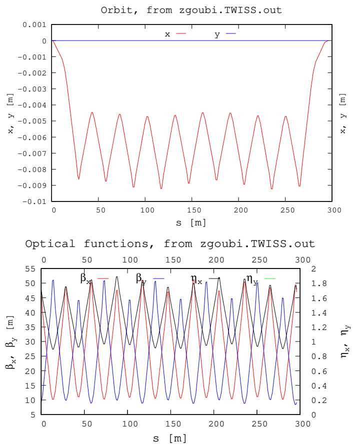
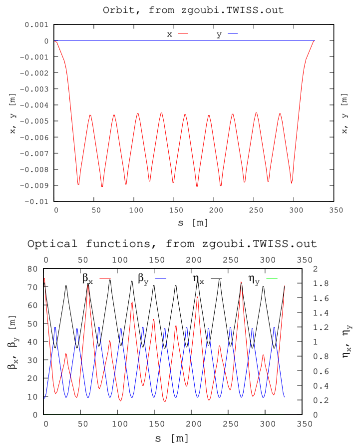
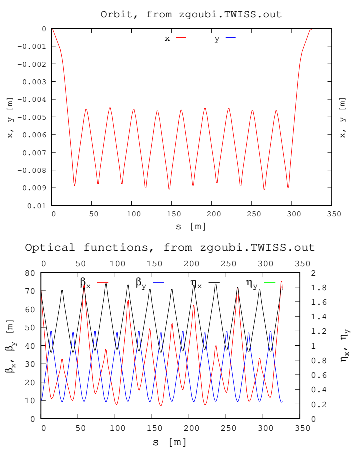


