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Spin resonance canceling lattice cell design principles

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Spin Resonance Canceling Lattice Cell Design principles*

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We show how to construct an accelerator lattice cell which minimizes all intrinsic spin resonance. We then apply this approach to various toy electron and proton accelerators, considering, AGS-like and CEPC/FCC-ee and FCC-hh like rings.

INTRODUCTION

The development of accelerators which can maintain high polarization for particle beams remains important for both nuclear and high energy physics experiments. Polarization is critical both for energy calibration and the actual interactions being probed. The use of magnets known as Siberian snakes[1] have opened up the high energy sector to polarized hadron beams, however it has its limitations. Siberian snakes work by rotating the polarization at regular azimuthal locations in the ring in such a way as to cancel the low order spin kicks developed in a given lattice. For protons a helical design using radial fields permit spin rotations which are relatively transparent optically and require field strengths that are relatively independent of energy. However for electrons the radial fields at low energies lead to orbit excursions beyond the beam aperture and at higher energies these same fields can cause excessive synchrotron radiative losses. As such solenoids are usually used for electrons, yet these can have significant impact on the optics and spin diffusion as well they are difficult to ramp to match an accelerating lattice and the peak fields for high energy electrons can become technically very challenging. For hadrons as the energy goes higher the net strength of the perturbing spin kicks grow larger and more snakes are required to counter these effects.

As such the development of lattice design approaches which can minimize the strength of spin resonances is desirable. To date these approaches have been confined to what is known as spin matching[2], where a spin resonance at a particular energy is suppressed by adjusting the strength of the quadrupoles in the lattice to minimize the net spin kick which contribute to the resonance strength. This technique has been used to good effect in several machines, such as the AGS and DESY. However, these approaches usually only work for a particular energy and to address spin resonances at other energies, magnets must be ramped into new relative strength configurations. This process if possible can be clumsy and challenging the magnet slew rates. In this paper we present a general design approach to supress spin resonances for all energies.

SPIN KICKS IN THE ARC CELLS

The transport of spin polarized beam across a standard arc focusing and defocusing lattice (FODO) introduces transverse spin kicks which can accumulate between dipoles. These spin kicks will, for an appropriate spin tune, add up coherently and lead to beam depolarization marked by the presence of an intrinsic spin resonance. However if the quadrupole's location and strength can be organized correctly the transverse spin kicks can cancel or be minimized for all spin tunes. This is somewhat similar to what is known as spin matching at a particular spin tune. However since the cancellation occurs between spin precessing dipoles, this makes the spin matching condition work for all energies and spin tunes. The development of the design for the future Electron-Ion Collider's (EIC's) rapid cycling synchrotron (RCS) required arc connecting regions which wouldn't contribute to the intrinsic spin resonances [3]. This was initially accomplished by ensuring that the betatron phase advance was an integer multiple of 2π, in the straight arc connecting lattice. In this paper we consider the case when it is not possible to achieve a full 2π phase advance in a given straight. In particular an arc cell where the drift distance between dipoles is too short.

SPIN PROPERTIES OF FODO CELL

A standard FODO cell contains two quadrupoles; a focusing and defocusing type with a drift between them. A popular construction of an arc cell has the focusing and defocusing quadrupoles with a dipole placed between them. In this situation there is no possibility of cancellation of the spin kick between dipoles. Another type of FODO cell includes the dipoles on either end of the focusing and defocusing string of quadrupoles. However in the case of only two quadrupoles, cancellation leads to an optically unstable lattice. A stable lattice can be achieved with the addition of a third quadrupole.

We can estimate the contributions to intrinsic spin resonances by considering the terms of the spin resonance

integral from [4].

$$\mathrm{w}_K = \lim_{N_T\to\infty} \frac{-1}{2\pi N_T} \int_0^{LN_T} \Big[(1+G\gamma)(z'' + \frac{iz'}{\rho}) - i(1+G)(\frac{z}{\rho})'\Big] e^{iK\theta(s)} \mathrm{d}s. \quad (1)$$

The dominant one is the first term in Eq. 1,

$$\int z'' e^{iK\theta(s)} ds = \sum_n k_n z_\beta = \sum_n k_n \sqrt{\beta_n}\cos(\mu_n + \phi) e^{iK\theta_n}. \quad (2)$$