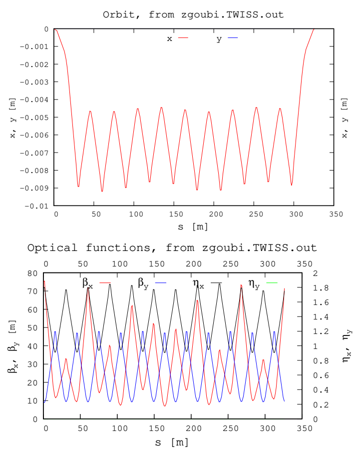
arc 3 - arc 2



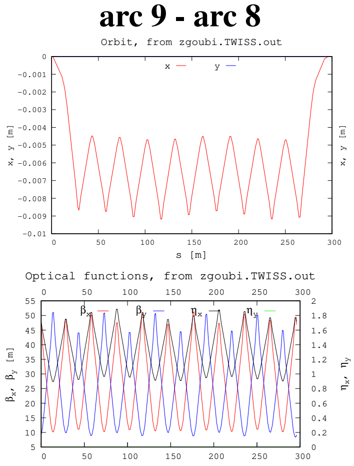
arc 1 - arc 12



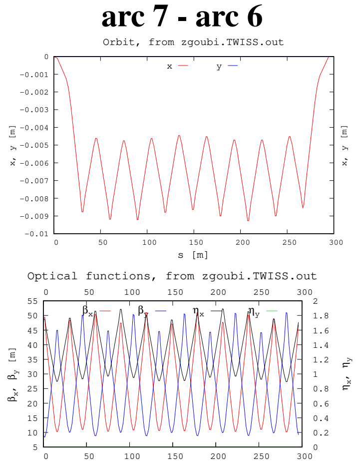
arc 11 - arc 10



arc 9 - arc 8

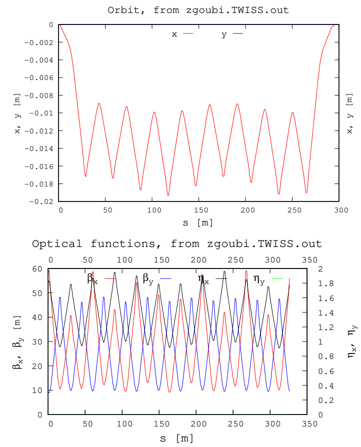


arc 7 - arc 6

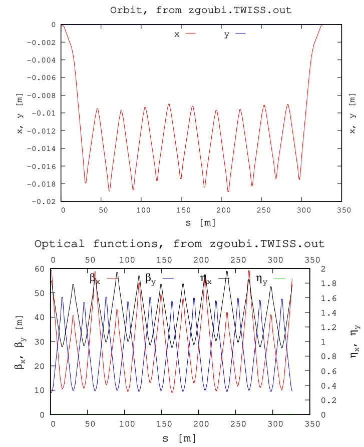


dB/B = +0.01

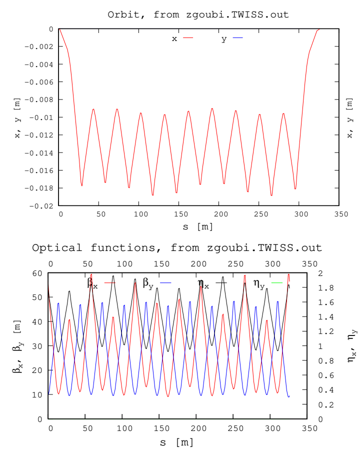
arc 5 - arc 4



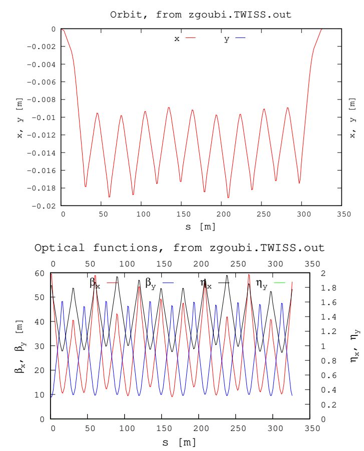
arc 3 - arc 2



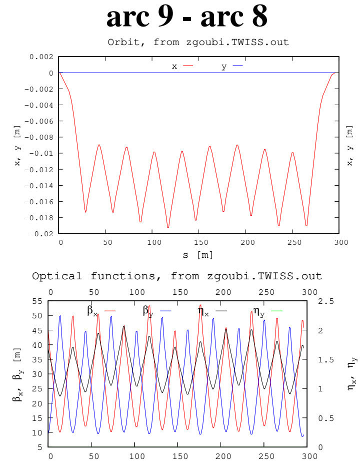
arc 1 - arc 12



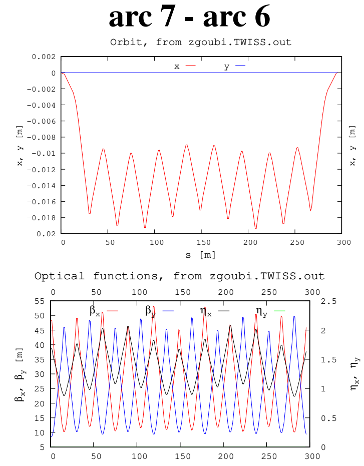
arc 11 - arc 10



arc 9 - arc 8

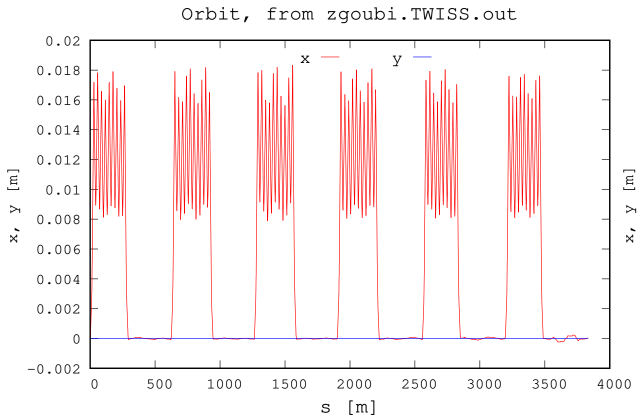


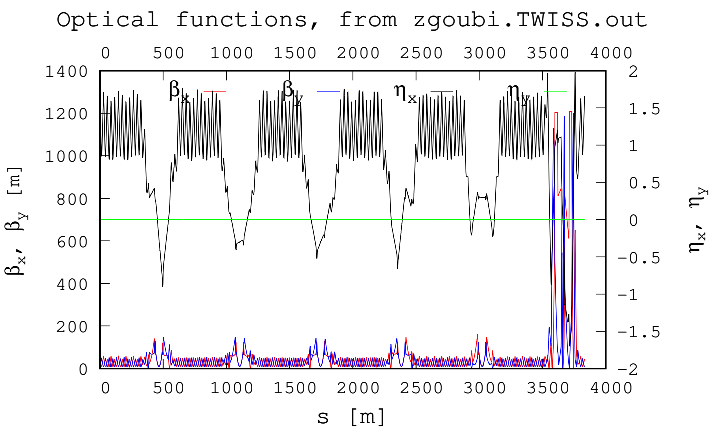
arc 7 - arc 6



RING, PERIODIC: ORBIT OFFSET AND OPTICAL FUNCTIONS

dB/B = -0.01





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• Twiss output file header

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@ ALFA           0.1797047550E-02
@ GAMMATR         23.58958038
@ Q1             0.2277204694       [fractional]
@ Q2             0.2099956230       [fractional]
@ DQ1           -0.7062536609E-01
@ DQ2             1.636508526
@ DXMAX           1.98563182E+00    @ DXMIN           -1.70185121E+00
@ DYMAX           0.00000000E+00    @ DYMIN            0.00000000E+00
@ XCOMAX          1.83365537E-02    @ XCOMIN          -2.30273605E-04
@ YCOMAX          0.00000000E+00    @ YCOMIN           0.00000000E+00
@ BETXMAX         1.20858961E+03    @ BETXMIN          4.18948943E-01
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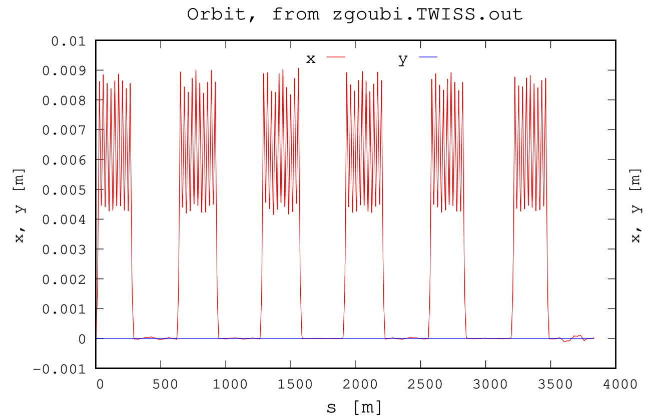

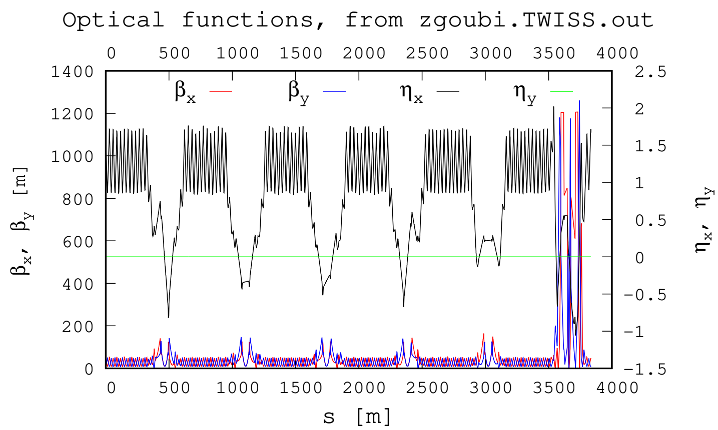

• Twiss output file header

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@ DQ1            0.9833096090
@ DQ2             1.051456183
@ DXMAX           2.02160047E+00    @ DXMIN           -1.05672804E+00
@ DYMAX           0.00000000E+00    @ DYMIN            0.00000000E+00
@ XCOMAX          9.24925380E-03    @ XCOMIN          -1.30232379E-04
@ YCOMAX          0.00000000E+00    @ YCOMIN           0.00000000E+00
@ BETXMAX         1.20589113E+03    @ BETXMIN          2.62914690E-01
@ BETYMAX         1.25992267E+03    @ BETYMIN          7.15564031E-02
@ XCORMS          3.22607098E-03
```

• SCALING settings to meet nominal tunes and chromaticities

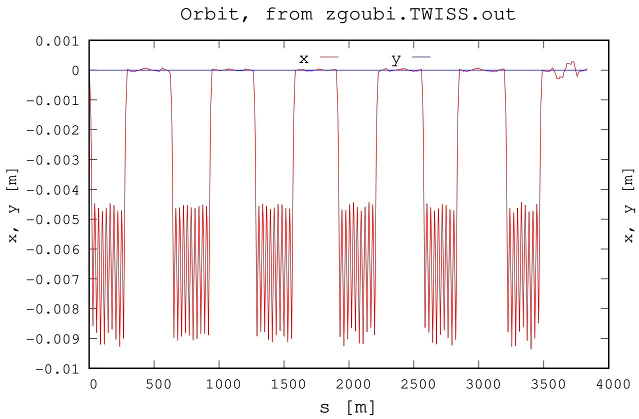
Input data file excerpt, shows that adjustment of the main quads (QF and QD) within a few permil is required to reset tunes to 28.228/29.21, adjustment of the sextupoles (SF, SD) within a few percent to reset chromaticities to 1/1

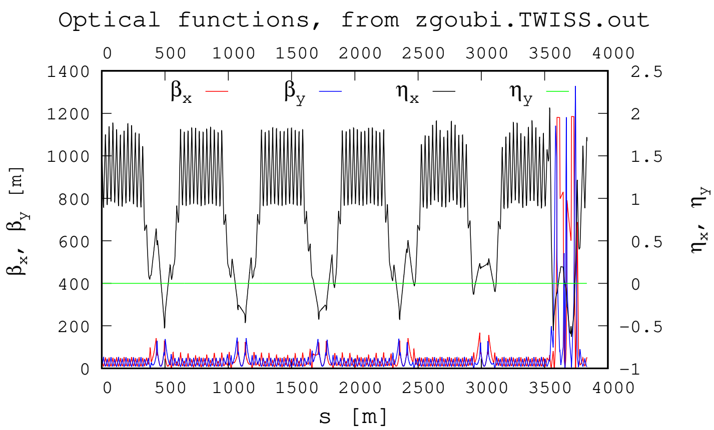
```
MULTIPOL QF*
-1
0.99700601
1
MULTIPOL QD*
-1
1.0021175
1
MULTIPOL SF*
-1
.9300000E+00
1
MULTIPOL SD*
-1
1.06000000E+00
1
```