Here we have dropped the constants and expanded the integral into a sum of the contributions from each element in the lattice indexed by n. We are using the standard Frenet-Serret coordinate system for z and s (see Fig. 1)and θ is the orbital angle. K is the spin resonance tune, β_n the vertical betatron function at n, μ_n the betatron phase at n and ϕ the initial betatron phase offset. The subscript n denotes values at each n^{th} element and k_n is the quadrupole k_1 value for the n^{th} quadrupole and we factored out the emittance and the common Gγ terms. A sufficient condition for the cancellation of the intrinsic resonance is if,

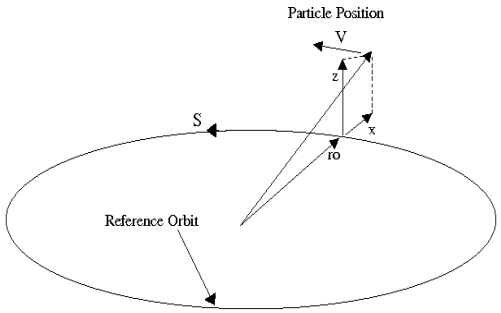

FIG. 1. The curvilinear coordinate system for particle motion in a circular accelerator. The unit vectors $\hat{x}$, $\hat{s}$ and $\hat{z}$ are the transverse radial, longitudinal, and transverse vertical basis vectors; and $\vec{r}_0(s)$ is the reference orbit.

$$0 = \sum_n k_n \sqrt{\beta_n}\cos(\mu_n)$$
$$0 = \sum_n k_n \sqrt{\beta_n}\sin(\mu_n). \quad (3)$$

The spin precessing terms only change in the dipole, so between dipoles they are common and can be factored out. Looking first at the FODO cell example in the thin lens approximation we consider,

$$\mathrm{QF\ \ O\ \ QD\ \ O} \quad (4)$$

Here QF and QD are focusing and defocusing quads respectively and O is a drift. In this case Eq. 2 would become,

$$0 = k_f\sqrt{\beta_f}\cos(\mu_f) + k_d\sqrt{\beta_d}\cos(\mu_d)$$
$$0 = k_f\sqrt{\beta_f}\sin(\mu_f) + k_d\sqrt{\beta_d}\sin(\mu_d) \quad (5)$$

Since we start the lattice with $\mu_f = 0$ it means that the sin equation of Eq. 4 would contain only one term, namely $k_d\sqrt{\beta_d}\sin(\mu_d)$. For this term to be zero either k_d or β_d would have to be zero or $\sin(\mu_d)$ which implies that $\mu_d = \pi$. In other words the phase advance across the whole cell would be 2π. So our cell with either have no defocusing quad, zero beta function at the quad or have an infinitely large beta function in the cell.

Thus using only two quads per cell wouldn't allow us to construct a viable intrinsic spin canceling cell. Introducing a third quadrupole between the dipoles yields,

$$M = \mathrm{QF_1\ \ O\ \ QD\ \ O\ \ QF_2\ \ O} \quad (6)$$

In this case Eq. 3 would become,

$$0 = k_1\sqrt{\beta_1}\cos(\mu_1) + k_2\sqrt{\beta_2}\cos(\mu_2) + k_3\sqrt{\beta_3}\cos(\mu_3)$$
$$0 = k_1\sqrt{\beta_1}\sin(\mu_1) + k_2\sqrt{\beta_2}\sin(\mu_2) + k_3\sqrt{\beta_3}\sin(\mu_3) \quad (7)$$

Since this is challenging to solve algebraically, we instead use MADX [5] to construct such a cell, now including two short dipoles at the end of the cell. In this case we have found solutions using MADX Simplex optimizer to give the following values for $k_2 = -0.397464$,$k_1 = 0.200315$ and $k_3 = 0.209916e$. Here the quad lengths are all 0.6 m and the drift length L= 0.10625 m and the dipole's bending radius of $\rho_0 = 7.343750003$. With this we built a ring with a periodicity of 6 and total circumference of 75.24 m. The results of Spin Resonance strength calculation for this lattice is shown compared to a typical FODO style lattice in Fig. 2.

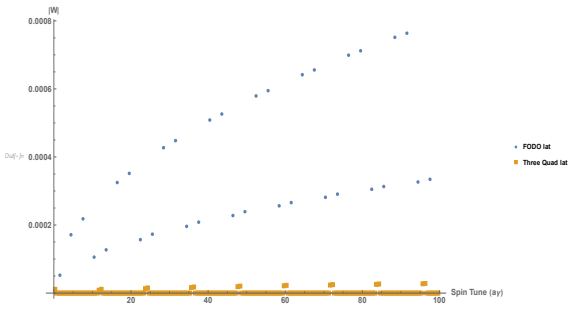

FIG. 2. Depol calculated spin resonances using special intrinsic spin resonance suppressing cell compared to a standard FODO cell with dipoles in-between