```
MULTIPOL QF*
-1
0.99901268
1
MULTIPOL QD*
-1
1.0001647
1
MULTIPOL SF*
-1
0.96960790
1
MULTIPOL SD*
-1
1.1139358
1
```

RING, PERIODIC: ORBIT OFFSET AND OPTICAL FUNCTIONS

dB/B = +0.005





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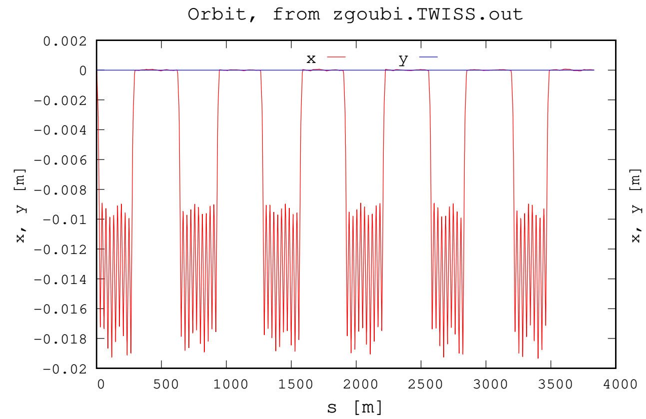
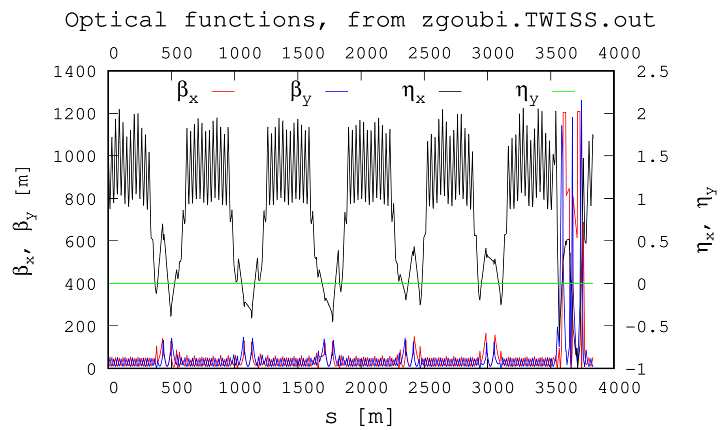
• Twiss output file header

```
@ LENGTH           3833.836925
@ ALFA            0.1913823426E-02
@ GAMMATR          22.85857046
@ Q1              0.2279428427        [fractional]
@ Q2              0.2100250895        [fractional]
@ DQ1            -0.6209486742
@ DQ2             -1.319173777
@ DXMAX            2.01237614E+00    @ DXMIN         -4.94022820E-01
@ DYMAX            0.00000000E+00    @ DYMIN          0.00000000E+00
@ XCOMAX           3.11324060E-04    @ XCOMIN        -9.40410447E-03
@ YCOMAX           0.00000000E+00    @ YCOMIN         0.00000000E+00
@ BETXMAX          1.17765078E+03    @ BETXMIN        2.22399129E-01
@ BETYMAX          1.32980438E+03    @ BETYMIN        7.11814740E-02
@ XCORMS           3.30470168E-03
```

dB/B = +0.01





• Twiss output file header