Using DEPOL to calculate and suppress the intrinsic yields higher accuracy and a bit larger suppression, yielding $k_2 = -0.401126257377$, $k_1 = 0.209148782538$

and $k_3 = 0.209150949859$ values. In Fig. 3 we can see how optimizing using Eq. 6 compares to direct optimization using the full spin resonance calculation with the DEPOL algorithm. In Tables I we looked at optimized

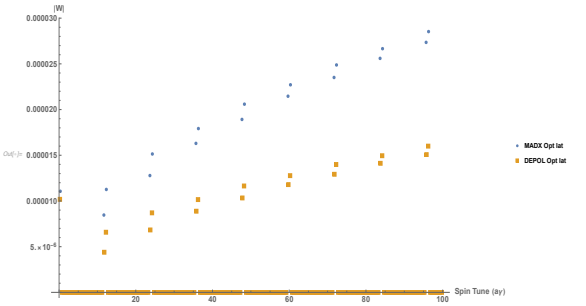

FIG. 3. Depol calculated spin resonances using special intrinsic spin resonance suppressing cell optimized using DEPOL algorithm compared to approximate resonance suppression.

quadrupole strengths for a larger bending radius of 117.5 using drifts from 2.225 to 6 m for various three quad combinations. In all cases we kept the dispersion below 1 m and max betatron function below 40 m.

TOY ACCELERATOR EXAMPLES

We now explore a few types of accelerators and how they might benefit using these design approaches. We first consider a machine similar to the Alternating Gradient Synchrotron (AGS) machine. Next we look at toy lattices on the scale of the proposed Future Circular Collider (FCC) for both electrons and hadrons. This is also very similar in scale to the proposed CEPC accelerator.

AGS sized machine

For the case of an AGS sized polarized electron machine the lower tunes permits the appearance of the $0+\nu$ spin resonance in the energy range for electrons accelerating to 18 GeV. However since the cells are designed to minimize the spin resonance contribution, its effect is negligible under an acceleration rate comparable to the RCS's (100 msecs). The strengths are detailed in Table. II. The radiated energy per turn of 115 MeV makes the RF power requirements challenging for such a machine. However a proton machine or one with a lower energy or higher bending radius appears possible.

For an AGS like proton machine, using a periodicity of 48 or higher could place all the intrinsic spin resonances outside of the standard AGS acceleration energy range. In Fig 4 we show a comparison of DEPOL calculated intrinsic spin resonances for the standard AGS lattice and the new AGS lattice.

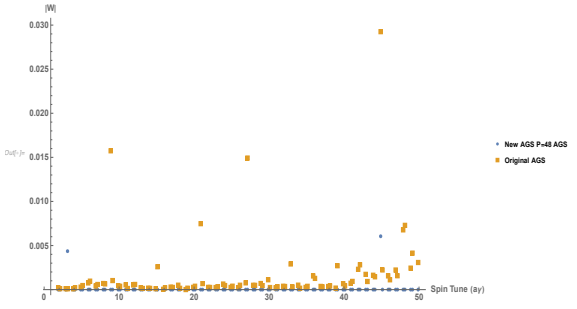

FIG. 4. Depol calculated spin resonances for the standard proton AGS machine and the new AGS like machine with periodicity of 48 and spin cell suppression.

FCC like machines

The proposed FCC-ee and Circular Electron Positron Collider (CEPC) are both positron and electron colliders with a circumference of around 100 km reaching top energies above 100 GeV. Considering the energy for the FCC-ee Booster with injection energy at 20 GeV and top extraction energy of 182.5 GeV in terms of $G\gamma$ this ranges from 45 to 414. Choosing a periodicity of 500 if the arc tunes are kept above 45 units and the construction circular with straight section confined to the 200 m long arc cell then all spin resonances can be avoided. If longer symmetry breaking straight sections are needed then an approach as was using the EIC's RCS to suppress and cancel the resonance contribution from these sections can be used. This would be easier to accomplish if sufficient room in the tunnel is made so that the booster would not need to bend around the experimental halls.

Additionally the FCC-hh is also proposed which will accelerate protons to energies from 3 TeV using the LHC as a booster to 50 TeV. In $G\gamma$ space this is 5732 to 95532. In this case a periodicity high enough to avoid spin resonances is not reasonable. However the arc cells could be construction with enough quadrupoles between the dipoles to achieve a major suppression of the spin resonances. In this case using four quadrupoles one focusing and three defocusing placed in an arc cell of 195.5 m long with a 136 degree phase advance in the horizontal and 21 degree in the vertical plane kept all intrinsics resonances in the energy range less than .80 at 10 mm-mrad normalized emittance. This is low enough that six snakes should be sufficient to control the intrinsic spin resonances.

This compares to an FCC-hh lattice which following a standard FODO lattice design using an arc cell of 195 m, which would give resonance greater than 7 at 10 mm-mrad normalized emittance. In Fig. 5 we can see a comparison between the two FCC-hh lattices.

TABLE I. lattices with spin minimized quad strengths with associated beta MAX and DX MAX values for various drift lengths

Bending Radius [m]	117.5					
Dipole to Dipole length	Drift length	k_1	k_2	k_3	DXMAX	BetaMAX
2.225	0.10625	0.526888771	-0.245684739	-0.261201752	1	30.6
3	0.3	0.432437535	-0.197949689	-0.211033385	0.955	40
4	0.55	0.479251608	-0.209103764	-0.227034009	0.548	40
5.05	0.8125	0.532619143	-0.219942013	-0.242838535	0.347	40
6	1.05	0.586348398	-0.229150176	-0.256696852	0.244	40
2.225	0.10625	0.286135935	-0.531632625	0.268731319	1	31
3	0.3	0.245191755	-0.445297425	0.22911557	0.892	40
4	0.55	0.297057375	-0.506501495	0.270952002	0.466	40
5.05	0.8125	0.379216596	-0.590811295	0.336069053	0.2522	40
6	1.05	0.524105113	-0.714907587	0.446500235	0.1356	40

TABLE II. Electron, AGS sized lattice with arc cells spin optimized

Bending Radius [m]	70.37
Circumference [m]	810.1955287
Q_x	11.85
Q_y	4.2
Dipole to Dipole length [m]	3.0
Drift length [m]	.3
KF	0.547
KD	0.245
KF1	0.265
BetaMAX [m]	32
DxMAX [m]	1
U_0 [MeV]	132
No. of Cells	115

TABLE III. Proton AGS sized lattice with arc cells spin optimized

Bending Radius [m]	29.37
Circumference [m]	808.56
Q_x	17.039
Q_y	3.12
Dipole to Dipole length [m]	12.8
Drift length [m]	2.08
KF	0.511
KD	0.181
KD1	0.0062
KD2	0.201
BetaXMAX [m]	31
BetaYMAX [m]	48.44
DxMAX [m]	0.46
No. of Cells	48

CONCLUSION AND FURTHER STUDY

Using intrinsic resonance canceling arc cells one can build up a whole ring with all sorts of broken symmetry and still avoid or greatly reduce the strong intrinsic depolarization. However one of the challenges is to build these cells in such a way that the beta functions and dispersion are controlled. Additionally their natural dynamic aperture and chromatic features should be studied to better understand the optimal configuration. In addition to building up the straight sections between the dipoles in IP regions of the planned RCS, these sort of cells might be worth considering for the arc cells as well especially for those lattices which do not lend themselves to using high periodicity.

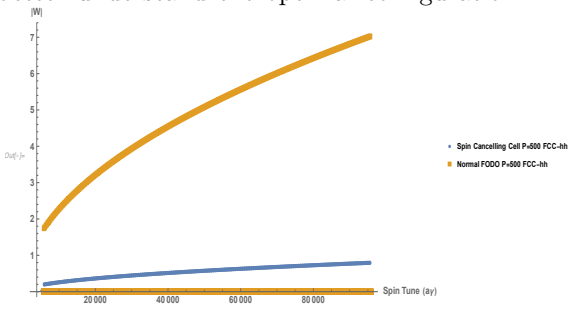

FIG. 5. Depol calculated spin resonances for a FCC-hh toy lattice using standard FODO cell construction versus an FCC-hh like lattice with arc cells which suppress the spin resonances.

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[1] Y. S. Derbenev and A. M. Kondratenko, Sov. Phys. Doklady **20** (1976).
[2] K. Steffen, (1982).
[3] V. H. Ranjbar, M. Blaskiewicz, F. Méot, C. Montag, S. Tepikian, S. Brooks, H. Witte, I. Marneris, V. Ptitsyn, and F. J. Willeke, Phys. Rev. Accel. Beams **21**, 111003 (2018).
[4] E. D. Courant and R. D. Ruth, *The Acceleration of Polarized Protons in Circular Accelerators*, Tech. Rep. BNL-51270 (BNL, 1980).
[5] CERN - BE/ABP Accelerator Beam Physics Group, "Methodical accelerator design: Mad-x v5.07,".