```
@ LENGTH           3833.808165
@ ALFA            0.1951694922E-02
@ GAMMATR          22.63570545
@ Q1              0.2278975092        [fractional]
@ Q2              0.2099605994        [fractional]
@ DQ1              3.072239478
@ DQ2              1.768544072
@ DXMAX            2.06469914E+00    @ DXMIN         -9.89428039E-01
@ DYMAX            0.00000000E+00    @ DYMIN          0.00000000E+00
@ XCOMAX           6.47080877E-05    @ XCOMIN        -1.93296832E-02
@ YCOMAX           0.00000000E+00    @ YCOMIN         0.00000000E+00
@ BETXMAX          1.20939509E+03    @ BETXMIN        2.48377242E-01
@ BETYMAX          1.26380114E+03    @ BETYMIN        7.12400366E-02
@ XCORMS           6.71836109E-03
```

• SCALING settings to meet nominal tunes and chromaticities

Input data file excerpt, shows that adjustment of the main quads (QF and QD) within a few permil is required to reset tunes to 28.228/29.21, adjustment of the sextupoles (SF, SD) within a few percent to reset chromaticities to 1/1

```
MULTIPOL QF*
-1
0.99620822
1
MULTIPOL QD*
-1
1.0014792
1
MULTIPOL SF*
-1
0.88229274
1
MULTIPOL SD*
-1
1.0165657
1
```

```
MULTIPOL QF*
-1
0.99837974
1
MULTIPOL QD*
-1
1.0012825
1
MULTIPOL SF*
-1
0.99723158
1
MULTIPOL SD*
-1
1.0290099
1
```

5 Trajectory Lengthening

Trajectory lengthening in the arcs, under the effect of orbit offset, for the four values $\mathrm{dB/B} = \pm 0.05$ and $\pm\, 0.01$, is displayed in Tab. 3, together with (i) the geometric compaction $\alpha_B = \frac{d\mathcal{C}/\mathcal{C}}{dB/B}$, (ii) the average radial offset $\mathrm{dR}\,/\,\mathrm{dB/B} = \mathrm{R} \times \alpha_\mathrm{B} = 610.17 \times \alpha_\mathrm{B}$.

Table 3: This table displays, as a function of dB/B: (i) trajectory lengthening in individual arcs (rows 3 to 8), (ii) summed up over all 6 arcs (row 9), (iii) geometric compaction α_B over the ring (row 11), (iv) radial offset (row 12).

		dB/B			
Arc		-0.01	-0.005	+0.005	+0.01
5-4	(mm)	8.438	4.314	-4.351	-8.938
3-2	(mm)	9.409	4.812	-4.821	-9.856
1-12	(mm)	9.498	4.831	-4.823	-9.867
11-1	(mm)	9.417	4.814	-4.819	-9.851
9-8	(mm)	8.443	4.316	-4.352	-8.937
7-6	(mm)	8.458	4.320	-4.350	-8.953
all arcs	(mm)	53.663	27.409	-27.520	-56.405
EIC Hadron ring ($\mathcal{C} = 3833.865$ m, R=610.17 m):					
α_B	(10^{-3})	-1.40	-1.430	-1.4356	-1.4712
$\mathrm{dR/dB/B}$	(m)	-0.854	-0.872	-0.876	-0.898

Table 4 shows the orbit length around the ring in the presence of orbit offset in the arcs (i.e., the length of the periodic orbits displayed in pp. 8, 9). This is a kind of "double check": the resulting trajectory lengthening (bottom line in Tab. 4) is expected to coincide the values in the "all arcs" row in Tab. 3 (the agreement is within 1$\sim$2%).

Table 4: Orbit length in the complete ring, from a "Twiss" computation (which yielded the graphs in pp. 8, 9), and difference with respect to the unperturbed optics. The table also summarizes the extreme orbit excursion in presence of the orbit offset ("XCOMAX" or "XCOMIN" values taken from the "Twiss file header", pp. 8, 9).

		dB/B				
		-0.01	-0.005	0	+0.005	+0.01
extreme orbit excursion	(mm)	18.336	9.249	0	-9.404	-19.329
orbit length $\mathcal{C}$	(m)	3833.91808	3833.89191	3833.86491	3833.83695	3833.80816
orbit lengthening $\mathcal{C} - \mathcal{C}_\mathrm{unperturbed}$	(mm)	+53.17	+27	0	-27.958	-56.745

Greater Azimuthal Orbit Offset Extent

By comparison with $|\alpha_B|$ values in Tab. 3, the momentum compaction is substantially greater (see p. 4, "Twiss file header"), $\alpha_p = 1.8738 \times 10^{-3}$. The bulk of the difference between both comes from the geometric orbit offset being zero in the IRs, whereas the chromatic orbit is not.

Namely, if the orbit offset is extended beyond the corrector pairs presently assumed (Tab. 2), into the IRs, and assuming the same extreme excursions along the arcs (pp. 6, 7), then

$$\alpha_B \to -\alpha_p$$

Referring to the α_p values of the present $-0.01 \le dB/B \le +0.01$ designs (assuming similar values upon extended orbit offset, pp. 8, 9), this gives the following majoring $d\mathcal{C}$ values (which cannot be reached, as in this limit the orbit offset extends over the IR):

		dB/B			
		-0.01	-0.005	+0.005	+0.01
Extreme orbit excursion in arcs	(mm)	+18.3	+9.2	-9.4	-19.3
$\alpha_\mathbf{B} \to -\alpha_\mathbf{p}$	(10^{-3})	-1.80	-1.84	-1.91	-1.95
dR /dB/B	(m)	-1.1	-1.12	-1.17	-1.19
Majoring $d\mathcal{C}$ values	(mm)	+69	+35	-37	-71

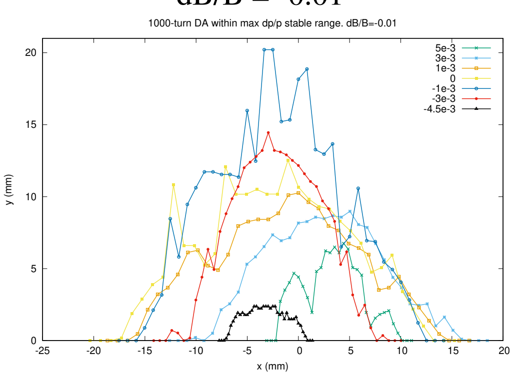
6 Short-Term DA

Short-term DA estimates have been performed using the rings of pp. 8, 9. They may be used for consistency checks against SimTrack.

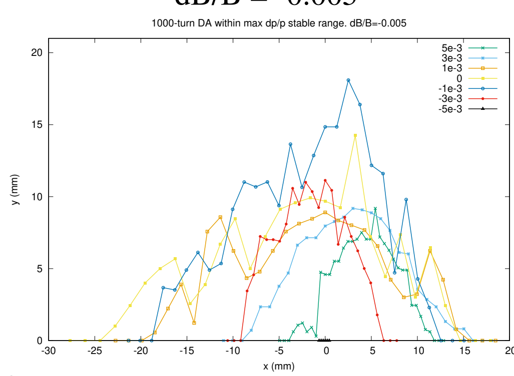
• For the 5 cases of concern here, $dB/B = 0,\ \pm 0.005,\ \pm 0.01$,

- DAs are computed for 1000 turns, for sample dp/p taken over the maximum stable dp/p range (max. $|dp/p|$ values explored in the figures below are at $\lesssim 0.5 \times 10^{-3}$ from stability limit),
- accuracy on x and y boundaries is 0.1 mm.

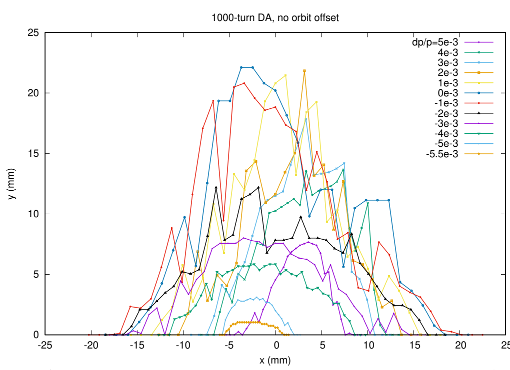
dB/B = -0.01



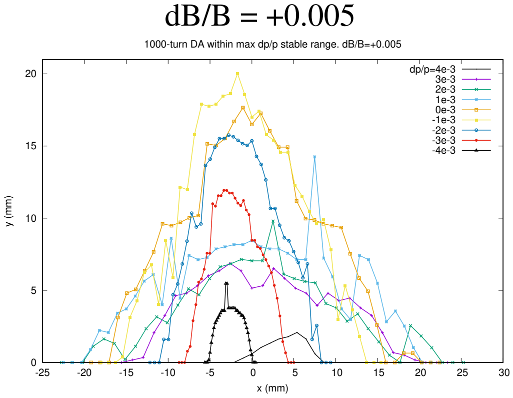
dB/B = -0.005



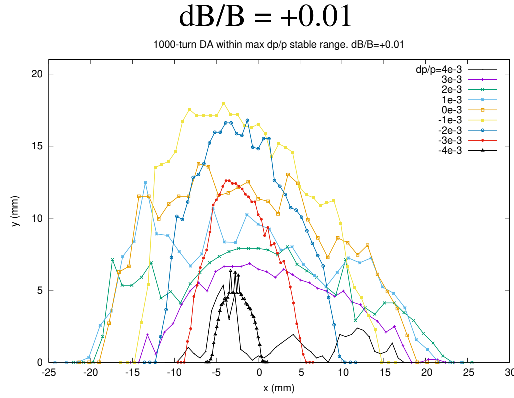
dB/B = 0



dB/B = +0.005



dB/B = +0.01



7 Conclusion

In conclusion, the following table recapitulates the extreme orbit excursion, trajectory lengthening, geometric compaction α_B, and radial shift $dR/dB/B$ values, in the hypothesis of the $\Delta x_{\rm offset} \propto \eta_x$ method described in Sec. 3, and in the case of identical $\Delta B/B$ in all 6 arcs:

		dB/B			
		-0.01	-0.005	+0.005	+0.01
Extreme orbit excursion	(mm)	+18.336	+9.249	-9.404	-19.329
Orbit lengthening	(mm)	+53.17	+27	-27.958	-56.745
$\alpha_{\mathbf{B}} = \dfrac{\mathbf{d}\mathcal{C}/\mathcal{C}}{\mathbf{dB/B}}$	(10^{-3})	-1.40	-1.430	-1.4356	-1.4712
$\dfrac{\mathbf{dR}}{\mathbf{dB/B}}$	(m)	-0.854	-0.872	-0.876	-0.898

The trajectory lengthening so obtained assume an azimuthal extent of the orbit offsets of about 250 m meters, between dedicated corrector pairs. A greater extent, toward the IRs, assuming the same extreme excursions in the arcs, would result in greater trajectory lengthening.

Arbitrary Radial Offset

As pointed out in the foregoing, the radial offset does not have to be applied to all 6 EIC hadron ring arcs, and does not have to be the same in all arcs concerned. This is apparent in the EIC ring input data file given in Appendix. The interest is in the possibility of simulating cases where greater (or smaller) aperture would be available in some arcs, compared to others. This obviously would assume that independent dB/B perturbation in the arcs of the EIC hadron ring is available. To that end, a decision must be made as to the implementation of this perturbation: the ”simplest” solution would be to use a single shunt supply on the main bus line powering all arc dipoles, and control the amplitude of dB/B via the current of that shunt. If the need arises for individual control of each arc of the EIC hadron lattices, then there would need to be one shunt supply per determined value of dB/B for each requested radial offset.

Simulation I/O Files

All the input files used for the simulations discussed here, as well as the output files so produced, can be found at the “EIC Task Force” sharepoint:

https://brookhavenlab.sharepoint.com/sites/EIC-Radial-Shift/Shared%20Documents/Forms/

Appendix

Multiple Different Rigidities

In the input data file below (which performs a “TWISS” computation), ’SCALING’ is a power supply command rack. It sets bending magnets, quadrupoles and sextupoles, and orbit offset steerers to proper field values. The 6 arcs and 6 IRs are in separate files, subject to ’INCLUDE’ (the equivalent of a “call” in MAD, with some additional subtleties).

In this example a case of three different rigidities is shown:

- the reference rigidity is 1 (dB/B=0) in all IRs, resulting in no orbit offset,

- it is perturbed by dB/B=0.005 in arcs 5-4, 3-2, 11-10, 9-8, 7-6, resulting in the orbit offsets displayed in p. 7

- it is perturbed by 0.0025 instead in arc 1-12, and there only, resulting by linearity in about half the orbit offset displayed under “arc 1 - arc 12” in p. 7.

Moving ’SCALING’ around as illustrated here, allows arbitrary change of the reference rigidity in any arc.

- **Complete EIC Hadron Ring:**

```
EIC Hadron Ring With 3 Different Rigities
'OBJET'
1000.000000
5
.001 .001 .001 .001 0. .0001
0. 0. 0. 0. 0. 1.

'SCALING'
1  33
BEND
-1
1.005
1
BEND IR*
-1
1.
1
MULTIPOL
-1
1.
1
MULTIPOL IR*
-1
1.
1
MULTIPOL QF*
-1
0.99700601
1
MULTIPOL QD*
-1
1.0021175
1
MULTIPOL SF*
-1
.9300000E+00
1
MULTIPOL SD*
-1
1.06000000E+00
1
MULTIPOL MULT
-1
-2.00000000E-01
1
MULTIPOL BI5_TH11   ! close orbit  ARC 5-4
-1
177.58894
1
MULTIPOL BI5_TH13   ! close orbit
-1
-1913.8606
1
MULTIPOL BI4_TH13   ! close orbit
-1
-1840.8107
1
MULTIPOL  BI4_TH11   ! close orbit
-1
333.82121
1
MULTIPOL BO3_TH10   ! close orbit  ARC 3-2
-1
307.70386
1
MULTIPOL BO3_TH12   ! close orbit
-1
-1844.2605
1
MULTIPOL BO2_TH12   ! close orbit
-1
-1911.7728
1
MULTIPOL  BO2_TH10   ! close orbit
-1
227.22489
1
MULTIPOL YO1_TH10   ! close orbit  ARC 1-12
-1
116.
1
MULTIPOL  YO1_TH12   ! close orbit
-1
-960.
1
MULTIPOL YO12_TH12   ! close orbit
-1
-923.
1
MULTIPOL  YO12_TH10   ! close orbit
-1
155.
1
MULTIPOL BO11_TH10   ! close orbit  ARC 11-10
-1
301.58831
1
MULTIPOL  BO11_TH12   ! close orbit
-1
-1821.0114
1
MULTIPOL BO10_TH12   ! close orbit
-1
-1932.9582
1
MULTIPOL  BO10_TH10   ! close orbit
-1
224.34123
1
MULTIPOL BI9_TH11   ! close orbit  ARC 9-8
-1
191.30175
1
MULTIPOL  BI9_TH13   ! close orbit
-1
-1918.7588
1
MULTIPOL BI8_TH13   ! close orbit
-1
-1854.5600
1
MULTIPOL  BI8_TH11   ! close orbit
-1
340.20023
1
MULTIPOL YI7_TH11   ! close orbit  ARC 7-6
-1
274.60244
1
MULTIPOL  YI7_TH13   ! close orbit
-1
-1830.7798
1
MULTIPOL YI6_TH13   ! close orbit
-1
-1854.8456
1
MULTIPOL  YI6_TH11   ! close orbit
-1
161.67522
1
'INCLUDE'
1
../arc5-arc4.TWISS.inc[#S_arc5-arc4:#E_arc5-arc4]
'INCLUDE'
1
../madIRs_fromGRD/ir4.inc[#S_ir4:#E_ir4]
'INCLUDE'
1
../arc3-arc2.TWISS.inc[#S_arc3-arc2:#E_arc3-arc2]
'INCLUDE'
1
../madIRs_fromGRD/ir2.inc[#S_ir2:#E_ir2]
'SCALING'
1  33
BEND
-1
1.0025
1
'INCLUDE'
1
../arc1-arc12.TWISS.inc[#S_arc1-arc12:#E_arc1-arc12]
'SCALING'
1  33
BEND
-1
1.0025
1
BEND IR*
-1
1.
1
'INCLUDE'
1
../madIRs_fromGRD/ir12.inc[#S_ir12:#E_ir12]
'INCLUDE'
1
../arc11-arc10.TWISS.inc[#S_arc11-arc10:#E_arc11-arc10]
'INCLUDE'
1
../madIRs_fromGRD/ir10.inc[#S_ir10:#E_ir10]
'INCLUDE'
1
../arc9-arc8.TWISS.inc[#S_arc9-arc8:#E_arc9-arc8]
'INCLUDE'
1
../madIRs_fromGRD/ir8.inc[#S_ir8:#E_ir8]
'INCLUDE'
1
../arc7-arc6.TWISS.inc[#S_arc7-arc6:#E_arc7-arc6]
'INCLUDE'
1
../madIRs_fromGRD/ir6.inc[#S_ir6:#E_ir6]

'FIT'
2
1 30 0 [-1.,1.]
1 31 0 [-10.,10.]
2   1e-8
3.1 1 2 #End 0. 1. 0
3.1 1 3 #End 0. 1. 0
'TWISS'
2 1. 1.
'END'
```